Orthogonalization to Reduce Overhead in MIMO Interference Channels

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Abstract—Interference channels are useful analytical models for distributed wireless communication networks with multiple simultaneously transmitting users. The degrees-of-freedom optimal transmit strategy for interference channels is interference alignment, which requires substantial channel state knowledge throughout the network. As the network grows, the sum capacity, theoretically, increases linearly. This result, however, neglects overhead from training and feedback. This paper accounts for overhead in the MIMO constant-coefficient interference channel with linear precoding and proposes an orthogonalization approach to maximizing sum throughput when overhead is considered. The optimization’s solution, assuming each group uses interference alignment, is found to require full channel state information and a brute-force search, so a greedy partitioning method with reduced CSI requirements is proposed.

I. INTRODUCTION

Interference channels are useful models for networks where non-causal sharing of data across multiple transmitters, such as for base station coordination, is infeasible. Such cases include ad hoc networks and cellular networks with low-bandwidth backhauls between base stations. Interference channels model the case of simultaneous point-to-point transmission by two or more transmitters such that the respective receivers observe the superposition of all transmissions in the network. The transmissions observed from transmitters not intentionally communicating with a given receiver are termed interference.

Recent work on interference channels has shown that, theoretically, the capacity of such networks increases linearly with the number of transmit/receive pairs in the network [1], [2]. In particular, by intelligently precoding the transmitted symbols, all the interference can be forced into a subspace of the received space at all receivers simultaneously. This precoding operation is termed interference alignment (IA). With two users, previous work has shown a loss in degrees of freedom when channel coefficients are not known at the transmitters [3], [4]. There is no prior work analyzing the interference channel without training for channel estimation at the receivers. All current methods for maximizing degrees of freedom for the interference channel require channel training and estimation at some node even if no feedback mechanism is employed. The number of total links grows with the square of the number of users in the network, meaning the overhead associated with training these links will outpace the capacity growth with many users. Similarly, the requirement of CSI, even if only at the receivers, is known to effectively reduce the degrees of freedom of a point-to-point block fading link [5]. Extending this model to the interference channel, overhead associated with training is expected to dominate an interference channel with many users, diminishing the promised capacity increase.

Prior work has considered the impact of imperfect CSI on the achievable sum rate of interference alignment [6], and the number of bits of limited feedback desired for single-antenna interference alignment [7]. Overhead due to training was neglected in both cases. Others have considered clustering a cellular network based on spatial proximity [8], but this clustering is done a priori and does not explicitly consider overhead. To our knowledge there is no prior work explicitly considering training overhead in MIMO interference channels.

This paper presents a model for analyzing overhead in MIMO interference channels and finds that the achieved sum rate with overhead of interference alignment will go to zero with a large number of users. We consider a fully connected interference channel, where spatial clustering is ineffective because of the proximity of all users. Thus, we propose to partition the users into orthogonally transmitting groups. The groups take turns transmitting, with interference alignment used as for transmission inside each group. Although such partitioning still results in an asymptotically zero sum rate, we show that for moderate number of users, partitioning can result in multiplicative gains in sum rate over applying IA to the entire network.

For the model outlined in this paper, the sum-rate-optimal partitioning is shown to be a highly complex optimization that requires global CSI and an exhaustive search over all possible partitioning combinations, calculating the IA precoders for each combination. We therefore propose a greedy algorithm that requires only channel quality information (CQI) on the link for each transmit/receive pair. Based on an approximation to the achievable sum rate for interference alignment using linear precoding, the proposed algorithm is shown to efficiently partition the network into IA groups.

The log refers to \( \log_2 \). Bold uppercase letters, such as \( \mathbf{A} \), denote matrices, bold lowercase letters, such as \( \mathbf{a} \), denote column vectors, and normal letters \( a \) denote scalars. The letter \( \mathbb{E} \) denotes expectation. \( \mathbb{C} \) is the complex field, \( \max\{a, b\} \) denotes the maximum of \( a \) and \( b \), \( \|\mathbf{A}\|_F \) is the Frobenius
norm of matrix $A$, and $|A|$ is the determinant of square matrix $A$. The identity matrix of appropriate dimension is $I$ and $|a|^{+} = \max\{a, 0\}.

II. SYSTEM MODEL

We consider a distributed synchronized network with $2K$ nodes, each with $M$ antennas. $K$ of the nodes have data to transmit to the other $K$ nodes, with no multiuser or cooperative transmission. In particular, transmitter $k \in \{1, \ldots, K\}$ has data destined only for receiver $k$. We assume a narrowband block fading model where the channel $H_{k,\ell}$ between transmitter $\ell \in \{1, \ldots, K\}$ and receiver $k$ is independently generated every $T$ transmission periods $\forall k, \ell$. We assume transmissions are frame and frequency synchronous. Thus, at any fixed moment in time, we have a $K$-user MIMO interference channel with $M$ antennas at each node, as illustrated in Figure 1. The assumption that all nodes have identical coherence times models the case where the nodes are fixed in relation to each other and a moving environment is causing time selectivity, for example, with fixed infrastructure near highways. Analysis for different coherence times for each link is left for future work.

