Design, Construction and Resonance Tracking of a Laboratory-Scale, Loudspeaker-Driven Thermoacoustic Cooler

Autumn Term 2013

DOI: 10.3929/ethz-a-010075245
Contents

Abstract v

List of Figures vii

1 Introduction 1
  1.1 Motivation ................................................. 1
  1.2 About This Thesis ........................................ 2

2 Fundamentals of Thermoacoustic Refrigeration 3
  2.1 Basic Principles of a Standing-Wave TAR .................. 3
    2.1.1 Standing Acoustic Waves in an Open/Closed Tube ... 3
    2.1.2 The Brayton Cycle and the Thermoacoustic Stack .... 4
  2.2 Main Components of a Standing-Wave TAR .................. 5

3 TAR Design, Manufacturing and Instrumentation 7
  3.1 General Requirements .................................. 7
  3.2 TAR Design using Linear Thermoacoustic Theory .......... 8
    3.2.1 Simplifying Assumptions ............................. 8
    3.2.2 Working Fluid and Operating Pressures .............. 12
    3.2.3 The Stack ........................................... 13
    3.2.4 Further Choices ..................................... 15
  3.3 Numerical TAR Optimization ................................ 17
    3.3.1 Acoustic Drivers .................................... 18
    3.3.2 Choosing a Loudspeaker and Duct Length Optimization . 18
  3.4 Mechanical Design and Manufacturing ..................... 20
    3.4.1 The Stack .......................................... 21
    3.4.2 The Heat Exchangers ................................ 21
    3.4.3 Resonator Aluminium Parts .......................... 22
    3.4.4 Resonator PVC Parts ................................ 23
  3.5 The TAR Test-Bed in a 19-inch Rack ..................... 25
    3.5.1 Sound-Proof Rack ................................... 25
    3.5.2 Waste Heat Management .............................. 25
  3.6 Measurement- and Control Instrumentation .............. 25
B Code Fragments 63
B.1 DeltaEC Code ............................. 63

C Engineering Drawings 69
Assembly Drawing in Isometric Perspective ................ 70
Assembly Drawing in Section Perspective .................... 71
Housing Cover .................................. 72
Loudspeaker Housing ................................ 73
Adapter Plate .................................... 74
Warm Duct ..................................... 75
Warm Duct View 2 ................................ 76
Stack Holder .................................... 77
Reducer Cone .................................... 78
Cold Duct Elements ................................ 79
Diverging Cone Part 1 ............................. 80
Diverging Cone Part 2 ............................. 81
Compliance Front End ................................ 82
PVC Screw for Sealing the Resonator ......................... 83
TAR Mounting Rail ................................ 84

D Experiment Settings 84
D.1 For Section 4.1: ............................. 84
D.2 For Section 4.2: ............................. 84
D.3 For Section 4.3: ............................. 85
D.4 For Section 5.2.1: ............................ 85

Bibliography 87
Abstract

This thesis describes the design, construction and control of a quarter-wavelength, loudspeaker-driven thermoacoustic refrigerator. The stack and resonator design is based on linear thermoacoustic theory combined with numerical simulations in the thermoacoustic design software DeltaEC. The mechanical layout focuses on experimental flexibility and ease of maintenance, which leads to a modular system design. Despite using air at ambient pressure as working fluid, the presented thermoacoustic cooler reaches more than 20 degrees Celsius temperature difference across the stack. Based on an experimental system characterization, two different performance criteria and control goals are identified. For tracking the system’s acoustic resonance frequency, a novel, harmonics-based control strategy is proposed, which does not require measurement nor estimation of the speaker state.
Zusammenfassung

# List of Figures

2.1 Standing acoustic waves in an open tube .......................... 4  
2.2 Brayton Cycle schematics for a gas parcel between two stack plates 4  
2.3 Hofler’s thermoacoustic refrigerator .............................. 6  
3.1 Schematics of a modified Hofler-style thermoacoustic cooler ..... 8  
3.2 TAR design process flowchart ..................................... 11  
3.3 Coefficient of Performance surface plot .......................... 15  
3.4 COP-maximizing stack center position for different stack lengths 16  
3.5 Expected stack performance diagram ............................. 16  
3.6 Comparison of different HiFi-speakers ............................ 19  
3.7 Exploded assembly drawing of the TAR’s main components ....... 20  
3.8 CAD drawing of a stack segment .................................. 21  
3.9 FEM analysis of a stack plate ..................................... 22  
3.10 The TAR - side view .............................................. 24  
3.11 The TAR - top view .............................................. 24  
3.12 Measurement board schematics ................................. 28  
3.13 The TAR test-bed .................................................. 29  
4.1 Pressure frequency response ........................................ 32  
4.2 Overall electrical impedance and conversion efficiency .......... 34  
4.3 Conversion performance for constant mechanical power 0.5 W. 35  
4.4 Temperature differences for a range of frequencies ............... 36  
4.5 Temperatures and relative COP for $f = 140$ Hz .................. 37  
4.6 TAR cool-down plot ................................................ 38  
4.7 Acoustic power in the second harmonic ........................... 39  
5.1 Phase-Locked-Loop flowchart ..................................... 44  
5.2 The accelerometer right before breaking off the speaker cone ... 44  
5.3 Corrupted acceleration raw measurements ........................ 44  
5.4 A closed tube with pressure amplitudes ........................... 45  
5.5 Frequency response of the second harmonics ..................... 47  
5.6 Flowchart of the harmonics-based frequency control principle... 48  
5.7 Convergence of the harmonics-based resonance control strategy 51  
5.8 Harmonics-based frequency tracking with different heat loads... 52
Chapter 1

Introduction

1.1 Motivation

It is a fact of thermodynamics that the states density, temperature and pressure are coupled in a gas: qualitatively, it can be said that pressure oscillations go along with temperature oscillations (if a constant system volume is assumed). In thermoacoustics, the interaction of pressure waves (sound) and heat in a working fluid can be used for energy conversion. A thermoacoustic refrigerator (TAR) uses mechanical power in the form of sound waves to pump heat across a specially designed thermoacoustic stack. A thermoacoustic engine (TAE) works the other way round: a temperature gradient across a stack produces sound-waves and thus transforms heat into acoustic power.

Although basic thermoacoustic effects have been known for a long time (first phenomenological descriptions in literature date back to the early 19th century) [1], a rigorous and complete mathematical description had not been presented until N. Rott published his linear thermoacoustic theory between 1969 and 1980 [2]. In the following years, increasing efforts were made to explore the theory, designs and applications of thermoacoustic machines [3–5]. It had been realized that – particularly for refrigeration – this unconventional technology features multiple advantages in comparison to standard vapor-compression refrigerators: no dynamic sealing and lubrication is required and there is no need for (usually harmful) refrigerants in the thermodynamic cycle [6], which makes thermoacoustic devices environmentally friendly and attractive for high-reliability applications. Furthermore, it is possible to steer and hold the temperature steadily at a desired setpoint using basic feedback control techniques, while conventional cooling technologies often rely on binary control schemes and oscillate about the setpoint [5].

These advantages make thermoacoustic cooling interesting for high-accuracy applications in cryogenics, for example. In comparison to conventional cryo-cooling technology, thermoacoustic refrigerators are relatively cheap and require
only little maintenance [5]. However, where every-day applications are concerned, thermoacoustic technology is currently not capable of competing with well-engineered vapor compression refrigerators. Considerable drawbacks of state of the art TARs are their low power density (conventional refrigerators reach high power densities by exploiting the enthalpy of the refrigerant’s phase change) and their comparably low efficiency.

Nevertheless, it is not unlikely that these drawbacks can be resolved through specific research efforts and in fact, there is a growing interest in the research community to make these devices more competitive [7].

Many works focus on the optimization of the acoustic driver, the stack geometry and the resonator shape. However, to the best of our knowledge, thermoacoustic energy conversion has not received much attention in feedback control research in the past decades. Phase-Locked-Loop controllers are often used to track the acoustic resonance frequency of TARs, e.g. in [8]. Relatively recent innovations are the sensor-less control of a TAR using open-loop model-based estimation [9], the adaptive controller presented in [10], or the extremum-seeking controller as presented in [11].

1.2 About This Thesis

Together with a previous project [12], this Master’s Thesis is supposed to initiate automatic control research in thermoacoustic refrigeration at ETH’s Automatic Control Laboratory. The work is structured as follows. Chapter 2 explains basic principles of standing-wave thermoacoustic refrigerators and introduces the main components of a TAR. The scope of Chapter 3 is the design and construction of a laboratory-scale thermoacoustic refrigerator, a suitable test-bed and the inclusion of feedback capabilities. A basic experimental characterization of the real system is given in Chapter 4. In Chapter 5, a brief summary of the current state of the art of automatic control in thermoacoustic refrigeration is provided. Then, a novel method for controlling the system to acoustic resonance, using pressure sensors exclusively, is described and discussed. Chapter 6 concludes this thesis with a summary and an outlook on future works. Engineering drawings and code fragments are provided in the appendices.
Chapter 2

Fundamentals of Thermoacoustic Refrigeration

2.1 Basic Principles of a Standing-Wave TAR

Thermoacoustic refrigerators can be subdivided into travelling-wave systems and standing-wave systems. In this thesis, only the latter are considered. Furthermore, considerations are restricted to devices which are powered by an electrodynamic driver, for example a common high-fidelity loudspeaker. For understanding such a TAR, two mechanisms are relevant: standing acoustic waves in a tube and the Brayton Cycle. In the following, qualitative explanations of both are given. Using Hofler’s 1986 refrigerator as example, the main components of a standing-wave thermoacoustic cooler are listed.

2.1.1 Standing Acoustic Waves in an Open/Closed Tube

Consider a simple, straight tube of length $L$ with an open end and a loudspeaker being attached to the other end (closed end). Assume that the driver generates a sinusoidal pressure wave of frequency $f$ as input signal. Standing acoustic waves of different orders will be observed as a function of $f$; they are due to the interference of the incident and the reflective wave. In a perfect, inviscid standing-wave, pressure and the gas parcel’s velocity are 90 degrees out of phase. Note that, as soon as energy is dissipated in the resonator, for example by frictional losses or a thermoacoustic process, pressure and flow will be in phase at resonance, c.f. Section 5.1.1.

Figure 2.1 qualitatively shows gas pressure and flow amplitudes for the first three resonance modes. The boundary conditions are the following: the open end is a pressure node and a velocity anti-node. The closed end is a pressure anti-node and a velocity node. Thus, for an open-closed configuration, the higher-order resonance frequencies are uneven integer multiples of the fundamental resonance frequency.
Chapter 2. Fundamentals of Thermoacoustic Refrigeration

Figure 2.1: Standing acoustic waves in tubes with pressure amplitudes indicated on the left and the corresponding gas velocities shown on the right. Open ends are pressure nodes, closed end are gas velocity nodes.

\begin{align*}
\lambda_1 &= 4L \\
\frac{f_1}{4L} &= \frac{a}{4L} \\

\lambda_3 &= \frac{4}{3}L \\
\frac{f_3}{a} &= \frac{3a}{4L} = 3f_1 \\

\lambda_5 &= \frac{4}{5}L \\
\frac{f_5}{a} &= \frac{5a}{4L} = 5f_1
\end{align*}

Figure 2.2: Illustration of the Brayton Cycle for a gas parcel between two solid plates.

2.1.2 The Brayton Cycle and the Thermoacoustic Stack

Consider a gas parcel oscillating back and forth in a thin pore of a solid, subject to the dynamics of the fundamental standing wave from Figure 2.1. Assuming that the gas is in good thermal contact with the solid, the interaction of the standing wave, the gas in the pore and the solid can be described as an idealized thermodynamic cycle process: the counter-clockwise Brayton Cycle. Its four steps are sketched in Figure 2.2 and described qualitatively in the following.

Assume that the gas parcel is displaced from its original location on the speaker-side of the pore to the right. The gas pressure decreases, which may be idealized as adiabatic expansion. Consequently, the gas parcel cools down. At the end of step one, it is assumed to be colder than the surrounding solid.
Isobaric heat transfer: the temperature difference between gas and solid causes heat transfer from the solid to the gas parcel.

The acoustic wave draws the gas parcel back to its original location on the left. Adiabatic compression takes place, the gas parcel is now warmer than the surrounding solid.

Isobaric heat transfer: in the last step, heat is transferred from the gas to the solid. The process restarts at step 1.

In this way, mechanical work in the form of acoustic waves produces a heat flow: heat is pumped against a temperature gradient. If the solid’s thermal conductivity is low enough to prevent rapid equalization of this thermal imbalance through heat conduction, a steady temperature gradient can be established.

This is the (very simplified) working principle of the thermoacoustic stack, and at the same time, it explains its name. By ‘stacking’ many pores in parallel, large heat flows can be produced. Note that even for a strong standing wave in air at ambient pressure, the gas displacement will typically be of a magnitude of a few millimeters only. By using a sufficiently long stack, large temperature differences can be reached (bucket-brigade principle). Since the stack can be considered as temporary storage for ‘packages’ of heat which are transported against the temperature gradient, two stack material requirements are obvious: first, a sufficiently large heat capacity is desirable. Second, the stack material should have a lower thermal conductivity than the working gas. However, it is important to mention that a too low thermal conductivity would be harmful for the process because it also affects the heat transfer between solid and gas. A trade-off between low heat conduction in the stack and a good heat transfer coefficient between solid and working fluid is required. Detailed considerations about the stack can be found in Section 3.2.3 of this thesis and in [5, 13].

### 2.2 Main Components of a Standing-Wave TAR

One of the most discussed loudspeaker-driven TARs in literature is Hofler’s 1986 refrigerator [14], which is sketched in Figure 2.3. In the following, it is used as an example to outline the main components of a thermoacoustic cooler.

The acoustic driver is used to start up and sustain a standing wave in the resonator. Since Hofler operated his device at high mean pressures, the design features a strongly modified loudspeaker. In a certain distance from the driver surface, the thermoacoustic stack is located. According to the cycle process which was given in Section 2.1.2, the end which faces the speaker becomes warm while the other end gets cold – with the stack being located as shown, heat is pumped towards the top end of the stack at the fundamental resonance frequency. The stack is accompanied by two heat exchangers (HX), which exchange heat with the
working gas. They are necessary to generate a practically measurable or usable heat flow. The warm HX and the driver housing are actively chilled by cooling water tubes. Thus, the warm end is set close to a reference temperature (for example ambient temperature).

