Stochastic dosimetry to manage uncertainty in numerical EMF exposure assessment

Stochastic Dosimetry to manage Uncertainty in Numerical EMF Exposure Assessment

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***Abstract*— In this paper we propose a novel approach which combines computational electromagnetics with statistics to statistically characterize the variations of the Radio Frequency (RF) exposure induced by inputs and affected by variability or uncertainty. Conventional numerical techniques such as the Monte Carlo Method, typically used to solve such a problem, are not useful in this case from a practical point of view since the computation time needed to assess the effect of the exposure is inordinately long for this type of problem. This novel approach consists of characterizing the statistical distribution of the output using a surrogate model which is employed in the numerical method. The bottleneck is encountered in the process of building a surrogate model by using a parsimonious approach, because an extensive set of computations are required by the Finite Difference in Time Domain (FDTD) method, despite the fact that the FDTD is a proven computationally efficient technique for modeling problems in bio-electromagnetism. The proposed method employs a truncated Generalized Polynomial Chaos Expansion scheme in conjunction with regression and Least Angle Regression (LARS) algorithms to identify the polynomial which has a significant influence and then to calculate the polynomial coefficient. The accuracy assessment of the surrogate model is carried out with the Leave-One-Out Cross Validation (LOOCV). In this paper this method is used to characterize the variation of the Specific Absorption Rate (SAR) induced in the head by a mobile phone having a variable position relative to the head.**

***Index Terms*— Human exposure, Specific Absorption Rate (SAR), Dosimetry, radiofrequency (RF), Finite Difference in Time Domain (FDTD), Generalized Polynomial Chaos Expansion, Least Angle Regression, Leave One Out Cross Validation.**

I. INTRODUCTION

Radio communication and wireless systems are part and parcel of our everyday life in the society in which we live today. Over the past three decades, we have witnessed an exponential increase in the use of smart mobile phones, tablets and home wireless LANs, and the emergence of all-pervasive wireless communication systems, such as body health monitoring sensors and large machine-to-machine communication networks, are contributing even further to the dizzying pace of this growth.

While the wireless communication systems have significantly affected our daily lives, they have also raised public concerns about possible health-related impacts of the exposure to electromagnetic fields (EMFs), in particular those linked to radio frequencies (RFs). Although, to-date no health risks have yet been proven [1] the risk perception is nevertheless regarded as important and the EMF exposure assessment of wireless devices and networks remains a key issue to be reckoned with for the deployments and operations of wireless networks.

During the past decades computational electromagnetics has taken advantage of the progress in computer technology and high performance computing. Large problems that were considered to be quite unmanageable even just ten years ago can be solved quite easily nowadays. Furthermore, significant acceleration of electromagnetic simulations can now be achieved by taking advantage of the availability of graphical processor units (GPUs) that did not exist only a few years ago. Given the availability of such enablers, computational electromagnetics schemes based on the Finite Difference Time Domain (FDTD) method in bio-electromagnetism has achieved great success and is being intensively used to not only design wireless systems but to also assess the Specific Absorption Rate (SAR) which quantifies the level of human exposure. Success in fabricating realistic human phantoms, including child models [2] that have details with millimeter-level resolutions, have encouraged the use of extensive simulations in engineering design of mobile devices and wireless systems.

Previous works have shown that human exposure depends on a large number of parameters, which can be variables and can be affected by uncertainties. For example, the dielectric properties used in the FDTD simulations are variables [3] [4] because of human variability and uncertainties linked to the measurement system; furthermore, the morphologies and human postures are also highly variable; furthermore, the anatomy evolves with age and this too contributes to the

variation. Last but not least, the sources of variability of the exposure induced by the wireless communication devices include the locations of the RF sources; the frequency bands that depend on their versatile use; and, on the technology as well.

The characterization of such variability requires a large number of simulations if we use conventional methods, such as the Monte Carlo method. Despite the fact that considerable progress has recently been made toward high performance calculation, as mentioned previously, the FDTD algorithm is still highly computer-intensive and, consequently, not entirely compatible with Monte Carlo methods, at least from a practical point of view, since the computation time required by the FDTD/Monte Carlo simulation is totally unrealistic. This, in turn, prompts us to seek an innovative approach to circumvent this problem and we describe below a novel method which combines computational electromagnetic with statistic tools towards this end. The technique is based on the use of a surrogate model, which substitutes the numerical method—for example the FDTD in the present application—to characterize the statistical distribution of the output. The bottleneck in this approach arises from the fact that it requires us to build an alternative model by using a parsimonious approach, which is considerably more time- and memory-efficient than the Finite Difference in Time Domain (FDTD) method, which is widely used in bio-electromagnetic type of simulations. We will use this new approach to characterize the variation of the Specific Absorption Rate (SAR) induced in the head by a mobile phone whose position varies statistically relatively to the head.