Communication is divided into frames of period $T$ symbols. At the beginning of each frame, the transmitters send mutually orthogonal training sequences to allow the receivers to estimate the channels. This training is necessary not only for coherent detection but also for CSI feedback required to exploit the full degrees of freedom in the network [3], [4]. Training plus feedback time is $\mathcal{L}(K, M) < T$ symbol periods such that in general we do not make assumptions about how many links must be estimated or how many symbols are required for estimation.

The data transmission portion of the frame begins after the first $\mathcal{L}(K, M)$ symbols and ends when the channel changes. Information theoretic results, which neglect overhead, suggest that all transmitters should send simultaneously to achieve the maximum degrees of freedom in the channel and thus approach its sum capacity with high transmit power [1], [2]. In particular, transmitter $\ell$ sends $S_\ell$ spatial streams to receiver $k$. At symbol period $n$, the signal observed by receiver $k \in \{1, \ldots, K\}$ is

$$y_k[n] = H_{k,\ell}F_\ell s_\ell[n] + \sum_{\ell \neq k} H_{k,\ell}F_\ell s_\ell[n] + v_k[n],$$

where $F_\ell$ is the $M \times S_\ell$ linear precoder used at transmitter $\ell$, $s_\ell$ is the $S_\ell \times 1$ vector of symbols sent by transmitter $\ell$, and $v_k$ is zero-mean white circularly symmetric zero-mean complex Gaussian noise with covariance matrix $\mathbb{E}v_kv_k^* = R_k$. The sum rate of the network in bits per transmission for a frame is then

$$R_{\text{sum}} = \left[\frac{T - \mathcal{L}(K, M)}{T}\right] + \sum_{k=1}^{K} \log |R_k + \sum_{\ell \neq k} H_{k,\ell}F_\ell F_\ell^* H_{k,\ell}^{-1}|.$$  

From (2) we observe that the overhead term $\mathcal{L}(K, M)$ effectively decreases the degrees of freedom in this network. Previous work has shown that at least $M$ symbols are required for estimation of an $M \times M$ MIMO channel [5]. Although there are $K^2$ MIMO links, the $K$ receivers can use the training from a given transmitter without any extra use of resources. Therefore, $\mathcal{L}(K, M) \geq KM$. Even assuming feedback requires no overhead, $R_{\text{sum}} = 0$ for $K \geq T/M$. In short, simultaneous transmissions requiring coherent CSIR, such as interference alignment, break down with large $K$.

To regain degrees of freedom for a given number of users, we propose to partition the users into groups that share the frame orthogonally in time or frequency. This concept is illustrated in Figure 2. Note that since the original $K$ users were modeled as a connected interference channel, where all receivers observe a signal from all transmitters above the noise floor, any subset of transmit/receive pairs, in isolation, may also be modeled as a connected interference channel. If the users are partitioned into $P$ index sets $\{K_p\}$, with $|K_p| = K_p$ users in the $p$th group, then the sum rate of the network

![Fig. 1. The MIMO interference channel. Each transmitter is paired with a single receiver. In the model considered in this paper, the channels $H_{k,\ell}$ are block fading with coherence time $T_{k,\ell}$.](image1)

![Fig. 2. Illustration of a partition of the $K$-user interference channel into two $K/2$-user interference channels transmitting orthogonally to each other.](image2)
becomes
\[
\hat{R}_{\text{sum}} = \sum_{p=1}^{P} \sum_{k \in \mathcal{K}_p} \frac{T/P - \mathcal{L}(K_p,M)}{T/P} \log \left| \mathbf{I} + \left( \mathbf{R}_k + \sum_{\ell \in \mathcal{K}_p \setminus k} \mathbf{H}_{k,\ell} \mathbf{F}_\ell \mathbf{F}_\ell^* \mathbf{H}_{k,\ell}^* \right)^{-1} \mathbf{H}_{k,k} \mathbf{F}_k \mathbf{F}_k^* \mathbf{H}_{k,k}^* \right|
\]
\quad (3)

We then aim to solve the following optimization:
\[
\begin{align*}
\text{maximize} & \quad \hat{R}_{\text{sum}} \\
\text{with respect to} & \quad P \in \mathbb{N}_1, K_p \in \mathbb{N}_1, \mathbf{F}_\ell \in \mathbb{C}^{M \times S_\ell}, \forall \ell \in \mathcal{P}
\text{subject to} & \quad \sum_{p=1}^{P} K_p = K, \\
& \quad \|\mathbf{F}_\ell\| \leq 1.
\end{align*}
\quad (4)

The solution to this optimization is computationally complex and involves not only a brute force search over every possible grouping, but also the calculation of the desired precoders for each grouping. Further, such an optimization requires full CSI at a central controller. In the next section we present a suboptimal greedy method for performing this grouping with only channel quality information (CQI).