In steady-state, a cooling power \( \dot{Q}_c \) of temperature level \( T_{\text{cold}} \) results at the cold heat exchanger. For research purposes, the cooling load is usually provided by an adjustable miniature heat source, for example a power resistor, which is directly coupled to the cold HX.

According to [14], the overall frictional losses on the resonator surface can be minimized by reducing the tube’s cross-section right after the cold HX (labeled ‘small-diameter section’ in Figure 2.3). Additionally, varying cross-sections are supposed to damp higher-order harmonics [5].

The design is also famous for the large ‘compliance volume’ at the end of the small-diameter tube. An open quarter-wavelength resonator has the disadvantage of noise emission at the non-sealed end. However, it allows for a shorter overall device length than a closed/closed half-wavelength resonator. The compliance is an approach to overcome this problem. Although the resonator is closed, the compliance volume ‘simulates’ an open end, thus both noise emission and overall length are reduced. In this design, the pressure node coincides approximately with the transition from the small-diameter section to the resonator sphere. Typically, the resulting device length will therefore be larger than \( \lambda/4 \), but still shorter than \( \lambda/2 \).

Figure 2.3: Hofler’s thermoacoustic refrigerator. Source: [14]
Chapter 3

Design, Manufacturing and Instrumentation of the Thermoacoustic Refrigerator

3.1 General Requirements

Technically mature thermoacoustic refrigeration systems which are able to provide large cooling powers at high temperature differences have been presented in [5]. However, these were mostly developed with a special focus on fundamental research in thermoacoustic theory. Several low-cost systems for thermoacoustic demonstrations have been described, an example can be found in [15]. The TAR in [7] served for the study of feedback control to find an optimal frequency operating point for the system. However, this system was built from simple off-the-shelf plastic parts and did not converge to a steady setpoint with $T_{\text{cold}} < T_{\text{room}}$ during operation. Since no published TAR design which matched the special requirements and needs for the future investigation of advanced feedback-control techniques was known to us, a new device had to be developed.

Due to its compactness and success, our TAR’s shape is roughly based on Hofler’s 1986 design. Additional requirements were simple and unambiguous handling, easy start-up within a few minutes and low maintenance efforts. Consequently, air at ambient pressure was the only reasonable choice for the working gas. Therefore, unlike in other designs, no pressure vessel, compressor, gas tank, high pressure sealing rings, etc. were required. Furthermore, temperatures below ambient temperature should be steadily maintainable. Thus, a good thermal management of the device was necessary: materials were to be chosen carefully and active cooling for the ambient temperature heat exchanger was desirable. The overall system should have low noise emission. Therefore, structure-eigenfrequencies at possible TAR operating frequencies were to be avoided by stiffening the construction where necessary. Furthermore, all parts should be
reusable for future projects. Thus, it was aimed at a highly modular design. As a spherical compliance volume was too difficult to manufacture, a diverging cone and a small half-sphere were used instead. Figure 3.1 shows a schematic diagram of the modified design and labels its main components.

![Schematic diagram of the modified design](image)

Figure 3.1: A modified Hofler-style thermoacoustic cooler

### 3.2 TAR Design using Linear Thermoacoustic Theory

#### 3.2.1 Simplifying Assumptions

In linear, one-dimensional thermoacoustics, standing waves of angular frequency $\omega$ are usually described in terms of the complex pressure amplitude $p(x) \in \mathbb{C}$ and volume flow rate $U(x) \in \mathbb{C}$ at the basis of a time-dependency of $\text{Re}[e^{i\omega t}]$. The original partial-differential wave equation [5] can then be expressed as a pair of coupled first-order differential equations, however with a large number of parameters which partly depend on flow rate, pressure and $x$ themselves:

$$\frac{dp(x)}{dx} = F(p(x), p_m, U(x), T(x), A(x), \omega, \ldots)$$

$$\frac{dU(x)}{dx} = G(p(x), p_m, U(x), T(x), A(x), \omega, \ldots)$$

(3.1)

where $p_m$ denotes the mean pressure, $T(x)$ is the temperature profile and $A(x)$ is the resonator’s cross-sectional area. The first equation is the momentum equation and the second the continuity equation. Based on them, plus the first and second law of thermodynamics, complex equations for thermoacoustic components, acoustic power, cooling power, etc. can be derived. As a presentation of these equations would be outside the scope of this thesis, it is referred to the standard works about the theory of thermoacoustic coolers and engines, such as [3] and [4].

Additionally, the thermoacoustic equations in their most general form can hardly be used to design a thermoacoustic apparatus. A more practically oriented
guide for designing a thermoacoustic stack and some fundamental parameters of a thermoacoustic refrigerator was presented by Tijani et al. in [16]. In the following Sections, the stack design rules from this 2002 paper are summarized and applied to a new design problem.

Table 3.1 shows the TAR design variables. First, two strongly simplifying assumptions are made [4,13]:

1. The boundary layer approximation: the plate half-spacing $y_0$ within the thermoacoustic stack is larger than the viscous- and thermal penetration depths, $y_0 > \delta_v$ and $y_0 > \delta_k$, with these being defined as

   $$\delta_k = \sqrt{\frac{K}{\pi \rho c_p f}}$$  \hspace{1cm} (3.2)

   $$\delta_v = \sqrt{\frac{\mu}{\pi \rho f}}.$$  \hspace{1cm} (3.3)

   The thermal penetration depth is a measure for the distance over which heat can be transferred while an oscillating gas parcel is close to a point of maximal displacement. The viscous penetration depth is a measure for frictional losses during oscillation.

   In other words: it is assumed that well-shaped boundary layers are formed on the stack plate surfaces.

2. The short stack approximation: the overall length of the thermoacoustic stack and the heat exchangers is small in comparison to the length of the acoustic wave. Differences in gas pressure and -velocity are negligible over the stack length and the stack does not disturb the acoustic field. Furthermore, the temperature difference over the stack is assumed to be much smaller than the average gas temperature, so that the caloric properties of the working gas remain constant within the stack.

   A detailed derivation of simplified thermoacoustic equations using these assumptions is given in [13].

   The design variables given in Table 3.1 are not independent. Therefore, it is common to introduce further simplifications: Swift argues in [4] that the heat conduction along the stack's $x$-axis is negligible for typical stack materials with low thermal conductivity. In [17], it is shown that many of the previously given non-independent stack parameters can be combined to dimensionless, linearly independent design variables. Relevant for this work are the drive ratio $D = \frac{p(x=0)}{p_{in}}$, the fluid’s Prandtl number $\sigma$ and the stack porosity $B = \frac{y_0}{\pi y_0 + \ell}$.

   Generally, there are many ways how the design of a new TAR can set up. It depends on the available manufacturing capabilities and materials, the desired system properties, etc. The flow-chart in Figure 3.2 shows how the stack and resonator design-process was structured in this work.
First, the working fluid, the mean pressure $p_m$ and the dynamic operating pressure $p(x = 0)$ were determined. Then, a geometry for the thermoacoustic stack was specified and an appropriate material was chosen along with a manufacturing technique. Depending on the manufacturing method, there was a trade-off between the achievable half-spacing $y_0$, the stack porosity $B$ and the technically feasible stack length $L_s$. It was desirable to choose the real plate half-spacing as close to the optimal spacing as possible subject to the corresponding manufacturing constraints for $B$ and $L_s$. The minimal achievable pore diameter then determined the optimal operating frequency and the approximate TAR length. The stack diameter was subsequently chosen subject to the conflicting goals of maximizing the system’s cooling power and keeping the manufacturing effort low. Afterwards, the stack’s Coefficient of Performance was maximized as a function its length and its position in the resonator, subject to the desired temperature difference $\Delta T$. Then, the heat exchangers’ dimensions were calculated. The shape and size of the compliance was determined by manufacturing constraints and a given cone apex angle. Finally, the thermoacoustic system was simulated in DeltaEC, where frictional losses and turbulence were taken into account as well. The simulation’s predictions were compared with the original design goals. The overall process was repeated until the predictions suited the system requirements well. Note that the first design stage did not take into account the acoustic driver. The choice of a loudspeaker went in parallel with the overall system’s fine tuning as described in Section 3.3.
3.2. TAR Design using Linear Thermoacoustic Theory

Figure 3.2: flow-chart of the stack and resonator design process.
Table 3.2: Physical properties of air at ambient pressure and 20 °C

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prandtl-number</td>
<td>$\sigma = 0.718$</td>
</tr>
<tr>
<td>Isentropic exponent</td>
<td>$\gamma = 1.37$</td>
</tr>
<tr>
<td>Thermal conductivity</td>
<td>$K = 0.0262 \frac{W}{\text{mK}}$</td>
</tr>
<tr>
<td>Isobaric specific heat</td>
<td>$c = 1004 \frac{J}{\text{kg K}}$</td>
</tr>
<tr>
<td>Specific gas constant</td>
<td>$R_s = 287.06 \frac{J}{\text{kg K}}$</td>
</tr>
</tbody>
</table>

### 3.2.2 Working Fluid and Operating Pressures

#### Gas Properties and Mean Pressure

In theory, the power density in a thermoacoustic refrigerator is proportional to the working fluid’s mean pressure [5]. Consequently, many pressurized TAR designs can be found in literature. However, the thermal penetration depth decreases with higher mean pressure, c.f. Equation (3.2). As it is explained in Section 3.2.3, the stack’s optimal pore dimensions are proportional to the thermal penetration depth and thus, increasing the mean pressure makes the optimal stack pores small and difficult to manufacture. Furthermore, a working gas for a thermoacoustic refrigerator should have a low Prandtl-number, which can be written as $\sigma = \frac{\delta^2}{\delta_k^2}$ in terms of the thermal- and viscous penetration depths. A low Prandtl-number corresponds to low viscous losses paired with good thermal contact between stack surface and working fluid [7]. Additionally, a TAR working fluid should feature a large isentropic exponent $\gamma$, c.f. [14].

In TAR designs in the literature, a trade-off between these requirements is typically realized by using noble gases (often Helium, Argon or mixtures) at operating pressures of 5 to 10 bar. However, this requires a sophisticated pressure vessel design and often the use of expensive sealing technology [18]. In this work, air at ambient pressure was used instead of noble gas in order to avoid construction difficulties. This lowered the achievable cooling power and temperature difference but drastically reduced the overall cost at the same time. It should be mentioned that this choice does not allow temperature setpoints below 0 °C in order to avoid air moisture freezing at the cold parts. Table 3.2 summarizes the relevant physical properties of air.

#### Dynamic Pressure and Drive Ratio

In the framework of the linear thermoacoustic theory it is assumed that the acoustic Mach-number is smaller than 0.1:

$$M = \frac{1}{\gamma} \cdot \frac{p(0)}{p_m} = \frac{1}{\gamma} \cdot D < 0.1 \ . \ (3.4)$$

For small drive ratios $D = p(0)/p_m$ in the order of a few percent, linear behavior of the TAR can be expected [3]. At a higher $D$, nonlinear effects such as conditional
turbulence may occur [5]. However, thermoacoustic coolers which were equipped with standard HiFi-speakers have never been reported to have reached drive ratios higher than a few percent in literature [7]. For that reason, we initially specified a dynamic pressure ratio of 0.03 for the following calculations. Note that, due to the design variables’ independence when using dimensionless equations, there would be no effect on stack- or resonator parameters in the linear theory, if 3% would either not be reached or even exceeded. Therefore, the direct influence of this parameter on the overall system layout is small.

3.2.3 The Stack

Geometry, Material, Manufacturing Technique and Optimal Operating Frequency

The thermoacoustic stack can be considered the central part of the entire device. Generally speaking, it consists of a solid with a large number of thin gas channels (pores). While the thermodynamic cycle process takes place, the solid is supposed to store heat temporarily without allowing for significant heat conduction along its $x$-axis. The optimal design of the pore geometry (shape and spacing) of a thermoacoustic stack has been discussed extensively in literature, detailed reference can be found in [19]. It turned out that an array of pins is the theoretically best geometry for a stack in terms of achievable cooling power. However, a pin array would be hard to manufacture. The second best choice is a stack of parallel plates, a detailed investigation of this geometry has been provided in [20].

Considering Equation (3.2), it can be seen that, with the working gas and pressure being fixed, the optimal TAR operating frequency $f$ is inversely proportional to the square of $y_0$. For the optimal spacing of the plates, a compromise between viscous losses and good thermal contact between the solid and the working gas has to be found. While the plate spacing should be as small as possible to allow for good overall thermal contact between the wall and the working gas, a too narrow plate spacing would result in increased friction, surface losses, unstable flows etc. It is widely accepted that in order to meet these requirements and not to perturb the acoustic field exceedingly, a plate half-spacing $y_0$ between $\delta_k$ and $2\delta_k$ should be selected [3]. A common choice is therefore $y_0 = 1.5\delta_k$. It was adopted in this project.

Using air at ambient pressure as working gas, and setting $y_0 = 1.5\delta_k$ into Equation (3.2), it can be solved for the corresponding optimal operating frequency, as a function of the stack manufacturing constraints. The achievable plate half-spacing is the limiting factor. In order to keep the total length of the thermoacoustic system short, it is desirable to minimize the achievable plate spacing.

In a previous thesis at the IfA [12], it was shown that Rapid-Prototyping and Rapid-Manufacturing are new ways to manufacture thermoacoustic components.
For example, parallel plate geometries can be modeled in CAD software and manufactured directly from volumetric data. In this project, it turned out that Selective Laser Sintering (SLS) with the material Accura 60 [21] provided the best commercially available trade-off between minimum wall-thickness \( l_0 = 0.5 \text{ mm} \) and minimum gap width between two plates \( y_0 = 0.6 \text{ mm} \). This corresponds to an operating frequency of \( f = 170 \text{ Hz} \). Moreover, this typical Rapid-Prototyping plastic material features a low thermal conductivity \( K_s \approx 0.2 \frac{\text{W}}{\text{mK}} \) but a high heat capacity \( c_s \approx 1.2 \frac{\text{kJ}}{\text{kgK}} \).

### Thermoacoustic Stack Dimensions

Having the operating frequency chosen as 170 Hz, the expected overall TAR length was at least a quarter wavelength, which is

\[
\frac{1}{4} \cdot \frac{a}{f} \approx 0.5 \text{ m}.
\]  \hspace{1cm} (3.5)

On the one hand, it was now necessary to keep the overall TAR as slim as possible because the entire design was based on a model of a one-dimensional acoustic wave. If the ducts became too thick, this assumption would not hold any more. On the other hand, it was desirable to increase the diameter in order to obtain a measurable cooling-power, which is proportional to the cross-sectional area of the stack. Third, mechanical stability and integrity had to be ensured. Additionally, the available wrought material and manufacturing constraints had to be taken into account. After several design iterations, the diameter of the thermoacoustic stack and the adjacent heat-exchangers was determined as \( d_{\text{stack}} = 84 \text{ mm} \).