The paper is organized as follows. In Section II, the Polynomial Chaos approximation of the random model response is presented. Section III, details the construction of the surrogate modeling based on planning experiments and truncated Generalized Polynomial Chaos Expansion. Section IV includes some numerical examples, which demonstrate the accuracy and the numerical efficacy of the proposed technique. Finally, some concluding remarks are provided in Section V.

II. POLYNOMIAL CHAOS METHOD

In the past, a number of works [5][6][7][8] have introduced methods based on stochastic finite elements with applications in mechanical and structural engineering and only very recently the method has been discussed in the context of computational electromagnetics in [9]. Polynomial Chaos (PC) expansion [8] method can be divided into two basic categories. The first of these is an intrusive method which requires that the governing equations of the solver algorithm be modified, and such manipulations can be very complex. In contrast to this, the PC expansion methods belonging to the second category only employ non-intrusive methods and use the solvers solely as black boxes without modifying them. The complexity of intrusive approaches explains the increasing attention that has been given to the non-intrusive methods that are easily generalizable since using solvers as black boxes.

The non-intrusive approaches can themselves be subdivided into two categories. The first one is these is based on the use of stochastic collocation approaches in which the polynomial approximation is constrained to precisely fit the model response at a suitable point set, and they rely upon well-established methods based on the Lagrange polynomial interpolation technique. The second category is comprised of spectral methods in which the polynomial chaos coefficients are estimated by using either spectral projections or least-square regressions. In this paper we will use these spectral methods to analyze the variations of the outputs of a dosimetric problem induced by the variations of the inputs.

Consider a mathematical model $\mathcal{M}$ which has M inputs and generates the output $\mathrm{y} = \mathcal{M}(\mathbf{x})$, and whose inputs $\mathbf{x} = \{x_1, x_2, \dots x_i .. x_M\}$ are affected by possible random variations or uncertainties. (In this paper we assume that $\mathcal{M}$ is given by the FDTD calculations. We denote the probability space by $(\Omega, \mathcal{F}, \mathcal{P})$, where Ω is the event space equipped with σ-algebra $\mathcal{F}$ and a probability measure $\mathcal{P}$. Note that $\mathcal{L}^2(\Omega, \mathcal{F}, \mathcal{P}, \mathbb{R})$ is the space of squared integrable real valued functions in which the inner product is defined by:

$$\langle \psi(\boldsymbol{X}), \phi(\boldsymbol{X}) \rangle = E\big(\psi(\boldsymbol{X})\phi(\boldsymbol{X})\big) = \int_{\mathbb{X}} \psi(\boldsymbol{X})\phi(\boldsymbol{X}) p_{\boldsymbol{X}}(\boldsymbol{x}) d\boldsymbol{x}$$

where E is the expectation operator. Using this formalism and assuming that the components of the input random vector are independent, then if the random response of the physical phenomena, $\mathrm{Y}(\omega) = \mathcal{M}(\mathbf{X}(\omega))$, has a finite variance then Y can be described, as shown in [8], by using an infinite modal expansion usually referred to as the polynomial chaos expansion .

$$Y(\omega) = \sum_{\boldsymbol{\alpha} \in \mathbb{N}^M} \beta_{\boldsymbol{\alpha}} \psi_{\boldsymbol{\alpha}}\big(\boldsymbol{X}(\omega)\big) \qquad (1)$$

In the above polynomial expansion, α is the multi-index, β_α's are the coefficients of the polynomial expansion and ψ_α ′s are the multidimensional polynomials. These polynomials are constructed by tensorizing univariate polynomials $\left\{\pi_j^{(i)}, j \in \mathbb{N}\right\}$ and are defined as:

$$\psi_{\boldsymbol{\alpha}}(\boldsymbol{x}) = \prod_{i=1}^{M} \pi_{\alpha_i}^{(i)}(x_i) \qquad (2)$$

where $\boldsymbol{\alpha}$ denotes the M-uplet $(\alpha_1, \alpha_2, \dots, \alpha_M) \in \mathbb{N}^M$