III. GREEDY PARTITIONING

To develop a greedy algorithm for partitioning the network, we must first define a selection function that assigns a value of placing a user in a group. This function would ideally be the sum rate increase of placing a user in a group. This is difficult in multiuser networks since the actual sum rate increase will depend on which future users are assigned to the group—knowledge that is unavailable in a greedy algorithm. Instead we resort to an approximation of this sum rate increase.

After partitioning the $K$-user interference channel into $P$ orthogonal groups, group $p$ will be a $K_p$-user interference channel that is restricted to utilizing only $1/P$ of the spectrum or coherence interval. Thus, interference alignment is a reasonable choice for precoder design in each group. Although interference alignment requires extensive CSI and calculation of precoders to find the exact sum rate, we note that the precoder solutions are independent from the direct links $\{\mathbf{H}_{k,k}\}, \forall k$. Thus, with interference alignment, the expected throughput will be approximately the rate obtained from randomly generating orthogonal precoders $\mathbf{Q}$ and combiners $\mathbf{\Phi}$ of correct rank drawn uniformly from the Grassmann manifold in the absence of interferers because of the lack of bias in direction through the channel realization in the algorithm. We then approximate the expected rate for user $k$ in group $p$ to be
\[
\bar{R}_{k,p} \approx \mathbb{E}_{\mathbf{\Phi},\mathbf{Q}} \log \left| \mathbf{I} + \mathbf{\Phi}^* \mathbf{H}_{k,k} \mathbf{Q} \mathbf{Q}^* \mathbf{H}_{k,k}^* \mathbf{\Phi} \right|
\approx \frac{\hat{d}(K_p,M)}{K_p} \log \left( 1 + \frac{\|\mathbf{H}_{k,k}\|_F^2}{MT} \right).
\quad (5)
\]
This approximation is justified via the plot in Figure 3. The difficulty with (6) is that the group size $K_p$ is, in general, unknown at the time user $k$ is being assigned. To remedy this, we define $K_O$, where
\[
\begin{align*}
od(K_O,M,T) & > d(K_O - 1, M,T) \\
& > d(K_O + 1, M,T).
\end{align*}
\quad (7)
\quad (8)
\]
Here, $d(K,M,T)$ is the degrees of freedom with overhead and is defined as
\[
\hat{d}(K,M,T) = \frac{T - \mathcal{L}(K,M)}{T} d(K,M).
\quad (9)
\]
Then we set $P = \text{round}(\frac{K}{K_O})$. Once $P$ is found, we can assign users to each group by their approximate rate function $\bar{R}_{k,p}$. The algorithm is summarized in Table I. Note that, this algorithm is based on a model with linear precoding, which does not likely result in a linear relationship between $K$ and $d(K,M)$ [11]. This algorithm can work for non-linear precoding [2], which may increase the degrees of freedom in a constant-coefficient interference channel, with an appropriate approximation of $\bar{R}_{k,p}$. This problem is beyond the scope of this paper.

IV. SIMULATIONS

This section presents numerical results comparing the greedy partitioning method of Section III. The simulations are done using iterative interference alignment with linear precoding [9], [10] with 100 iterations, although the IA
The coherence interval is $T = 30$ and $M = 2$ antennas per node.

precoders can be found with any IA solution. The degrees of freedom using this method has been conjectured to be $d(K, M) = 2MK/(K + 1)$ [11].

The first simulation gives the sum rate versus the number of users $K$ for greedy partitioning and IA applied to the entire network with $L(K, M) = KM$, which is the minimum amount of overhead required for training [5]. The coherence interval $T = 30$, and each node is equipped with $M = 2$ antennas. The plot is shown in Figure 4. The greedy partitioning sum rate does not have a monotonic relationship with $K$ since each group cannot have exactly $K_0$ users unless $K/K_0$ is an integer. Nevertheless, the suboptimal partitioning’s sum rate drastically outperforms applying IA to the entire network.

The second simulation, whose plot is illustrated in Figure 5, shows the sum rate performance of the greedy partitioning method and the exhaustive partitioning method for $K = 3$ users for various $T$. As with the previous simulation, $M = 2$ antennas are at each node. With a small coherence interval, the two perform very similarly as the greedy method partitions the network into 3 groups with one user that transmits interference-free. With larger $T$, the exhaustive search outperforms the greedy method because it still partitions the network into three interference-free groups. Partitioning into one group would result in non-zero interference with only 100 iterations of the iterative IA design, reducing the achievable sum rate. With perfect IA precoders and large $T$, the sum rate of the partitioning methods approaches the sum rate of IA without overhead.

V. CONCLUSIONS

We have demonstrated the importance of considering overhead associated with training and feedback in practical design for the interference channel. In particular, as the network grows, the sum rate with overhead of IA goes to zero. To increase sum rate with a finite number of users, we propose partitioning the network into orthogonally transmitting groups.

Although the optimum partition requires a complex brute-force search with global CSI, we propose a greedy algorithm that requires only direct-link CQI, and much of the gains of an exhaustive search can be made.

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REFERENCES