The calculation of the optimal stack length and position can be considered as a constrained optimization problem. The constraint is the desired temperature difference. Recalling that the temperature of the cold side of the stack should be set a few degrees above the freezing temperature of water and considering that the warm side had to become warmer than ambient temperature in order to lead away waste heat, a stack temperature difference of \( \Delta T = 30 \degree \text{C} \) was guessed.

In [16], Tijani et al. described a stack optimization procedure using a dimensionless formulation, where the stack’s Coefficient of Performance

\[
COP = \frac{\dot{Q}_c}{W}
\]  \hspace{1cm} (3.6)

is maximized. In the following, we make use of their results. The dimensionless procedure was implemented in MATLAB. Given the temperature difference \( \Delta T \), the stack’s COP can be expressed as a function of the stack length \( L_s \) and the stack center position \( x_s \). Figure 3.3 shows a surface plot of the stack’s COP as a function of these two variables and the constraint \( \Delta T = 30 \degree \text{C} \). It can be seen that, for every stack length, there is a corresponding stack center position which optimizes the stack’s COP. The curve of optimal x-L- pairs is separately displayed.
3.2. TAR Design using Linear Thermoacoustic Theory

Figure 3.3: The optimal x-L-pairs (black line) which maximize the stack’s Coefficient of Performance over a set of different stack lengths $L_s$ and stack center positions $x_s$.

In order to choose a reasonable stack length, the predicted COP, the cooling power and the required acoustic power were calculated for every optimal x-L-pair, see Figure 3.5. Note that a high performance would be achievable for rather small cooling powers. In contrast, as the desired nominal cooling load increases, the fraction of required acoustic power to pump that amount of heat increases and the stack’s COP lowers. While the cooling power increases linearly with the stack length in the beginning, it starts to bend down at a stack length of approximately 6 cm. Since this provided a good trade-off between cooling-power and performance, the stack length for the TAR was chosen $L = 6$ cm. Figure 3.4 shows the corresponding stack center position $x_s = 4.7$ cm.

3.2.4 Further Choices

The Heat Exchangers

Cold HX: according to [16], the optimal “length of the heat exchanger is determined by the distance over which heat is transferred by gas” and “corresponds to the peak to peak displacement of the gas at the cold heat exchanger location”. The heat exchanger length therefore results as

$$l_{\text{coldHX}} = \frac{2p(0)}{\omega \rho_m a} \sin \left( \frac{\omega x}{a} \right).$$  \hspace{1cm} (3.7)
Figure 3.4: COP-maximizing stack center position for different stack lengths. The dashed black line indicates the eventually chosen pair $L_s = 6$ cm and $x_s = 4.7$ cm.

Figure 3.5: Expected stack performance. This diagram shows the predicted cooling power $\dot{Q}_c$, the required acoustic power $W$ and the corresponding COP of the thermoacoustic stack as a function of the stack length.
Hence, a length \( l_{\text{coldHX}} \) of 3.5 mm was chosen in this work.

**Warm HX:** where the length of the warm heat exchanger is concerned, it is argued in [16] that the warm HX should have about double the length of the cold HX. Thus, \( l_{\text{warmHX}} = 7 \text{ mm} \).

**The Cold Duct**

In [14], it was derived that a minimization of frictional losses in the resonator can be achieved by choosing a diameter ratio

\[
\frac{d_{\text{cold-duct}}}{d_{\text{stack}}} \approx 0.54. \tag{3.8}
\]

For that reason, we determined \( d_{\text{cold-duct}} = 45 \text{ mm} \). The optimal length of the cold duct was computed in a numerical simulation, see Section 3.3.

**The Compliance Volume**

As a spherical compliance volume was too difficult to manufacture, the compliance was designed as a diverging cone which is closed with a half-sphere. From a theoretical point of view, it was desirable to have a rather large volume in order to simulate an open end. However, manufacturing capabilities were limited to a maximum part diameter of 120 mm, which eventually determined the half-sphere’s size. Furthermore, the cone’s apex angle was set to \( 10^\circ \) according to [16], in order to avoid stall and turbulence at the transition to the cold duct.

### 3.3 Numerical TAR Optimization

The next step in the design procedure was accomplished using the “Design environment for low-amplitude thermoacoustic Energy Conversion”, *DeltaEC* [22]. DeltaEC performs a numerical integration of a one-dimensional wave-equation subject to the device geometry, TAR operating parameters, temperatures and user-defined boundary conditions. The boundary conditions are not restricted to the front or back end of the simulated device. In fact, DeltaEC uses a shooting algorithm to match guessed variables to user-specified targets which can be distributed all over the device. With the basic TAR parameters calculated in Section 3.2, a DeltaEC model could be set up. The shooting algorithm was used to determine the remaining unknown system parameters, for instance the optimal length of the cold duct, subject to constraints (for example \( f = 170 \text{ Hz} \)). The simulation environment was used to investigate friction and turbulence effects. The differential equations used in DeltaEC are more general than those used for the prior design steps. Therefore, the results sometimes differed significantly from what was expected from the simpler design rules. The design process shown in Figure 3.2 was therefore repeated until a good agreement between the design
goals and the DeltaEC predictions was achieved. In the following, it is described how DeltaEC was used to find a suitable acoustic driver and to fine-tune some design variables.

3.3.1 Acoustic Drivers

In experimental TAR designs in the literature, the drivers are often highly modified HiFi speakers or specially designed acoustic drivers for thermoacoustic applications which are perfectly matched to the acoustic system in terms of moving mass, stiffness, $B \times L$ product, etc. The reason is that commercial HiFi-speakers have a comparably low electro-acoustic conversion efficiency (usually a few percent) because they are optimized for operating at a broad range of frequencies. Due to their low efficiency, they produce a lot of waste-heat which additionally perturbs the thermoacoustic system. In TAR designs which use pressurized vessels, the efficiency of the driver can often be increased by including a gas-spring in the back of the speaker [13]. However, for ambient pressure, the gas spring’s required piston area would drastically exceed the cross-sectional area of the entire resonator. Therefore, for this project, there was no other option but choosing a commercial HiFi speaker for the first experimental setup.

3.3.2 Choosing a Loudspeaker and Duct Length Optimization

From ten different commercially available HiFi speakers which were recommended by a distributor, the one which matched the task best was selected using numeric simulations. For all investigated speakers, parameters were obtained from the manufacturers. The waste-heat flows of the different speakers were modeled in DeltaEC. It was assumed that an external cooling cycle would immediately remove 70% of the speaker’s waste heat from the speaker housing. The rest would increase the temperature of the working gas and the temperatures of the warm duct and the warm HX. Thus, it would affect the TAR’s performance.

Setting the desired operating frequency, cooling load and temperature difference as targets and using the length of the cold duct, the required speaker input power etc. as guesses, the performance of each speaker-resonator combination was assessed. Two performance criteria were used: the predicted overall device COP

$$COP_{tot} = \frac{\dot{Q}_c}{P_{el}}$$

(3.9)

and the predicted stack COP, relative to Carnot’s efficiency

$$COP_{rel} = \frac{(T_{hot} - T_{cold})\dot{Q}_c}{T_{hot}W}.$$  (3.10)
3.3. Numerical TAR Optimization

Figure 3.6: Ten different speakers were tested for their ability to drive the TAR efficiently. The loudspeaker Monacor MSH115HQ delivered the best predicted relative stack COP and overall device COP.

The stack center position $x_s$ was discretized and used as an additional optimization variable. Then, for every speaker, $COP_{rel}(x_s)$ was maximized subject to the targets. Figure 3.6 shows the resulting relative COPs for all tested speakers plotted against the expected overall COPs. The loudspeaker Monacor MSH115HQ [23] performed best with a predicted relative COP of 9.6% and a predicted overall device COP of 7.5%. In addition to the maximal COP, it was found that the stack center position $x_s$ should be enlarged to 7 cm under influence of the waste heat flow from the speaker, which is partially conducted towards the stack trough the gas.

Appendix B.1 gives the DeltaEC code for the finally obtained optimized TAR, all final design variables, guesses, targets, a plot of the predicted pressure and flow magnitudes and the predicted acoustic power flow.
3.4 Mechanical Design and Manufacturing

From the final DeltaEC simulation results, the dimensions of all mechanical TAR parts were derived. Important aspects during the mechanical design process were easy maintenance, high modularity and low noise emission.

All parts were developed using a CAD software package. Figure 3.7 shows an exploded assembly drawing of the final design and labels the main components. A set of engineering drawings for all parts (except for the stack and the heat exchangers) and two assembly drawings are given in Appendix C. The following Sections deal with more detailed construction aspects of the different components. Most parts were manufactured by the Mechanical Workshop of ETH’s Department of Information Technology and Electrical Engineering.

Figure 3.7: Exploded assembly drawing of the TAR’s main components.
Figure 3.8: Volume model of a stack segment. Two support ribs perpendicular to the plates stiffen the construction.

3.4.1 The Stack

The stack material and its properties were discussed in Section 3.2.3. Selective Laser Sintering was chosen as manufacturing method. The overall stack length is 60 mm. Since a viscous support material had to be removed after the sintering process, building a stack of the required dimensions would not have been feasible without enlarging the plate spacing. Because of that, multiple identical, 20 mm long stack modules, which could be aligned precisely using a j6/H7 transition fit, were designed. Since the stack plates are long, thin-walled elements and since the plastic material is not very stiff, a problem arose with their mechanical frequency response. Modeling the broadest stack plate as two-sided clamped plate and computing its low-order eigenmodes using an FEM analysis showed that the acoustic resonance frequency 170 Hz was likely to interfere with eigenfrequencies of some plates. This might have excited strong vibrations or even might have caused damage. Figure 3.9 shows a qualitative illustration of the first three eigenmodes of the broadest plate and the corresponding frequencies. As a counter-measure, two support ribs, which are orthogonal to the plates, were included. It was found by similar FEM simulations that this shifts the first stack eigenfrequencies beyond 800 Hz, which is considered harmless. Figure 3.8 shows a volume model of one stack segment.

3.4.2 The Heat Exchangers

The heat exchangers were designed very similar to the stack segments. The plugs and sockets of the heat exchangers were designed as j6/H7 transition fits to ensure precise alignment. Both heat exchangers were manufactured from an Aluminum-Silicium alloy with low thermal resistance by Selective Laser Melting. Critical parts, such as the cable feed-through, were post-processed manually.
Warm HX

To make the transition from the warm duct to the stack smoother, the warm heat-exchanger’s plates were designed slightly thinner than the stack plates. Two thick support-ribs which are perpendicular to the plates and which are located at the same positions as the stack’s support ribs, improve heat conduction to the warm duct and to the external cooling cycle. To ensure good thermal coupling, heat-conduction paste was put between the warm duct and the warm HX. All edges on the front end were chamfered in order to minimize flow resistance and to minimize the risk of turbulence at the transition.

Cold HX

The cold heat exchanger must be insulated well and at the same time it must be in good contact with the stack. In our design, the cold heat exchanger was fitted into the reducer cone. During assembly, when the reducer cone and the stack holder were bolted together, the cold heat exchanger was pressed onto the stack. Thus, a tight connection between the reducer cone, all HX- and stack segments and the warm duct was obtained. A power resistor, which serves as adjustable heat load, was attached to the cold HX. A 360 degree groove along the HX’s contour hides power resistor- and sensor cables. Two milled gaps lead them into the resonator chamber. All edges on the back end of the cold HX were chamfered in order to assure a smooth transition of the air flow into the reducer cone.

3.4.3 Resonator Aluminium Parts

Generally speaking, it should be aimed at removing heat from the warm parts while insulating the cold parts. At the same time, it is important to use a non-magnetic material in order to avoid undesired coupling effects with the loudspeaker. For that reason, and because of its superior thermal conductivity, all major warm parts (the speaker housing, the adapter plate and the warm duct) were made of Aluminum.
3.4. Mechanical Design and Manufacturing

The speaker housing features cooling fins and a separate cover which provides connectors for sensors and the power supply. The warm duct has two interfaces for external heat exchangers which can be used to cool the TAR actively. There are threads for mounting the TAR in a rack and a thread for a pressure transducer. The warm duct was polished inside to minimize friction.

The adapter plate was connected to the warm duct via a transition fit and a bolted connection. Then, the adapter plate was bolted together with the loudspeaker and the speaker housing. It turned out that no additional rubber O-rings were required to keep the noise level low. Figure 3.10 shows a close-up view on the TAR: external heat-exchangers are attached to the warm duct and four pressure sensors are mounted. On the right, there is a BNC connector for a sensor within the speaker housing.

3.4.4 Resonator PVC Parts

All parts which become cold, namely the stack holder, the reducer cone, the cold duct and the compliance, were made of PVC. This material has a low thermal conductivity, it is inexpensive and it is easy to machine.

The stack holder was designed such that both heat exchangers and three stack segments can be pressed onto each other by tightening the bolted connection between the stack holder and the reducer cone. For easy assembly, there’s a little clearance (loose fit) between the stack segments and the stack holder. It is symmetric for intuitive assembly, two H7/h7 transition fits center both the warm duct and the reducer cone.

As mentioned before, the reducer cone was designed such that it bears the cold HX and interlocks all stack- and HX segments at the same time. There are holes for sensor- and power resistor cables. It was connected to the cold duct by a fine thread.

The cold duct was designed in a modular way. In this work, two segments were built. Each segment features a flat surface with three regularly spaced threads for pressure transducers. The diverging cone and the compliance half-sphere were glued together. The compliance features another thread for an optional pressure transducer. Since there are more holes than pressure transducers, all redundant holes can be sealed with specially designed PVC screws, c.f. Figure 3.10. The whole resonator is shown in Figure 3.11.
Figure 3.10: A close-up view on the TAR, seen from the side. External copper heat-exchangers are attached to the warm duct. There is a BNC connector for a sensor within the speaker housing. Four pressure transducers are mounted at different threads. The unused threads can be sealed with custom-made PVC screws.

Figure 3.11: The assembled thermoacoustic refrigerator with pressure sensors.
3.5 The TAR Test-Bed in a 19-inch Rack

3.5.1 Sound-Proof Rack

In order to avoid a negative influence of heat convection onto the performance of the TAR, the system was mounted vertically, such that the warm end is on the top. As a consequence, the cold air streams down into the compliance while the warm air rises up to the speaker end, where the external cooling is located.