The univariate polynomials comprise a family of orthonormal polynomials with respect to the margin probability density functions (PDF):

$$< \pi_j^{(i)}(X_i), \pi_k^{(i)}(X_i) > \; = \mathrm{E}\left(\pi_j^{(i)}(X_i), \pi_k^{(i)}(X_i)\right) = \delta_{j,k} \quad (3)$$

where $\delta_{j,k}$ is the Kronecker delta. Let us denote $\mathrm{f}_{\mathrm{X_i}}(\mathrm{x_i})$ as the marginal PDF of the random variable $\mathrm{X_i}$. The independence of the input random variables enables us to write the PDF of X as

$$\mathbf{f}_{\mathbf{X}}(\boldsymbol{x}) = \prod_{i=1}^{M} \mathrm{f}_{\mathrm{X_i}}(x_i) \qquad (4)$$

In engineering, inputs with uniform or Gaussian distributions are often used, and the corresponding polynomial families are the Legendre and Hermite Polynomials families, respectively.

The PC coefficients can be estimated by using spectral projections or via the use of least-square regressions. The "projection" approach takes advantage of the orthogonality of the chaos polynomials. Assuming that the polynomials are normalized, the coefficients $\beta_{\boldsymbol{\alpha}}$ can be derived by using a projection operation, in common with the procedure followed in the modal approach in electromagnetics.

Using (1), (2) and (4) the coefficients $\beta_{\boldsymbol{\alpha}}$ can be estimated by evaluating the integral that represents the projection of M(x) on $\psi_m(x)$

$$\beta_{\boldsymbol{\alpha}} = \int \mathcal{M}(\boldsymbol{x})\, \psi_{\boldsymbol{\alpha}}(\boldsymbol{x}) f_{\boldsymbol{X}}(\boldsymbol{x}) dx \tag{5}$$

Since the relationship between y and x ($note$: $y = \mathcal{M}(\mathbf{x})$) is only known through the FDTD simulations at this step, the integral calculation has to be done by using the Monte Carlo Method and a sparse grid [9],[10],[11] to reduce the computational cost. For instance for a problem with 4 inputs, the assessment of the coefficients of the polynomials having order below 4 requires 401 FDTD simulations. This approach is suitable if the complexity of the problem is not very high, as is the case when there are only a few polynomials involved in the PC expansion, or if our objective is to only assess the values of a few coefficients. Otherwise, for the case of dosimetry, for instance, the method is not really well suited; instead, as we will show in the following section the method based on least-square regressions and cross validation is more efficient as we will show in the following section.

III. SURROGATE MODELING USING TRUNCATED GENERALIZED POLYNOMIAL CHAOS EXPANSION

As pointed out in the previous section, the computational time required by the FDTD simulations is quite high, and this in turn precludes its direct use with conventional statistical schemes such as the Monte Carlo method for statistical dosimetry. On the other hand, a truncation of the Generalized Polynomial Chaos Expansion could be an efficient way to ease the computational burden if the assessment coefficient is performed using least-square regressions and cross validation that require a reasonable computational effort.

Let us consider a model $\hat{y}$, which is an alternative candidate for the model constructed by using a truncation of the infinite polynomial expansion (1):

$$\hat{Y} = \sum_{k=0}^{P-1} \beta_k \psi_k(\boldsymbol{X}) \tag{6}$$

where P is the number of polynomials involved in the truncation. The estimation of the coefficient β_k can be performed by using the well-known regression approach to computing the coefficients which minimize the mean-square error of approximation of the model response. Let us note that $\boldsymbol{y} = \{y^1, .., y^i, .. y^N\}$ is the vector representing the output of simulations driven by the M inputs, $\boldsymbol{x}^i = \{x_1^i, x_2^i, .. x_j^i, .. x_M^i\}$. As pointed out earlier, the components of the vector $\mathbf{y}$, $y^i = \mathcal{M}(\mathbf{x}^i)$ are derived by using the FDTD simulations.

Using the above notations, the coefficients derived by using the regression analysis can be written:

$$\widehat{\boldsymbol{\beta}} = \{\boldsymbol{\Psi}^T \boldsymbol{\Psi}\}^{-1} \boldsymbol{\Psi}^T \boldsymbol{y} \tag{7}$$

where $\boldsymbol{\Psi}$ is a matrix whose elements ψ_k^i that can be obatined numerically from the relationship $\psi_k^i = \psi_k(\boldsymbol{x}^i)$. Of course, not unexpectedly, the quality of the model depends n the truncation, and we will further discuss this point later. For now, we add the remark that it also depends on how well we have represented the vector pair $\{\boldsymbol{x}, \boldsymbol{y}\}$.