The TAR was mounted upright in a 19-inch Aluminium rack, which is shown in Figure 3.13. The rack was equipped with sound-proof panels, the front panel can be removed to access, show or modify the device. Outside of the sound-proof box there are stations for measurement and control instrumentation.

3.5.2 Waste Heat Management

The thermoacoustic refrigerator exhibits two major heat sources. First, the warm heat exchanger, which delivers the heat flow from the heat pumping process. Second, the waste heat from the acoustic driver. In the presented TAR design it is accounted for both at the same time by cooling the warm duct, which is thermally well coupled to the warm HX and the speaker housing. According to DeltaEC simulation results for extreme cases, the expected waste heat flow is up to 70 Watts. Considering the relatively small available surfaces, this demanded for a high power density cooling solution, for example a water cooling cycle: two Alphacool HF 14 copper heat-exchangers, which were mounted to the warm duct, provide a maximum volume flow rate of 10 liters/min. The heat exchangers can be supplied with water from an external water reservoir, an Alphacool VPP655 water pump was installed in the 19" rack. An external water-chiller (Hailea Ultra Titan 150) can be used to regulate the cooling water to a desired temperature with a tolerance of ±1 °C. Thus, even experiments with high input powers could be made without over-heating the device and potentially damaging the speaker and the stack.

3.6 Measurement- and Control Instrumentation

3.6.1 Data Acquisition and Signal Generation

For data acquisition and signal generation, two National Instruments USB-6356 X Series [24] DAQ boards were used. Each provided eight 16 bit analog input channels with ±1 V, ±5 V or ±10 V range and 1.25 mega-samples per second update rate and two analog output channels with ±10 V voltage range. One device was used for data acquisition, the other one was used for signal generation only. All measurements were made in differential mode. While the USB-boards
were generally easy to set up and operate, we had to get along with a non-deterministic delay due to the USB-bus. As a consequence, no real-time control was possible with this setup. However, the control loop could be closed using block-wise data transfer and processing with a low update rate, since system’s thermal dynamics were slow enough.

### 3.6.2 Power Amplifier

The driver signals generated by the NI DAQ board’s analog output were amplified using a two-channel \textit{t.amp TA450 MK-X} audio-amplifier (2×125 W/8Ω). The acoustic driver has a nominal peak input power of 100 W (RMS), but it turned out that for long-running thermoacoustic experiments, the electric input power had to be limited to 50 W in order to avoid loudspeaker damage. With 125 W per channel, the power amplifier provided enough headroom to prevent signal clipping.

### 3.6.3 Control Software

In this work, \textit{National Instruments Signal Express 2013} was used for pure open-loop DAQ tasks. \textit{NI LabView 2013} was employed for advanced data-acquisition purposes and closed-loop control. LabView is a visual, data-flow controlled programming language. It is capable of multi-threading, which is especially useful for in-the-loop applications with high computational effort. \textit{Mathworks Matlab/Simulink} was tried as well, but did not lead to satisfactory results. Details about that issue are outlined in Appendix A.1.1.

### 3.6.4 Sensors and Additional Actuators

**Accelerometer**

A light-weight \textit{Kistler Type 8778A500 500g Ceramic Shear Accelerometer} [25], which was glued onto the back of the speaker membrane, was used to measure the acceleration of the acoustic driver. It was operated using a \textit{Kistler Coupler Type 5108A} [26]. The sensor was connected to a BNC mounting sleeve in the speaker housing cover. Supposing that the loudspeaker oscillates in a perfect sinusoidal way, the negative acceleration is directly proportional to the speaker cone displacement. Assuming that the speaker cone gets maximally displaced at its center and assuming that the displacement is zero at its edge, the accelerometer had to be placed at 2/3 of the distance from the center to the edge in order to get an estimate for the average speaker cone acceleration.
Pressure Transducers

*Omega PXM459-070HCG-10V* [27] piezo-resistive pressure transducers were used to measure the pressure waves at different points of the resonator. Their measurement range is up to ±70 mbar relative to ambient pressure with 0.08% accuracy. The sensor positions can be varied according to the user’s interest, the precise thread locations relative to the speaker are given in Appendix B.

Speaker Voltage and Current

To allow measurements of the speaker voltage and current, a measurement board (Figure 3.12) was soldered, which additionally serves as central router between DC sensor power supplies, the DAQ board and the temperature sensors. The board itself connects the power amplifier and the speaker and closes the speaker circuit. A voltage divider with \((R_{sm} + R_{lg})/R_{sm} = 11\) was used to measure the voltage across the speaker. A 10 mΩ high-precision shunt-resistor \(R_s\) was employed to measure the current through the speaker. An *Analog Devices AD628* (MSOP8) high common-mode difference amplifier was used to amplify the voltage drop across the shunt-resistor. \(R_1 = 1\) MΩ and \(R_2 = 10.2\) kΩ were chosen such that the total amplifier gain was 10. The capacitor \(C (18\) µF) implements a low-pass filter with cut-off frequency at approximately 800 Hz. All employed resistors have 1% tolerance.

Thermometers

Three *TSic 501F* (TO92) IC temperature sensors were used to measure the temperatures of the cold HX, the warm HX and the environmental temperature in the rack. The sensors show an optimum accuracy of ±0.1 °C within 5 °C to 45 °C measurement range and have minimal self-heating. One sensor was glued on each heat exchanger’s surface. Due to their small size, it is assumed that they do not perturb the sound waves. The supply voltage- and sensor-wires of the thermometer on the warm HX’s surface first pass through the stack and then pass through a hole (with sealing) in the reducer cone, together with the wires from the cold thermometer. The reference temperature thermometer was placed loosely in the rack. Supply voltage and signal wires are routed over the measurement board in Figure 3.12.

Additional Heat Load

In order to impart an additional, controllable heat load into the cold side of the system, a 1%- precision 100 Ω power-resistor (5 W) was placed on the cold HX in safe distance to the thermometer. The heat load could either be controlled by direct computer signal generation via the NI DAQ boards, where a constant amplification through a t.amp PM40C amplifier was provided, or manually by a
Figure 3.12: Schematics of the measurement board, which connects all sensors except for the accelerometer and the pressure transducers.

regulated DC power supply. A good thermal contact between power resistor and heat exchanger was ensured by adding heat conduction paste. A compact type of power resistor had to be chosen in order to keep the flow resistance small.
Figure 3.13: The TAR, vertically mounted in the 19-inch rack. The external water-cooling equipment is not shown.
Chapter 4

Basic Experimental System Characterization

A series of experiments was made in order to compare the theoretically predicted properties and performance of the TAR with the performance of the real system. The acoustic resonance and electro-acoustic conversion efficiency are examined in Section 4.1. Different notions of performance are discussed in Section 4.2. Data of a simple cool-down experiment is shown in Section 4.3. An investigation of the pressure wave’s harmonic content concludes this Chapter in Section 4.4.

4.1 Resonance and Electro-Acoustic Conversion Efficiency

4.1.1 Acoustic Resonance

To characterize the fundamental resonance behavior of the TAR system, several experiments were made. Two pressure sensors were installed, one directly at the speaker membrane, one at the first thread in the cold duct, right behind the stack. For all tests in this Section, the speaker was driven with sinusoidal signals between 100 Hz and 260 Hz. Table D.1 in Appendix D summarizes the experimental settings. For identifying frequency domain responses, for example from the accelerometer to the pressure sensors, the Sinusoidal Correlation Method as described in [28] was used.

Figure 4.1 shows a magnitude- and a phase plot for the frequency response from the accelerometer to the pressure sensors. At resonance, the magnitude peaks and the phase difference between speaker cone acceleration and pressure becomes approximately -90°. It can be seen that the real system’s resonance frequency is in good agreement with the theoretically predicted resonance frequency (170 Hz). In this experiment, the deviation was about 2 Hz. Note that the actual resonance frequency also changes as a function of environmental conditions like

31
(a) Magnitude plot of the frequency response from the accelerometer to the pressure sensors before and after the thermoacoustic stack. The magnitudes peak at the fundamental resonance frequency, 168 Hz in this experiment.

(b) Phase plot of the frequency response from the accelerometer to the pressure sensors before and after the thermoacoustic stack. At resonance, the phase difference between acceleration and the pressure wave is -90°.

Figure 4.1: Frequency response plots for the transfer behavior from the speaker cone acceleration to two pressure sensors (one in front of the speaker surface and one after the stack). Resonance occurs close to the theoretically predicted frequency 170 Hz.
temperature, air humidity, etc. The ratio between the pressure amplitudes at the chosen sensor locations is in good agreement with DeltaEC simulation results (Appendix B.1).

4.1.2 Impedance

An experimentally determined magnitude plot of the acoustic driver’s electrical impedance as seen by the power amplifier is shown in Figure 4.2a. While the characteristic impedance plot provided by the manufacturer [23] has a global resonance peak at 89 Hz, the speaker being enclosed in the speaker housing and driving the acoustic load shows a different behavior. In the frequency range of acoustic resonance, the impedance magnitude is almost constant and close to the nominal impedance 8 Ω. There are two local impedance maximums at 125 Hz and 245 Hz - an additional degree of freedom has been added by coupling the speaker and the acoustic system together. Optimally, the impedance peaks should be matched to the acoustical resonance frequency in order to allow for an efficient electro-acoustic energy conversion [13]. Since no measures were taken to manipulate the speaker’s characteristic parameters in this work, it would have been nothing but a coincidence if an impedance maximum had been close to the system’s acoustic resonance. The contrary is the case, the region of best mechano-acoustic conversion (Figure 4.1a) is closer to a local impedance minimum. Various techniques exist to match the impedance of the driver and the acoustic system – in Appendix A.2.1, it is referred to some.

4.1.3 Acoustic Power and Electro-Acoustic Conversion Efficiency

The acoustic power $\dot{E}_{ac}$ at the entrance to the thermoacoustic resonator can be calculated as a function of the pressure amplitude $p(0)$, the gas velocity amplitude $u(0)$ and their phase difference $\phi$ [4]. While the pressure amplitude can be measured directly, the latter can be inferred from the amplitude $a_0$ of the acceleration measurements and it can be stated that

$$\dot{E}_{ac} = \frac{A}{4\pi f} |p(0)| \cdot |a_0| \sin(\phi)$$

with $A$ being the duct’s cross-sectional area and $f$ being the frequency.

The overall electro-acoustic conversion efficiency is defined as the acoustic power divided by the electrical input power. Figure 4.2b shows an experimentally obtained electro-acoustic conversion efficiency plot for the TAR. It can be seen that the maximum acoustic power did not result at the acoustic resonance frequency, it was achieved close to 140 Hz instead. The characteristic shape of this plot has been observed before by other researchers [13,29] and is due to the mismatch of the impedances of the different subsystems.
(a) Total electric impedance magnitude of the acoustic driver as seen by the power amplifier. The characteristic impedance plot provided by the manufacturer [23] shows a single resonance peak at 89 Hz. However, the coupling between speaker and resonator shifts the first maximum up to 125 Hz and induces a second maximum at 243 Hz. Near the acoustic resonance frequency, the electrical impedance is close to the nominal speaker impedance 8 Ω.

(b) Electro-acoustic power conversion efficiency at the speaker surface, calculated for a constant electrical input power. It can be seen that the maximum acoustic power does not result at the acoustic resonance frequency, but close to 140 Hz instead. This is a result of the mismatch between acoustic impedance and speaker impedance.

Figure 4.2: Experimentally determined impedance of the electro-acoustic system (a) and the resulting acoustic power in the duct (b).
4.2 Thermal Performance

In the general case, when the impedance of the electro-acoustic driver and the acoustic impedance of the thermoacoustic resonator are not matched, different notions of performance have to be distinguished. In general, the overall electro-thermal performance is not comparable to the thermoacoustic performance. In the following, the main differences are shown by experimental results.

4.2.1 Mechano-Thermoacoustic Conversion Performance

Figure 4.3a and Figure 4.3b show measured temperature differences across the stack and the corresponding relative COP for constant 0.5 Watts of mechanical power in the acoustic driver. The mechanical power at different frequencies was measured and controlled in terms of acceleration. The largest temperature difference and the highest relative COP were obtained near the fundamental acoustic resonance frequency. Note that the electro-acoustic conversion efficiency was not optimal at this operation point according to Figure 4.2b. The experimental parameters are summarized in Table D.2.

4.2.2 Overall Electro-Thermal Conversion Performance

The overall electro-thermal conversion performance is depicted in Figure 4.4. The nominal electric input power was held constantly at 30 Watt over a range of different frequencies and heat loads on the cold HX. For a constant electrical
input power, the highest cooling effect was not obtained at the acoustic resonance frequency. The largest temperature differences were obtained for frequencies in the region of 140 Hz. All four curves are similar in shape, but shifted on the temperature axis as a function of the additional heat load supplied by the power resistor.

Comparing the extrema and the shapes of the plots in Figures 4.4 and 4.2b, and comparing the result with others [7], it seems reasonable that the frequency of highest temperature differences coincides with the frequency of the highest acoustic power in the system. When choosing a frequency of operation for the TAR, it is therefore advisable to operate it near the maximum acoustic power frequency in order to reach a high overall conversion efficiency: the achievable temperature difference is almost twice the temperature difference at the acoustic resonance frequency.

Running the system open-loop at 140 Hz with 30 W input power and different heat loads led to the plots shown in Figure 4.5. Small heat loads could be cooled to lower temperature levels, yielding a low relative COP. The highest relative COPs were obtained for heat loads near 3 Watts at small temperature differences and reach up to 9.5% of Carnot’s limit.
4.3 A Cooling-Down Experiment

Figure 4.6 shows a typical cooling-down experiment temperature plot for a constant electrical input power of 50 W and the driving frequency being close to the maximum acoustic power frequency. During this 30-minutes experiment, the room temperature stayed constantly at 23.2 °C. The thermoacoustic effect caused a rapid drop of the cold HX’s temperature during the first couple of minutes. Subsequently, the cold side gradually approached a steady-state at approximately 8 °C. At the beginning of the experiment, the warm HX’s temperature rose similarly fast, however the external cooling cycle took effect soon and limited the temperature to roughly 30 °C. The slight changes of the warm side’s temperature for $t > 5$ min were due to uncontrollable speed changes of the water pump and the binary control scheme of the external water-cooler. The thermal mass of the system’s cold side is large enough that the temperature changes of the external cooling cycle did not affect the temperature of the cold side in a measurable way.

For comparison, a scenario with identical parameters was simulated in DeltaEC, assuming that 70% of the loudspeaker waste heat caused by loudspeaker losses were removed immediately by the external heat exchangers. In the simulation, the cold HX reached a temperature of almost +2 °C. At the same time, the warm HX’s temperature only reached +26 °C. So, for both heat-exchangers, the predicted steady-state temperatures were lower than the experimental results. Besides the imperfectness of the real-world system, it turned out that the cooling performance of the external heat-exchangers was slightly over-estimated in the beginning (c.f. assumptions in Section 3.3.2). Assuming that a bigger fraction of the loudspeaker’s waste-heat reached and influenced the thermoacoustic stack, a better agreement between measured and simulated data could be obtained.
4.4 Harmonics

Usually, higher-order harmonics are considered an unwanted side-effect in thermoacoustic machines. They are excited by nonlinear mechanical and/or gasdynamical effects and often draw energy from the fundamental mode. According to the standard literature, c.f. [5], higher-order harmonics may represent a non-negligible amount of the acoustic power present in the system, especially at high drive ratios. Various approaches have been presented for suppressing harmonic content, either by appropriate resonator-shaping or by active cancellation [30].

In the TAR designed in this work, harmonics are present, even audible, but they play a negligible role for the thermoacoustic effect. Of all the detectable harmonic content, the second harmonic accounted for the biggest fraction. The acoustic power transmitted across the stack by the second harmonic was measured using the two-microphone-method as described in [5]. The system’s fundamental mode was strongly driven at frequencies ranging from 110 Hz to 190 Hz (40 Watt electric input power). The amplitudes and phases of the second harmonic were measured close to the speaker surface and behind the thermoacoustic stack. Figure 4.7 shows the acoustic power transmitted by the second harmonic. The energy flow induced by the harmonic pointed into negative direction for frequencies in the range of the overall acoustic power maximum (c.f. Figure 4.2b) and at fundamental frequencies higher than 170 Hz. In contrast, the second harmonic’s power flow pointed into the positive direction between 150 Hz and 170 Hz.

The total harmonic distortion (THD) at the speaker surface, measured in terms of the pressure amplitudes, Equation (4.2), where $p_1$ denotes the amplitude of the fundamental pressure wave and $p_h$ denotes the amplitude of the $h^{th}$
harmonic, always stayed below 1%:

\[
\text{THD} = \sqrt{\sum_{h=1}^{H} p_h^2} \leq 1\%
\] (4.2)

Harmonics up to fourth order \((H = 4)\) were investigated. Even at the highest achievable drive ratios, harmonics could be considered harmless in this work.

Figure 4.7: Acoustic power transmitted across the stack by the 2\textsuperscript{nd} harmonic, calculated by the two-microphone method as given in [5]. The electric input power was 40 W, other experiment parameters were similar to those in Table D.1. Note that the energy flow induced by the harmonic points in negative \(x\)-direction for frequencies in the range of the acoustic power maximum (c.f. Figure 4.2b) and at frequencies higher than 170 Hz.
Chapter 5

Control of the TAR

In typical state-of-the-art TAR designs which use electro-acoustic drivers with purely sinusoidal input signals, two variables have to be regulated according to the desired control objective: the driving frequency $f$ and the fundamental wave’s pressure amplitude $p_1(x = 0)$, abbreviated as $p_1$ in the following.

The latter is usually controlled by regulating the peak input voltage to the driver. An advantage of thermoacoustic refrigerators in comparison to standard vapor-compression refrigerators is that, given a certain heat load at the cold HX, its temperature can be regulated precisely to a desired setpoint by adjusting the electric input voltage (as long as the system is powerful enough to provide the required cooling capacity and temperature difference). This is often done using basic PI-controllers. However, controlling the temperature and the input voltage is not the scope of this thesis and it is referred to other works, for example [9].

For the second control variable, the frequency $f$, on which we focus in this Chapter, two cases can be identified. Depending on the control objective of the user, $f$ can either be chosen so that

1: the mechano-thermoacoustic conversion performance is maximized, compare Section 4.2.1, or

2: the overall electro-thermoacoustic conversion performance is maximized, compare Section 4.2.2.

These control objectives would be identical for a system with matched resonator- and speaker impedance. It is known that the optimal operation frequencies change with operational parameters of the TAR, such as the heat load, the temperature difference across the stack and the pressure amplitude $p_1$. 
5.1 Current State of the Art and Associated Problems

For each control objective, particular control strategies have proven useful in existing TAR designs. The most prominent ones are briefly summarized in the following:

5.1.1 Phase-Locked-Loop Controllers

For a quarter-wavelength resonator, it can be shown that the phase difference between pressure and particle velocity must be zero at resonance, or equivalently, the phase difference between pressure and acceleration of the speaker cone must be 90°. A derivation of this relationship can be found in [7]. A similar, experimentally achieved result is shown in Figure 4.1b of this work. Current state of the art for control objective 1, maximizing the performance of the mechano-thermo-acoustic part, is the Phase-Locked-Loop (PLL) controller, which forces the pressure and acceleration into a quadrature phase relationship.

A flowchart of the PLL-controller as used in this project is given in Figure 5.1. The pressure $p_1$, measured at the speaker surface, and the acceleration measurement are first multiplied and then fed into a low-pass filter (LPF), which averages a large number of samples. It generates an error signal $e$ which is zero if pressure and acceleration signals have a phase difference of 90°, which is positive for smaller phase differences and negative for bigger phase differences. Thus, it can be used as input signal to a PI-controller, whose output $\Delta V$ is added to a initially guessed starting value $V_0$. The resulting total $V_{tot}$ regulates a Voltage-Controlled-Oscillator (VCO). The VCO produces a sinusoidal AC signal $u(t)$ with the frequency being proportional to $V_{tot}$, which is subsequently fed into the power amplifier and drives the speaker. The initial guess for $V_0$ must be so that the resulting initial frequency is sufficiently close to the true acoustic resonance frequency. The PI controller gains can be easily tuned manually, a good compromise between overshoot, lock-on time and robustness can for example be obtained by the Ziegler-Nichols tuning method [31].

Advantages of PLL-controllers are their simplicity, their fast lock-on characteristics and their high steady-state precision. Ryan’s PLL on a real-time platform in [7] achieved a steady-state precision of $\Delta f = 0.25$ Hz and a lock-on time of “a few seconds”. The PLL in this work, implemented in LabView and using the National Instruments USB data-acquisition boards, locked onto the acoustic resonance with a steady-state precision of $\Delta f = 0.5$ Hz and a lock-on time of about one minute, which is still sufficiently faster as the plant’s thermal dynamics (c.f. Figure 4.6). A general disadvantage of PLLs is that they require knowledge about speaker displacement, velocity or acceleration, which usually requires costly sensors.
5.2. Harmonics-Based Frequency Control Algorithm

5.1.2 Extremum-Seeking Controllers

State of the art for objective 2, maximizing the acoustic power, are Extremum-Seeking controllers. This type of regulators typically uses gradient-ascent algorithms to find and track an operating frequency with a local acoustic-power maximum. While these control algorithms make it possible to track an optimal operating point which is off the acoustic resonance frequency, they are often slower in convergence than PLLs. Moreover, computing the acoustic power at the speaker surface usually requires a pressure transducer and an accelerometer, compare Equation (4.1). Two different extremum-seeking controllers are compared in [7], others can be found in [10, 11].

5.1.3 Accelerometer Problems

In TAR designs with a metallic driver surface, e.g. in [13], acceleration sensors can be permanently attached to the driver by threaded mounting. For a HiFi speaker with a non-rigid cone, however, it turned out that a permanent mounting is difficult: The sensor was glued to the cone using either Loclite 496 or Loclite 460 instant glue. Although the accelerometer’s mass is only about 2 grams, it was found that its interaction with the speaker cone is enormous. For example, at \( f = 500 \) Hz and intermediate driving, the inertia of the acceleration sensor imparts an additional force of about 10 Newton into a very small area of the speaker cone. Typically, the sensor already became loose after a few minutes of operation and the acceleration measurements were distorted heavily. In most trials, the sensor fell off the speaker cone after a short period of time, such that no continuous operation of the TAR was possible. Figure 5.2 shows a photograph of the accelerometer right before breaking off. Figure 5.3 shows an example for a corrupted acceleration measurement, where the true acceleration signal was distorted by oscillations of the partly loose accelerometer. Measurements like the one presented worsened the performance of the PLL controller drastically.

5.2 Second Harmonic-Based Frequency Control Algorithm for Acoustic Resonance

As explained in the previous Section, it turned out that the acceleration sensor was an error-prone and rather unreliable part of the control system. For that reason, and because it is of natural economic interest to reduce the variety of sensors in a technical system, a new approach to identify and track the acoustic resonance frequency is presented in this work. It is based on locking the phase difference between the second 2nd harmonics which are measured with microphones or pressure transducers at the front and at the back end of the resonator.
Figure 5.1: Flowchart of a PLL-controller for tracking the acoustic resonance frequency, based on acceleration and pressure measurements.

Figure 5.2: The accelerometer right before breaking off the speaker cone: it merely sticks to the paper cone at its top right edge. Therefore, the acceleration sensor can oscillate in different modes than the speaker cone.

Figure 5.3: An example for a corrupted acceleration raw measurement. The loose accelerometer oscillated in its own modes which are clearly visible in the plot.
5.2. Harmonics-Based Frequency Control Algorithm

5.2.1 Key Idea

Simple Physical Motivation

Consider a closed tube as shown in Figure 5.4. It is well known that the pressure amplitude \( p_i(x,t) \) of a standing wave of order \( i \) can be written as the superposition of an incident wave and a reflected wave. As a strong simplification, we lump all occurring ‘losses’ in the acoustic wave (friction, turbulence, etc.) into an impedance discontinuity which is supposed to occur at the right end of the tube \( (x = L) \) and which affects the intensity of the reflected wave by a reflection-coefficient \( \Lambda \). Then, we write the sum of the incident and reflected wave as

\[
p_i(x,t) = \tilde{p}_i \cos (\omega_i t - k_i x) + \Lambda \tilde{p}_i \cos (\omega_i t + k_i x) . \tag{5.1}
\]

For \( \Lambda = 0 \), we get a pure traveling wave, for \( \Lambda = 1 \) we obtain an ideal standing wave, and for \( 0 < \Lambda < 1 \) we obtain a pseudo-standing wave with a net power flow to the right.

Using the General Phase Shift Theorem [32], Equation (5.1) can be rewritten as

\[
p_i(x,t) = \tilde{p}_i \sqrt{1 + \Lambda^2 + 2\Lambda \cos (2k_i x)} \cdot \cos (\omega_i t - \theta (k_i, x)) \tag{5.2}
\]

with overall pressure amplitude \( p_i(x) \). Figure 5.4 depicts standing-wave pressure amplitudes for the fundamental mode \( p_1(x) \) and for the second harmonic \( p_2(x) \).

The phase angle \( \theta (k_i, x) \) results as

\[
\theta_i (k_i, x) = \text{Arg} \left( (1 - \Lambda) \sin (k_i x) + j (1 + \Lambda) \cos (k_i x) \right) . \tag{5.3}
\]

Assuming that the system is dispersion-less, the wave-number \( k_i \) can be expressed as a function of the \( i^{\text{th}} \)-mode’s wavelength \( \lambda_i \)

\[
k_i = \frac{2\pi}{\lambda_i} , \tag{5.4}
\]

thus \( \theta_i (k_i, x) \rightarrow \theta_i (\lambda_i, x) \).
Now consider the pressure phases at the two opposing ends $x = 0$ and $x = L$ for a given wavelength $\lambda_i$. We calculate their phase difference $\Delta \theta_i$ as

$$
\Delta \theta_i = \theta (\lambda_i, 0) - \theta (\lambda_i, L)
= \text{Arg} (j (1 + \Lambda)) - \text{Arg} \left( (1 - \Lambda) \sin \left( 2\pi \frac{L}{\lambda_i} \right) + j (1 + \Lambda) \cos \left( 2\pi \frac{L}{\lambda_i} \right) \right).
$$

(Equation 5.5)

Evaluating expression (5.5) shows that the phase difference between the pressure wave at the speaker surface and the pressure wave at the sealed end is expected to be zero or $180^\circ$ for

$$
\frac{L}{\lambda_i} = 0, 1, 2, \ldots.
$$

(Equation 5.6)

In other words, if we measure the pressure waves at the speaker end and the pressure waves at the compliance, we expect their phase difference to be zero or $180^\circ$ for the fundamental mode, the second harmonic, the third harmonic, etc. The key idea is to use the phase error signal $\Delta \theta_i$ as an input signal for a feedback controller which eliminates the phase difference and thus drives the system to the point of acoustic resonance. If the expected phase difference is $180^\circ$, it can be transformed into zero by negating the pressure measurements of one sensor, which is assumed in the following.

**Behavior of the Real System**

It must be emphasized that in the presented TAR, the higher-order waveforms are naturally present in the standing wave if it is driven at a frequency in the range of the fundamental acoustic resonance frequency. Due to the nonlinear characteristics of gas-dynamics, they are even audible at low input powers and they are detectable with the pressure transducers in any case – as an integer multiple of the driving frequency. According to Equations (5.5) and (5.6) one would expect zero phase difference for the fundamental mode and its higher-order harmonics. However, in our TAR, that relationship could not be confirmed for the fundamental wave. Instead, it showed a phase difference of $\Delta \theta_1 \approx 5^\circ$ at frequencies close to the acoustic resonance. Therefore it could not be used for control as proposed in the previous Section. Two reasons could explain that behavior: first, the TAR was originally designed as a quarter-wavelength resonator, but above theory requires measurements in the compliance. The first part of the resonator could be at resonance with the fundamental mode while there is a slightly different behavior in the compliance. Second, at this point in time, we could not quantify the true effects of the thermoacoustic stack and the heat exchangers on the acoustic field of the fundamental mode. Although we ensured to have an appropriate plate spacing during the design process, the boundary-layer approximation might not capture all relevant effects.
5.2. Harmonics-Based Frequency Control Algorithm

(a) Magnitude plot showing the relation of the second harmonic’s pressure amplitude in the compliance to its pressure amplitude close to the speaker surface. There is a peak at the acoustic resonance frequency.