It is worthwhile to point out that the calculations indicated above should be performed in a way such that any bias is avoided. While we can use a fully random process for these computations, we cannot guarantee that the space filling would be uniform in this case. The Latin Hypercube Sampling approach[12][13], known as LHS, is often used for planning the experiments. This method generates a sample of plausible set of parameter values from a multidimensional statistical distribution while taking care of a uniform filling of space.

As stated previously, the complexity of the problem is unknown a priori; hence the accuracy of the model which we build by using a truncation approach needs to be tested and compared with a target value. To achieve this objective, we can use methods such as bootstrap [14] or Leave-One-Out Cross Validation (LOOCV) algorithms. The latter is a very intuitive method, which is often implemented to assess the accuracy of a model. Let us consider N experiments $y^i = \mathcal{M}(\mathbf{x}^i)$. One can use N-1 experiments to build a model and the last one experiment to test the accuracy of the model. Let us consider the FDTD outputs of N simulations, represented by $\boldsymbol{y} = \{y^1, .., y^i, .. y^N\}$, corresponding to the output the N sets of M inputs ($\boldsymbol{x}^i = \{x_1^i, x_2^i, .. x_j^i, .. x_M^i\}$). If we remove one element for this set, $\{y^1, .., y^i, .. y^N\} - \{y^j\}$, we can build M sets of vectors having M-1 elements that can be used to build a surrogate model. We can then use $\{y^j, \boldsymbol{x}^j\}$ to assess the difference between the prediction and the sample. Performing this operation for M samples enables us to assess the actual mean-square error of the model and to check its quality. If the accuracy is below our expectation, then we would introduce a new sample, via FDTD simulations, to build an advanced surrogate model.

Depending on the complexity of the physical problem, not all the polynomials involved in the polynomial chaos expansion would be equally important. Therefore, it is of great interest, from computational efficacy point of view, to iteratively build a sparse polynomial chaos expansion. The objective is to select the most important polynomials, taking into account the constraint of a constant cardinal of the polynomials basis. The sparse polynomial chaos, based on least angle regression method [15] and Least absolute shrinkage and selection operator method [16] (often used under the same "LARS"), are well adapted for engineering problems. They can be used to identify the polynomials having a significant influence on the sensibility indices and the statistical distribution of the output.

Among the methods that can be used to perform the sensitivity analysis which provides the relationship between

the uncertainty of the output and those of the inputs, the analysis of variance [17], known as ANOVA, with the "Sobol" decomposition [18] is often used. With this approach, the response of a process having finite variance and independent inputs can be decomposed into main effects and interactions, and the global variance can be decomposed into partial variances. Thanks to the orthonormality of the polynomial chaos basis, the global variance $\widehat{D}$ and the partial variance $\widehat{\mathrm{D}}_{\mathrm{i}_1,\ldots\mathrm{i}_\mathrm{s}}$, are respectively given by:

$$\widehat{D} = Var\left(\hat{Y}\right) = \sum_{k=0}^{N-1} \beta_k^2 \quad ; \quad \widehat{\mathrm{D}}_{\mathrm{i}_1,\ldots\mathrm{i}_\mathrm{s}} = \sum_{\alpha \in \mathcal{T}_{\mathrm{i}_1,\ldots\mathrm{i}_\mathrm{s}}} \beta_\alpha^2 \tag{8}$$

Where $\mathcal{T}_{i_1,\ldots i_s} = \{\alpha: \alpha_k > 0, , k \in (i_1, \ldots i_s)\}$

The Sobol indices are given by

$$S_{i_1,\ldots,i_s} = \frac{\sum_{\alpha \in \tau_{i_1,\ldots i_s}} \hat{\beta}_\alpha^2}{\sum_{0<|\alpha| \le p} \hat{\beta}_\alpha^2} \tag{9}$$

The total sensitivity indices S_i^T which quantify the total effect of an input parameter on the output are as the sum of all partial sensitivity indices given by:

$$S_i^T = 1 - \frac{\widehat{D}_{-i}}{\widehat{D}} \tag{10}$$

where $\widehat{\mathrm{D}}_{-\mathrm{i}}$ is defined as the sum of all $\widehat{\mathrm{D}}_{\mathrm{i}_1,\ldots\mathrm{i}_\mathrm{s}}$ that do not include the index i.