(b) The phase difference $\Delta \theta_2$ for the second harmonic with the pressure waves being measured at the speaker surface and the compliance. The phase difference becomes zero at the fundamental acoustic resonance frequency. It can be seen that by starting at an initial frequency guess $f_0$ in a $\pm 20$ Hz interval about the fundamental resonance frequency, it is possible to find the acoustic resonance frequency using $\Delta \theta_2$ as error signal for a feedback controller.

Figure 5.5: Frequency response of the second harmonics, from speaker surface to the compliance. Seven datasets were averaged, details can be found in Table D.5.
Figure 5.5 shows experimental results for the second harmonic’s phase difference $\Delta \theta_2$. The system was driven at frequencies from 110 Hz to 190 Hz. The harmonic content was extracted by means of a Fourier-Transform. For the second harmonic, a zero-crossing of $\Delta \theta_2$ was found at the fundamental acoustic resonance frequency, see Figure 5.5b. Starting at an initial frequency guess $f_0$ in a $\pm 20$ Hz interval about the fundamental resonance frequency, it is therefore possible to find and track the true acoustic resonance frequency using $\Delta \theta_2$ as error signal.

A corresponding magnitude plot is given in Figure 5.5a, it shows a maximum at the acoustic resonance frequency - the second harmonic’s amplitude in the compliance is almost three times as large as the second harmonic’s amplitude at the speaker surface.

It has already been mentioned in Section 4.4 that the Total Harmonic Distortion in the measured pressure wave is so low that we can consider it as harmless w.r.t. the heat-pumping process. But with an experimentally determined SNR-ratio of at least 50 for the second harmonic, it can be used as reliable error variable for feedback control.

5.2.2 Algorithm

In the following, an algorithm for a second-harmonic-based frequency control of the TAR is proposed, which uses the phase difference $\Delta \theta_2$ as error signal for a PI-controller and a Voltage-Controlled Oscillator to generate a sinusoidal driving signal. Figure 5.6 shows a flowchart of the closed control loop.
Initialization

0. Make a starting frequency guess \( f_0 \) such that \( f_0 \) is within a sufficiently narrow interval (here: \( \pm 20 \) Hz) about the nominal TAR operating frequency and select \( V_0 \) accordingly. Wait for the start-up-transients to decay.

Data Acquisition and Pre-Processing

1. With both pressure sensors (at the speaker surface and at the compliance), acquire \( N \) samples with sampling frequency \( f_s \) simultaneously. Thus, obtain two discrete-time series \( p(0, m) \) and \( p(L, m) \), \( m = 0, 1, 2, \ldots, N - 1 \).

2. Remove the means, obtain \( \bar{p}(0, m) \) and \( \bar{p}(L, m) \) with \( m = 0, 1, 2, \ldots, N - 1 \).

3. Use one of the time series, for example \( \bar{p}(0, m) \), to find the maximum integer number of cycles contained in the signal. Let the integers \( m_s \) and \( m_e \) denote the corresponding indices of the first and the last relevant zero-crossing of \( \bar{p}(0, m) \).

4. For both signals, extract an integer number of cycles by defining \( n := m - m_s \) for \( m_s \leq m \leq N - 1 \), thus obtaining two reduced time-series \( \bar{p}(0, n) \) and \( \bar{p}(L, n) \) with \( n = 0, 1, 2, \ldots, (m_e - m_s - 1) \).

Extracting the Second Harmonic

The second harmonic can be extracted in different ways. In [30], a bandpass-filter was used - it was applicable because the THD was very high. In this work, as the THD is rather small, the harmonic content is extracted by means of a limited-bandwidth Discrete Fourier Transform, the isolation of relevant frequencies, and a subsequent Inverse Discrete Fourier Transform:

5. Compute the cross correlations of both time-series \( \bar{p}(0, n) \) and \( \bar{p}(L, n) \) with complex sinusoids of frequencies \( \frac{k}{m_e - m_s} \). Choose \( k \) such that the frequency band where the second harmonic is expected to appear is covered. Assuming that the second harmonic will be at two times the fundamental frequency, we can for example choose \( k \in \{ \lfloor 2f/f_s \rfloor - h, \lfloor 2f/f_s \rfloor + h \} \) with \( h \) being the search interval’s half-width parameter, which can be chosen as desired. We consequently obtain

\[
P_{k}(0) = \sum_{n=0}^{(m_e - m_s - 1)} \bar{p}(0, n) \cdot e^{-j \frac{2\pi kn}{m_e - m_s}} \quad (5.7)
\]

and

\[
P_{k}(L) = \sum_{n=0}^{(m_e - m_s - 1)} \bar{p}(L, n) \cdot e^{-j \frac{2\pi kn}{m_e - m_s}} \quad (5.8)
\]

which is a Discrete Fourier Transform over a limited bandwidth.
6. Given the sequences of complex numbers $P_k(0)$ and $P_k(L)$, with $k \in \{\lfloor 2f/f_s \rfloor - h, \lfloor 2f/f_s \rfloor + h \}$, isolate a subset $K$ of the most dominant frequencies in the considered interval, e.g. those for which the magnitude surpasses 50% of the maximal observed magnitude.

7. Perform the inverse DFT over $K$, thus finally extracting the second harmonic’s dominant content:

$$ p_2(0, n) = \frac{1}{(m_e - m_s)} \sum_{k \in K} P_k(0) \cdot e^{j \frac{2\pi kn}{(m_e - m_s)}} \quad (5.9) $$

and

$$ p_2(L, n) = \frac{1}{(m_e - m_s)} \sum_{k \in K} P_k(L) \cdot e^{j \frac{2\pi kn}{(m_e - m_s)}} \quad (5.10) $$

**Quadrature Demodulation**

There are several approaches to find the phase difference $\Delta \theta_2$ between $p_2(0, n)$ and $p_2(L, n)$. For example, if the harmonic’s peak is close to the center of one of the DFT’s ‘bins’, the phase difference could be taken directly from the complex numbers $P_k(0)$ and $P_k(L)$. The phasor-values could be interpolated by zero-padding if desired and the algorithm could continue directly at step 10. If the second harmonics intensities are distributed over several bins, a Quadrature-Demodulation technique [33] can be used to determine the phase-difference between two signals of identical frequency with sub-sample precision:

8. Perform a Discrete Hilbert Transform on either $p_2(0, n)$ or $p_2(L, n)$ in order to obtain a 90° phase-shifted signal, here for example:

$$ \tilde{p}_2(0, n) := \mathcal{H}\{p_2(0, n)\} = h(n) * p_2(0, n) \quad (5.11) $$

with

$$ h(n) = \frac{1 - \cos(\pi n)}{\pi n} \quad . \quad (5.12) $$

9. $p_2(L, n)$ is now multiplied pointwise with $p_2(0, n)$ as well as with $\tilde{p}_2(0, n)$. The averages of the resulting two vectors directly indicate the phase difference between both pressure signals with sub-DFT-sample precision. Defining the variables $t_1$ and $t_2$ as

$$ t_1 = \frac{1}{m_e - m_s - 1} \sum_{n=0}^{m_e-m_s-1} p_2(0, n) \cdot p_2(L, n) \quad (5.13) $$

and

$$ t_2 = \frac{1}{m_e - m_s - 1} \sum_{n=0}^{m_e-m_s-1} \tilde{p}_2(0, n) \cdot p_2(L, n) \quad , \quad (5.14) $$

the phase difference $\Delta \theta_2$ between $p_2(0, n)$ and $p_2(L, n)$ results as

$$ \Delta \theta_2 = \text{Arg} (t_1 + jt_2) \quad . \quad (5.15) $$
5.2. Harmonics-Based Frequency Control Algorithm

![Graph showing convergence of the second harmonic-based resonance control strategy.](image)

Figure 5.7: Convergence of the second harmonic-based resonance control strategy. Starting at an initial frequency guess of 180 Hz, the system reached a steady-state close to 167.3 Hz after approximately 3 minutes.

PI-Controller and VCO

10. The error signal $\Delta \theta_2$ can be directly supplied to a PI-controller. The resulting control signal $\Delta V$ is added to the initial guess $V_0$, which subsequently drives a Voltage-Controlled Oscillator. The VCO’s output signal of frequency $f$ is amplified by a power amplifier with adjustable gain and subsequently drives the TAR’s speaker. Once the new frequency $f$ has been set, restart at step 1.

5.2.3 Experimental Results

The algorithm described in Section 5.2.2 was implemented in LabView. The sampling frequency was chosen as $f_s = 20$ kHz. $N = 30000$ samples were acquired at each iteration. Together with the time-delay caused by the computation, this resulted in an overall update rate of approximately 0.6 Hz. The VCO was configured so that a 1 V change in input voltage directly transformed into a 1 Hz frequency change at the output. Thus, the output of the PI-controller could be directly interpreted as a frequency difference to be applied. An initial frequency $f_0 = 180$ Hz was assumed.

A typical result for running the system with this harmonics-based controller is shown in Figure 5.7. The system reached a steady-state at approximately 167.3 Hz after 180 seconds. Note that the shown plot was recorded immediately after the system had reached its thermal equilibrium in another experiment with identical settings. If the system experienced a ‘cold start’, it typically took up to half an hour until it reached a steady-state. In the long run, the steady-state
Figure 5.8: Harmonics-based frequency tracking under the influence of different heat loads.

precision in the recorded experiment was close to the frequency spacing of the DFT (0.66 Hz). Similar results were obtained for experiments with different drive ratios: the controller drove the system to its acoustic resonance for both very low drive ratios and pressure amplitudes at the limit of the acoustic driver.

Figure 5.8 shows experimental results for the resonance frequency tracking when additional heat loads were applied. After +1 Watt was imparted at the cold HX, the frequency lowered about 1 Hz. After it was switched off again, the frequency returned to its initial steady-state. With a +1.5 Watt additional heat load, the frequency decreased about 1.5 Hz.

5.2.4 Discussion

Without doubt, the harmonics-based frequency control approach for tracking the acoustic resonance frequency requires significantly more computing power than the traditional Phase-Locked-Loop, which can even be implemented in form of electrical circuits without a computing unit. Another disadvantage is that the achievable steady-state precision depends directly on the number of recorded samples – therefore, it is inversely proportional to the control loop’s update rate. With a few minutes lock-on time, the harmonics-based approach is slower than a PLL, however, it is still significantly faster than the thermal dynamics of the plant. Based on the experiments conducted so far, it can be presumed that the steady-state precision of the new approach (0.66 Hz) is in a similar range as the steady-state precision of a PLL, which achieved 0.5 Hz in a previous experiment. Both are sufficient to track the relatively broad acoustic resonance peak, c.f. Figure 4.1a. The biggest advantage of the new approach is that it does not require an accelerometer, which are still quite costly at the required precision,
in comparison to small microphones. Pressure sensors turned out to be more reliable than accelerometers for continuous TAR operation. Furthermore, it must be emphasized that the PLL control strategy is sensitive to (probably difficult to measure) sensor-inherent phase delays, while the presented harmonics-technique is completely independent of those.

However, there are many points, which need to be investigated more closely:

- In this Chapter, we worked with an update rate of approximately 0.6 Hz, however the slow plant dynamics might allow for even longer sample periods. It should be investigated how much the sample period can be increased subject to the thermal dynamics in order to minimize the steady-state error of the control.

- Although the phasor values are already available after the first DFT in the algorithm, significantly better results were obtained experimentally by performing an Inverse Discrete Fourier Transform to extract the time-domain data of the second harmonic and subsequently applying a Hilbert Transform and a Quadrature Demodulation instead. It should be examined how different interpolation methods, e.g. zero padding or weighted phasor-value interpolation, etc., affect the control performance.

- A validation/comparison using a different controller (for example a PLL controller) of the resonance lock-on behavior and the tracking results is desirable. This could not be done in this thesis since the required experiment durations were not achievable with the accelerometer in combination with the loudspeaker’s paper cone.
Chapter 6

Conclusion and Outlook

6.1 Summary and Conclusion

The intention of this work was to initially start up research in thermoacoustic refrigeration at ETH’s Institute for Automatic Control. First, a laboratory-scale thermoacoustic refrigerator was designed and constructed, including feedback capabilities and a test-bench. The key design requirements were high modularity, compactness, low noise emission, good thermal management, low amount of maintenance and reliability. Last but not least, the system should be mobile for presentation purposes. Therefore, a slightly modified, quarter-wavelength Hofler-type design was chosen, using air at ambient pressure as working fluid. The stack- and resonator design parameters as well as a suitable HiFi loudspeaker were determined using a design procedure which combined well established design considerations based on linear thermoacoustic theory and advanced numeric simulations with the thermoacoustics simulation software DeltaEC. Using a number of simplifying assumptions and forming dimensionless groups, the major design variables were made independent of each other and could be individually chosen subject to optimization criteria and key requirements. The technologically achievable stack plate spacing was found to be one of the key variables besides the working gas, eventually it determined the optimal acoustic resonance frequency (170 Hz) and the resonator shape. For the thermoacoustic stack and the heat exchangers, a new modular design was introduced. The stack consists of many parallel plates and was eventually manufactured by stereolitography, while both inner heat exchangers were laser-melted. All mechanical parts were designed in CAD software, critical parts were additionally optimized with regard to harmful structural resonance modes. The TAR features a great variety of interfaces for sensors. Pressure sensors, an accelerometer for the speaker cone and sensors for measuring voltage and current through the speaker were included. The device was controlled using National Instruments USB-DAQ boards. The TAR and most of the feedback instrumentation was embedded into a 19-inch rack as test-
bench. It was equipped with sound-proof panels and an external water-cooling cycle, which ensures stable experiment-conditions and prevents the system from heating up too much, even for very high electrical input powers. In summary, this enables the researcher to a great variety of thermoacoustic experiments in an everyday laboratory environment without excessive noise exposure.

The system was nominally designed for a temperature difference of 30 °C. Under laboratory conditions, a temperature difference of 22 °C could be recorded. The acoustic resonance frequency was found to coincide well with theoretical predictions. However, it turned out that the largest temperature differences can be achieved at the frequency of maximum acoustic power, which is approximately 30 Hz off the acoustic resonance frequency. The reason is that the acoustic impedance of the resonator and the HiFi speaker’s impedance were not matched. Thus, two different control goals were identified. First, maximizing the mechano-thermoacoustic conversion performance, which is maximal at the acoustic resonance frequency. Second, maximizing the overall electro-thermoacoustic conversion performance, which is maximal at the frequency of highest acoustic power.

For the first control objective, a novel control strategy was presented, which relies on extracting the second harmonic from the pressure wave at two opposite locations of the resonator. Based on the insight that the phase difference between the second harmonics – measured at different resonator locations – must be zero if the system is at resonance, this phase difference can be used as error signal for a controller. The control strategy was implemented in LabView.

In summary, the two main contributions of this thesis are the following: a ready-to-operate thermoacoustic refrigerator was designed and constructed. Feedback capabilities were included and its performance was proven. Motivated by the absence of reliable acceleration measurements, the fundamentals for a novel acoustic resonance tracking algorithm were provided, which uses higher-order harmonic content of the signal and showed promising results in first experiments.
6.2 Future Work

Although first positive results were achieved in this thesis, many questions remain open. In the following, some prospects for automatic control in thermoacoustic refrigeration are briefly outlined, and a short preview of a possible future thermoacoustics project at ETH’s Institute for Automatic Control is given.

6.2.1 How can Automatic Control Help to Improve Thermoacoustic Refrigeration?

Currently, thermoacoustic refrigeration is not a highly active research field in automatic control. However, we see many questions where some well-known methods from automatic control science can help to accelerate the development of thermoacoustic devices.

The precise mathematical modeling of thermoacoustic processes – in particular of higher-order phenomena where harmonics are involved – is still very challenging and complicated. System Identification techniques, as they are state of the art in automatic control, may help to derive models of a TAR’s thermal characteristics and heat pumping performance, and in particular, how these depend on parameters like the drive ratio or the heat load.

While it is often assumed that a sinusoidal wave is the optimal input to the acoustic driver, we do in fact not know if this is the case. Most of the theory of thermoacoustic refrigeration assumes sinusoidal waves as optimal. To validate this assumption, Iterative Learning could be used, in order to maximize the cooling- or conversion performance, depending on the shape of the input signal.

The list of possible topics could be extended with many other items, for example model-based estimation, harmonics suppression, etc.

6.2.2 Future Work at the IfA

First, a validation of the harmonics-based controller presented in Section 5.2 needs to be carried out. For example, its tracking results could be compared to the results of a well-established controller (for example a PLL). This could not be done in this thesis, because it was hard to attach the accelerometer rigidly to the paper cone for the required experiment times.

Second, in order to increase the control performance and in order to make the implementation of more advanced controllers possible, the National Instruments USB-DAQ boards should be replaced by a Real-Time DAQ and -computing system. This question is addressed in more detail in Appendix A.1.1.

We have seen in Section 4.2 that two different performance goals have to be distinguished for systems where speaker- and acoustic impedance are not matched. An accelerometer-less method for tracking the acoustic resonance, was presented in this work. For tracking the frequency of the best overall efficiency,
gradient-ascent algorithms have been implemented in other works (e.g. [7]). However, determining the acoustic power at the speaker surface requires precise knowledge about the speaker state. So far, acceleration sensors or expensive displacement sensors have been used by others [9]. We assume that, provided an acoustic driver model, pressure wave measurements and knowledge about the input voltage and current through the driver, it should be possible to approximate the speaker state by Recursive Estimation. All required measurement instrumentation has been provided in this work. The speaker model could either be obtained by System Identification or by modeling. A starting point could be the well-known Thiele-Small speaker model [34], for which the parameters can be estimated from impedance measurements by least-squares. An open-loop algorithm for speaker state estimation close to the acoustic resonance was given in [9], and it is likely that this could be extended to a recursive algorithm and to acoustic power estimation.
Appendix A

Some Engineering Aspects about the System

A.1 Collection of some Pitfalls

A.1.1 Matlab/Simulink and the National Instruments USB-6356 DAQ Card

Problem Description

It was tried to use the National Instruments USB-6356 DAQ card in combination with Mathwork’s MATLAB/Simulink to acquire data and to generate analog output signals. However, when a signal generation block (like a discrete sine wave) was connected to the analog output block of Simulink’s DAQ library, some errors occurred: the generated output was not continuous and showed an erratic behavior. Periodic breaks of one to two seconds occurred in the output signal. This could partly be fixed by introducing a large buffer-block before the analog output block. But then, this buffer introduced a considerable, non-deterministic time-delay into the closed-loop system, which made it difficult to tune the controllers. Additionally, even with a large enough buffer, the signal generation only worked randomly: approximately in one of five trials using Simulink Accelerator Mode, which compiles the system in order to speed up execution. When using Normal Mode, the simulations did not run at all.

Insights

The root cause of this discontinuous output behavior was that the USB-6356’s buffer was not filled fast enough. On our platform, the Simulink simulation was not able to keep pace with the required data output rate. So either the simulation should have been sped up dramatically or the data output rate should have been lowered. Both solutions were unrealistic – the latter particularly because high
Appendix A. Some Engineering Aspects about the System

Sampling rates were required in order to capture the sound waves with sufficient resolution. The fundamental problem was that the analog input/output block acquired data in real-time. However, the Simulink model would not run at real time. There would always be a significant delay (depending on the buffer size) between the input and the output. Things got complicated here since the delay was non-deterministic – this was due to the fact that the operating system itself was not real-time, and moreover Simulink was not designed for achieving tasks like this. In Simulink, both the DAQ toolbox and NI USB devices were not supported for real-time applications to date.

Another reason for the erratic behavior was that the I/O device’s buffer was not cleared between different simulations in accelerator mode. It turned out that this was an unknown bug in Simulink. At each run, the .mexw32 file generated by the compiler had to be deleted manually before re-compiling the simulation.

Solutions

The easiest solution was to use the NI-USB-6356 board with NI LabView instead of Simulink. Still, there was a non-deterministic latency of about 0.2 seconds between signal generation in the PC and execution on the board, due to the USB bus. When using USB boards, it is advised to keep the programs small in order to execute smoothly. Because of the high required sample rates, larger programs might fail. Another option is to buy a real-time capable PCI card, for example the NI PCI-6251 (or better). I/O cards like that will work with both Simulink and LabView.

A.2 Some Mechanical Design Suggestions

A.2.1 Commercially Available Linear Alternators for Thermoacoustic Applications

Despite its low electro-acoustic conversion efficiency, a commercial HiFi speaker was considered as the best choice for a first laboratory-scale TAR design. The following section outlines why the considered TAR design cannot be combined with a commercially available linear motor for thermoacoustic devices, e.g. [35].

Linear motors in thermoacoustic applications can reach a very high conversion efficiency – up to almost 90% – by exploiting electro-mechanical-acoustical resonance. However, most commercially available acoustic drivers are designed to run at frequencies $< 100$ Hz in a pressurized environment, where strong gas-spring forces on the piston faces can be exploited to improve efficiency. In an ambient-pressure environment, gas-spring forces will generally be small, so achieving comparably high operating frequencies like 170 Hz at high drive ratios is challenging. For instance, consider that a D=3% pressure wave is to be reached against at-
mospheric pressure. That implies a pressure amplitude of 3000 Pa, which will not generate a large spring force. For example, the natural spring force of the QDrive 1s102D motor \[35\] is 33 kN/m and the moving mass is about 0.5 kg. Thus, to reach an operating frequency of 170 Hz, a large additional spring stiffness would be required. To obtain this from a 3000 Pa pressure wave, a piston diameter about half a meter would be required. Constructing a gas spring with this diameter does not make sense.

It should be kept in mind that achieving high drive ratios would be required if nonlinear phenomena like strong higher order harmonics or conditional turbulence were to be examined. As the resonator is designed in a highly modular way, the TAR from this work can be upgraded to better drivers easily. A common way is to “tune” a commercially available HiFi speaker by supplementary construction measures, like adding a mechanical spring or a rigid metal plate to replace the speaker cone. Indeed, in \[13\], a detailed procedure for fitting a loudspeaker to a given acoustic system is described. In a modified acoustic driver, a better way to attach the accelerometer could be provided.

### A.2.2 Resonator and Stack Issues

**Reducer Cone**

One mechanical part of the resonator that could still be improved is the reducer cone, which is shown in the engineering drawing on page 78. First, its front end should be designed a little thicker in order to minimize heat leakage. Second, the cutouts which are designed to pass through the sensor and power resistor cables, should be sealed properly with large O-rings. In the current setup, provisional rubber bands were used as sealing, but they are insufficient. Air leaks through the cutouts at high driving ratios.

**Heat Exchangers**

It turned out that Selective Laser Melting is an excellent way to manufacture arbitrary heat exchanger geometries, but it also exhibits a few disadvantages: the metal is a little bit brittle and difficult to finish. Additionally, surface roughness seems to vary drastically among different manufacturers. In the end, the ones we got were rougher than expected, especially at the inner surfaces. This could be responsible for a significant amount of the overall losses in the resonator.
Appendix B

Code Fragments

B.1 DeltaEC Code

```
TITLE Thermoacoustic Refrigerator Final Design Simulation

BEGIN
1.0000E+05 a Mean P Pa
170.00 b Freq Hz G
292.71 c TBeg K G
216.73 d |p| Pa G
0.0000 e Ph(p) deg
0.0000 f |U| m^3/s
0.0000 g Ph(U) deg
air Gas type

1
SURFACE Back End of Driver Housing
sameas 3a a Area m^2 216.73 A |p| Pa
0.0000 B Ph(p) deg
3.6469E-07 C |U| m^3/s
180.00 D Ph(U) deg
0.0000 E Htot W
-3.9521E-05 F Edot W

ideal Solid type

2
DUCT Cylindrical Driver Housing
sameas 3a a Area m^2 Mstr 206.25 A |p| Pa
0.2629 b Perim m 2a 1.9547E-02 B Ph(p) deg
0.1000 c Length m 8.9647E-04 C |U| m^3/s
5.0000E-04 d Srough -90.126 D Ph(U) deg
0.0000 E Htot W
-2.3516E-04 F Edot W

ideal Solid type

3
VESPEAKER Acoustic Driver: Monacor MSH1115HQ
5.5000E-03 a Area m^2 3000.0 A |p| Pa
5.8000 b R ohms -90.148 B Ph(p) deg
```
Appendix B. Code Fragments

5.3000E−04 c L H 8.9142E−04 C |U| m^3/s
5.3500 d BLProd T−m −90.148 D Ph(U) deg
5.8000E−03 e M kg 28.959 E Htot W
2000.0 f K N/m 1.3371 F Edot W
0.0000 g Rm N−s/m 28.959 G WorkIn W
18.852 h |V| V G 18.852 H Volts V
93.746 i Ph(V) deg G 3007.7 K |Px| Pa
ideal Solid type 3.0858 I Amps A

RPN Cooling water tube removes 70 percent of loudspeaker heat
!We assume that 70 percent of the loudspeakers waste heat are absorbed by
!its housing. Subsequently, the heat exchangers on the ambient
temperature duct absorb it.

0.7000 a G or T 9.6237 A ChngeMe
9.6237 A ChngeMe
Htot Edot − inp * sto Htot rcl − =H2k

RPN Enforce Resonance
!Force the phase of U and p to be equal
0.0000 a G or T =5A 0.0000 A null

RPN Enforce desired resonance frequency
!170 Hz as resonance frequency provide the best trade−off considering the
!stack−manufacturing capabilities and overall device length. By setting this
!as target, the correct corresponding length of the cold duct can be
!computed as a guess.

170.00 a G or T =6A 170.00 A null

RPN Enforce desired Drive Ratio
!Enforcing the Drive Ratio here makes it possible to guess the therefore
!required input voltage for the speaker

3000.0 a G or T =7A 3000.0 A |p|

RPN Insert the radius of the ambient temperature duct here
!Counter−check the resulting diameter of the cold duct!

4.2000E−02 a G or T 5.5418E−03 A A
4.2000E−02 a G or T 5.5418E−03 A A
4.0000E−02 c Length m 5.1451E−03 C |U| m^3/s
5.0000E−04 d Srough −170.36 D Ph(U) deg
ideal Solid type 1.3225 F Edot W

DUCT Ambient temperature duct
sameas 8A a Area m^2 2976.6 A |p| Pa
sameas 8B b Perim m −90.303 B Ph(p) deg
4.0000E−02 c Length m 5.1451E−03 C |U| m^3/s
5.0000E−04 d Srough −170.36 D Ph(U) deg
ideal Solid type 1.3225 F Edot W
!------------------------------------------------------------ 10
RPN  Particle Displacement (after speaker)
!Counter-check if the acoustic driver is capable of delivering this
!displacement.

RPN  Particle Displacement (after speaker)

0.0000  a G or T  8.6920E-04  A ChngeMe
9C 2  / pi / 0b / 9a /

HX  Ambient temperature heat exchanger
sameas  9a  a Area  m^2  2963.2  A |p|  Pa
  6.0000E-03  c Length  m  5.7325E-03  C |U|  m^3/s
sameas  12d  d y0  m  -172.16  D Ph(U)  deg
   -11.802  e HeatIn  W  G  -2.1782  E Htot  W
   290.00  f SolidT  K  =11H

HX  Ambient temperature heat exchanger

sameas  9a  a Area  m^2  2963.2  A |p|  Pa
  6.0000E-03  c Length  m  5.7325E-03  C |U|  m^3/s
sameas  12d  d y0  m  -172.16  D Ph(U)  deg
   -11.802  e HeatIn  W  G  -2.1782  E Htot  W
   290.00  f SolidT  K  =11H

HX  Cold heat exchanger
sameas  8A  a Area  m^2  2709.3  A |p|  Pa
  3.0000E-02  c Length  m  1.0234E-02  C |U|  m^3/s
sameas  12d  d y0  m  -177.39  D Ph(U)  deg
   2.1782  e HeatIn  W  G  -1.0214E-14  E Htot  W
   275.13  f SolidT  K  =13H

HX  Cold heat exchanger

sameas  8A  a Area  m^2  2709.3  A |p|  Pa
  3.0000E-02  c Length  m  1.0234E-02  C |U|  m^3/s
sameas  12d  d y0  m  -177.39  D Ph(U)  deg
   2.1782  e HeatIn  W  G  -1.0214E-14  E Htot  W
   275.13  f SolidT  K  =13H

HX  Cold heat exchanger

sameas  8A  a Area  m^2  2709.3  A |p|  Pa
  3.0000E-02  c Length  m  1.0234E-02  C |U|  m^3/s
sameas  12d  d y0  m  -177.39  D Ph(U)  deg
   2.1782  e HeatIn  W  G  -1.0214E-14  E Htot  W
   275.13  f SolidT  K  =13H