IV. APPLICATION IN NUMERICAL DOSIMETRY

The tissue exposure to a radio frequency (RF) electromagnetic field is quantified by using the specific absorption rate (SAR), which is defined as the ratio of the electromagnetic power absorbed by human tissues and the mass of these tissues. Locally the SAR is given by

$$\mathrm{SAR} = \frac{\sigma \mathrm{E}^2}{2\rho} \tag{11}$$

In this formulation σ, ρ and E represent, respectively, the conductivity of the body tissue (S/m), the mass density of the tissue (kg/m3) and the peak electric field strength in the tissue (V/m). International bodies such as ICNIRP or IEEE have defined limits that are quantifying exposure using the SAR over whole the body or over a mass of 10 grams. The electric field strength can be assessed either by using measurements or by employing numerical methods such as the FDTD, thanks to the progress in high performance computing. The FDTD enables us to consider specific tissues exposures, for instance exposure of brain tissues.

Considerable amount of effort has been invested during the last 20 years toward the development of experimental methods and measurement facilities that enable us to perform tests to ascertain that the safety limits of the devices are indeed within compliance prior to releasing these devices into the marketplace.

For mobile phones operating in close proximity of the human head, technical standards have been defined for both the methodologies as well as protocols involving two test positions, referred to as the cheek and tilt of the phones [16]. While following this approach is useful for carrying out compliance tests, the approach is not helpful for characterizing the distribution of the exposure linked to the phones whose use is variable, as is typically the case in practice.

As we have already pointed out earlier, despite significant recent progress in computational electromagnetics, the highly computer-intensive nature of the FDTD, it is not well suited for such characterization performed by using the Monte Carlo method. The PC can be used to overcome this limitation by using a surrogate model to analyze the influence of variable position of a phone on the SAR10g (i.e., the maximum SAR over 10 grams in the head) induced in the "duke" head model [19][20].

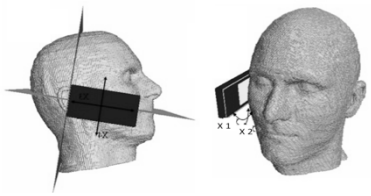

Fig. 1. Configurations used to assess the influence of the position of the phone on the head exposure

Figure 1 shows the configurations investigated in this work. The phone position relatively to the head is assumed to be governed by four uncertain and independent input parameters, $\mathrm{X}_1, \mathrm{X}_2, \mathrm{X}_3, \mathrm{X}_4$, namely rotations and translations, uniformly distributed over $[0°, 30°]$, $[-15°, +15°]$ and $[5\ mm, 30\ mm]$, $[-10\ mm, +10\ mm]$, respectively. The $\mathrm{X}_1, \mathrm{X}_2, \mathrm{X}_3, \mathrm{X}_4$ inputs are transferred to the range [-1, 1] by using iso-probabilistic functions [18], so that they can be used with PC and Legendre [19] polynomial types of basis. The LHS method has been used to select the suitable FDTD simulations to be carried out. Figure 2 shows the probability distribution function (PDF) of these simulations,

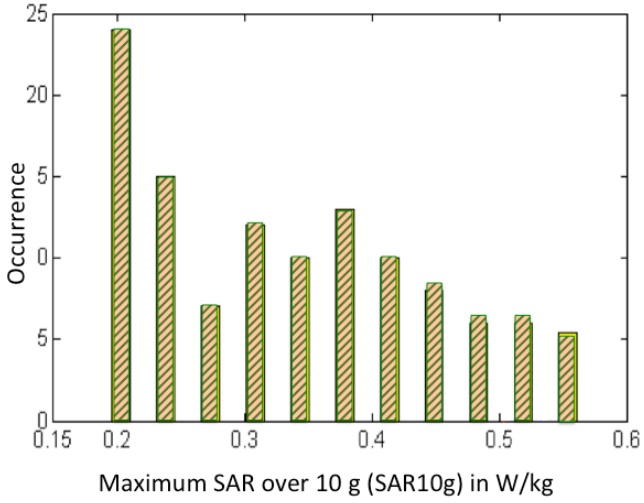

Fig. 2. PDF of FDTD simulations

The FDTD simulations have been used to construct a sparse and truncated PC expansion, first by using the hyperbolic index sets [23], and second by employing LARS [15]. Figure 3 shows that the PDF of SAR obtained by using these different surrogate models are equivalent even though, as explained previously, the computational effort associated with a non-sparse PC is significantly higher. The LARS is much more efficient than other sparse approaches. In the present example a LOOCV accuracy of 1% is obtained with a 7th order and 71 simulations (15 significant polynomial coefficients to compare with the 330 that are required when we use a full GPCE). We obtain a 0.1% accuracy with 103 simulations (78 simulations added iteratively to the 25 initial ones when using the NLHS) and, in this case, 27 polynomials are involved in the surrogate model.