HX  Cold heat exchanger

sameas  8A  a Area  m^2  2709.3  A |p|  Pa
  3.0000E-02  c Length  m  1.0234E-02  C |U|  m^3/s
sameas  12d  d y0  m  -177.39  D Ph(U)  deg
   2.1782  e HeatIn  W  G  -1.0214E-14  E Htot  W
   275.13  f SolidT  K  =13H

HX  Cold heat exchanger

sameas  8A  a Area  m^2  2709.3  A |p|  Pa
  3.0000E-02  c Length  m  1.0234E-02  C |U|  m^3/s
sameas  12d  d y0  m  -177.39  D Ph(U)  deg
   2.1782  e HeatIn  W  G  -1.0214E-14  E Htot  W
   275.13  f SolidT  K  =13H

HX  Cold heat exchanger

sameas  8A  a Area  m^2  2709.3  A |p|  Pa
  3.0000E-02  c Length  m  1.0234E-02  C |U|  m^3/s
sameas  12d  d y0  m  -177.39  D Ph(U)  deg
   2.1782  e HeatIn  W  G  -1.0214E-14  E Htot  W
   275.13  f SolidT  K  =13H

HX  Cold heat exchanger

sameas  8A  a Area  m^2  2709.3  A |p|  Pa
  3.0000E-02  c Length  m  1.0234E-02  C |U|  m^3/s
sameas  12d  d y0  m  -177.39  D Ph(U)  deg
   2.1782  e HeatIn  W  G  -1.0214E-14  E Htot  W
   275.13  f SolidT  K  =13H
Appendix B. Code Fragments

\[ \text{inp } 0.54 \ast ; \text{inp } 0.54 \ast 2 \ast \pi \ast ; \text{inp } 0.54 \ast \text{sqrd } \pi \ast \]

\[ \text{RACT Cold Duct} \]
\[ \text{sameas 15A a Area } m^2 \ 86.569 \ A |p| \ Pa \]
\[ \text{sameas 15B b Perim } m \ 96.971 \ B \ Ph(p) \ deg \]
\[ 0.2406 \ c \ Length \ m \ G \ 1.4651E-02 \ C \ |U| \ m^3/s \]
\[ 5.0000E-04 \ d \ Srough \ -177.92 \ D \ Ph(U) \ deg \]
\[ -1.0214E-14 \ E \ Htot \ W \]
\[ \text{ideal Solid type} \]
\[ 5.4070E-02 \ F \ Edot \ W \]

\[ \text{CONE End cone} \]
\[ \text{The geometric properties are set by manufacturing constraints and the fact \that the cone's half-angle should be below 10 degrees.} \]
\[ \text{sameas 16a a AreaI } m^2 \ 1230.4 \ A |p| \ Pa \]
\[ \text{sameas 16b b Perimi } m \ 92.130 \ B \ Ph(p) \ deg \]
\[ 0.2500 \ c \ Length \ m \ 5.3000E-03 \ C \ |U| \ m^3/s \]
\[ \text{sameas 18A d AreaF } m^2 \ -177.96 \ D \ Ph(U) \ deg \]
\[ -1.0214E-14 \ E \ Htot \ W \]
\[ \text{ideal Solid type} \]
\[ 4.9568E-03 \ F \ Edot \ W \]

\[ \text{RPN Ins. end rad of cone. Calc Area and Perimeter} \]
\[ \text{Manufacturing constraint: maximal radius 54mm} \]
\[ 5.4000E-02 \ a \ G \ or \ T \ 9.1609E-03 \ A \ CrossA \]
\[ 0.33929 \ B \ Perim \]
\[ \text{inp 2 } \ast \pi \ast; \text{inp sqrd } \pi \ast \]

\[ \text{DUCT Compliance Part 1 \shortend} \]
\[ \text{sameas 18a a Area } m^2 \ 1246.1 \ A |p| \ Pa \]
\[ \text{sameas 19a b Perim } m \ 92.127 \ B \ Ph(p) \ deg \]
\[ 2.5000E-02 \ c \ Length \ m \ 3.1322E-03 \ C \ |U| \ m^3/s \]
\[ 5.0000E-04 \ d \ Srough \ -177.96 \ D \ Ph(U) \ deg \]
\[ -1.0214E-14 \ E \ Htot \ W \]
\[ \text{ideal Solid type} \]
\[ 2.9826E-03 \ F \ Edot \ W \]

\[ \text{RPN Calc. parameters of compliance Volume} \]
\[ \text{sameas 18a a G or T} \ 3.2946E-04 \ A \ ChngeMe \]
\[ 1.8322E-02 \ B \ ChngeMe \]
\[ 20a sqrd 2 \ast \pi \ast; \text{20a 3 }-0.666 \ast \pi \ast \]

\[ \text{COMPLIANCE Compliance Part 2 \shortend} \]
\[ 1.3343E-02 \ a \ SurfAr \ m^2 \ 1246.1 \ A |p| \ Pa \]
\[ \text{sameas 20A b Volume } m^3 \ 92.127 \ B \ Ph(p) \ deg \]
\[ 5.6041E-18 \ c \ |U| \ m^3/s \]
\[ -148.35 \ D \ Ph(U) \ deg \]
\[ -1.0214E-14 \ E \ Htot \ W \]
\[ -1.7207E-15 \ F \ Edot \ W \]

\[ \text{HARDEND Boundary conditions} \]
\[ 0.0000 \ a \ R(1/z) \ =22G \ 1246.1 \ A |p| \ Pa \]
\[ 0.0000 \ b \ I(1/z) \ =22H \ 92.127 \ B \ Ph(p) \ deg \]
0.0000 c Htot W =22E 5.6041E−18 C |U| m^3/s
−148.35 D Ph(U) deg
−1.0214E−14 E Htot W
−1.7207E−15 F Edot W
−1.0197E−16 G R(1/z)
1.8004E−16 H I(1/z)

RPN CoP of the Thermoacoustic stack
13e 12F /

RPN Overall cooling power is net power plus tail dissipation
13e 13F +

RPN COP vs Carnot's COP
24A 3F / 13H 11H 13H − / /

RPN Total device length
9c 11c + 12c + 13c + 14c + 16c + 17c + 18a +

RPN Cooling Power/ Electrical Input Power
13e 3G /

RPN Overall cooling power is net power plus tail dissipation
13e 13F +
Appendix C

Engineering Drawings

The following mechanical engineering drawings are included:

- Assembly drawing in isometric perspective (p. 70)
- Assembly drawing in section perspective with pressure sensor positions and overall dimensions (p. 71)
- The housing cover (p. 72)
- The loudspeaker housing (p. 73)
- The adapter plate which connects loudspeaker, loudspeaker housing and warm duct (p. 74)
- The warm duct (pp. 75f.)
- The stack holder (p. 77)
- The reducer cone (p. 78)
- The cold duct elements (p. 79)
- Part 1 of the diverging cone (p. 80)
- Part 2 of the diverging cone (p. 81)
- The compliance front end (p. 82)
- The custom-made PVC screw for sealing the resonator (p. 83)
- The mounting rail for mounting the resonator into the 19" rack (p. 84)

No 2D-drawings of the stack and the heat exchangers were made. CAD (step)-files and/or additional views of other parts can be obtained from the author upon request.
Material: Aluminium
All dimensions in [mm]

<table>
<thead>
<tr>
<th>Designed by</th>
<th>Checked by</th>
<th>Approved by</th>
<th>Date</th>
<th>Date</th>
</tr>
</thead>
<tbody>
<tr>
<td>Marius Gritthaler</td>
<td></td>
<td></td>
<td></td>
<td>13.08.2013</td>
</tr>
</tbody>
</table>

IIA ETHZ
Thermoacoustic Resonator
Housing Cover
Edition | Sheet
1 / 1
Material: Aluminium
All dimensions in [mm]

- 30° cut-ins with distance 3mm
- 4x M4 (10mm)

Design: IFA ETHZ
Thermoacoustic Resonator
Loudspeaker Housing
Edition 1
Sheet 1/1
Material: Aluminium
All dimensions in [mm]

IITA ETHZ Thermoacoustic Resonator
Adapter Plate Edition Sheet

Designed by: Markus Gritthaler
Checked by: Approved by: Date: Date: 13.08.2013

R1, polished
Countersink for M3 screw head
Material: Aluminium
All dimensions in [mm]
All milled radii R7
Polish inner surfaces

I1A ETHZ
Thermoacoustic Resonator
Warm Duct
Edition
Sheet 1/2

Designed by: Markus Grithaler
Checked by: 13.08.2013
Approved by: 
Date: 

A-A (1:1)

PRODUCED BY AN AUTODESK EDUCATIONAL PRODUCT
Threads for mounting of external heat exchangers - in similar fashion at the opposing surface
Material: PVC
All dimensions in [mm]
All chamfers 0.3 mm
Material: PVC
All dimensions in [mm]
Roughness of inner surfaces as low as possible
All chamfers 0.5 mm, no chamfers along the gas flow
1) Required number of items: 2
   Please screw the two ducts together before milling of the plain surface for pressure sensor placement
2) Material: PVC
3) All dimensions in [mm]
4) Roughness of inner surfaces as low as possible
5) Avoid chamfers at the duct’s inner surface

- M5x1.5-0.9
- M5x1.5-6H
- G 1/4-19
- G 1/4-19
- G 1/4-19
- M5x1.5-6H
- G 1/4-19
- G 1/4-19
- G 1/4-19
- M5x1.5-6H

Thermoacoustic Resonator

IIA ETHZ
Cold Duct

Date: 11.08.2013
Material PVC
Roughness of inner surfaces as low as possible
All dimensions in [mm]

- Designed by: Markus Glithofer
- Checked by: 
- Approved by: 
- Date: 09.08.2013
Material: PVC
Roughness of inner surfaces as low as possible
All dimensions in [mm]
Material: PVC
Roughness of inner surfaces as low as possible
All dimensions in [mm]
Required no. of items: 8
Material: PVC
All dimensions in [mm]

IfA ETHZ
Thermoacoustic Resonator
PVC Screw

Designed by
Markus Gifthaler

Checked by

Approved by

Date

Date

12.08.2013

Edition

Sheet

1 / 1
35mm x 15 mm St-35 Steel Rectangular Tube
Appendix D

Experiment Settings

D.1 For Section 4.1:

Table D.1: Experiment Data for Section 4.1:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>generation sampling frequency</td>
<td>20 kHz</td>
</tr>
<tr>
<td>acquisition sampling frequency</td>
<td>20 kHz</td>
</tr>
<tr>
<td>acquired block size</td>
<td>60000 samples</td>
</tr>
<tr>
<td>transient decay time(^1)</td>
<td>1 s</td>
</tr>
<tr>
<td>frequency range</td>
<td>100 Hz - 260 Hz</td>
</tr>
<tr>
<td>frequency spacing</td>
<td>1 Hz</td>
</tr>
<tr>
<td>electric input power</td>
<td>10 W</td>
</tr>
<tr>
<td>external heat load</td>
<td>off</td>
</tr>
<tr>
<td>cooling water temperature</td>
<td>23-24 °C</td>
</tr>
<tr>
<td>environmental temperature</td>
<td>23.2-23.4 °C</td>
</tr>
</tbody>
</table>

D.2 For Section 4.2:

Table D.2: Experiment Data for Section 4.2.1:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>generation sampling frequency</td>
<td>20 kHz</td>
</tr>
<tr>
<td>transient decay time(^2)</td>
<td>30 min</td>
</tr>
<tr>
<td>frequency range</td>
<td>150 Hz - 190 Hz</td>
</tr>
<tr>
<td>frequency spacing</td>
<td>5 Hz</td>
</tr>
<tr>
<td>mechanical power(^3)</td>
<td>0.5 W</td>
</tr>
<tr>
<td>external heat load</td>
<td>1 W, 2 W</td>
</tr>
<tr>
<td>cooling water temperature</td>
<td>23-24 °C</td>
</tr>
<tr>
<td>environmental temperature</td>
<td>22-23.5 °C</td>
</tr>
</tbody>
</table>

\(^1\)Time delay between start of data generation and start of data acquisition
Table D.3: Experiment Data for Section 4.2.2:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>generation sampling frequency</td>
<td>20 kHz</td>
</tr>
<tr>
<td>transient decay time</td>
<td>30 min</td>
</tr>
<tr>
<td>frequency range</td>
<td>120 Hz - 180 Hz</td>
</tr>
<tr>
<td>frequency spacing</td>
<td>5 Hz</td>
</tr>
<tr>
<td>electrical input power</td>
<td>30 W</td>
</tr>
<tr>
<td>external heat load</td>
<td>1 W, 2 W, 3 W, 4 W</td>
</tr>
<tr>
<td>cooling water temperature</td>
<td>23-24 °C</td>
</tr>
<tr>
<td>environmental temperature</td>
<td>21.5-23 °C</td>
</tr>
</tbody>
</table>

D.3 For Section 4.3:

Table D.4: Experiment Data for Section 4.3:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>generation sampling frequency</td>
<td>20 kHz</td>
</tr>
<tr>
<td>frequency</td>
<td>140 Hz</td>
</tr>
<tr>
<td>electrical input power</td>
<td>50 W</td>
</tr>
<tr>
<td>external heat load</td>
<td>off</td>
</tr>
<tr>
<td>cooling water temperature</td>
<td>23-24 °C</td>
</tr>
<tr>
<td>environmental temperature</td>
<td>23.2 °C</td>
</tr>
</tbody>
</table>

D.4 For Section 5.2.1:

Table D.5: Experiment Data for Section 5.2.1:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>generation sampling frequency</td>
<td>30 kHz</td>
</tr>
<tr>
<td>acquisition sampling frequency</td>
<td>30 kHz</td>
</tr>
<tr>
<td>acquired block size</td>
<td>300000 samples</td>
</tr>
<tr>
<td>transient decay time</td>
<td>1 s</td>
</tr>
<tr>
<td>number of datasets averaged</td>
<td>7</td>
</tr>
<tr>
<td>frequency range</td>
<td>110-190 Hz</td>
</tr>
<tr>
<td>frequency spacing</td>
<td>1/3 Hz</td>
</tr>
<tr>
<td>electrical input power</td>
<td>30 W</td>
</tr>
<tr>
<td>external heat load</td>
<td>off</td>
</tr>
<tr>
<td>cooling water temperature</td>
<td>21-22 °C</td>
</tr>
<tr>
<td>environmental temperature</td>
<td>20.8-21 °C</td>
</tr>
</tbody>
</table>

\(^2\)Time delay between start of wave generation and acquisition of the temperature.

\(^3\) Measured with the accelerometer
Bibliography