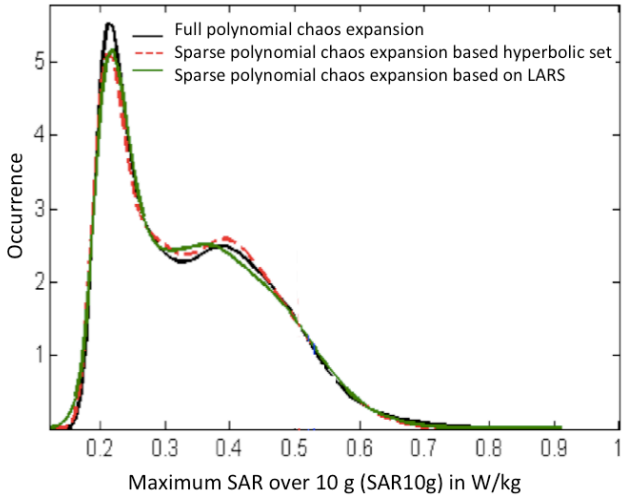

Fig.3: Pdf of SAR obtained using the surrogate model

Figure 3 is also of interest from an application point of view, since the shape of this PDF was not predictable before we carried out the study. A sensitivity analysis has been carried out to identify the important parameters.

The coefficients assessed with PC, LARS and LOOCV have been used to carry out the sensibility analysis. Figure 4 shows the Sobol indices. This sensitivity analysis shows that S1 S3, S12 and S13 are the most significant indices, and that the S1 index has a very strong influence. This serves to confirm the fact that the angle X_1 governing the angle toward the cheek is the most influential parameter. This analysis is important for RF dosimetry, as it helps to know on which parameters we should focus on when studying' the exposure to RF-EMF from mobile phones.

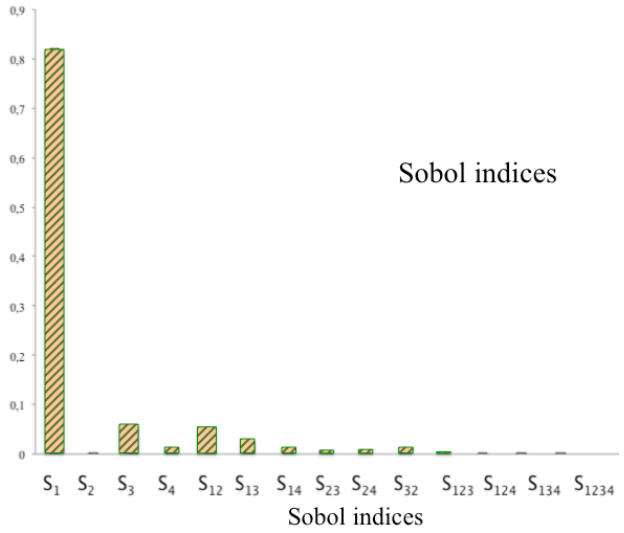

Fig 4. Sobol indices

V. Conclusions

This paper has presented a novel approach for efficiently assessment of the statistical distribution of the outputs of computer-intensive electromagnetic numerical calculations with inputs that are affected by variability. The proposed approach consists of building a meta-model based on the sparse Polynomial Chaos representation using LARS algorithm. This model is then employed to characterize the variation of the Specific Absorption Rate (SAR) induced in the head by a mobile phone whose position relatively to the head is variable. The proposed method enables us to significantly reduce the computational effort by performing fewer simulations number and carrying out a sensitivity analysis. The results show that the method is suitable for computational electromagnetics and that it has the potential to develop stochastic dosimetry techniques for complex configurations which cannot be handled by usual deterministic computational methods. Another example in which the technique described this work has been found useful has been described in a companion paper in which a Frequency Selective Surface (FSS) whose cells have random perturbations. The interested reader is referred to [24] for further information and details.

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