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Working Paper

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Publication date:
2007-02

Permanent link:
https://doi.org/10.3929/ethz-a-005354261

Originally published in:
Economics Working Paper Series 07/65

CER-ETH - Center of Economic Research at ETH Zurich

Economics Working Paper Series

ETH
Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

Knowledge Codification and Endogenous Growth*

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February 2007

Abstract

The usual models of endogenous growth treat knowledge codification as a by-product of R&D and as costless. In contrast to this, one can observe great efforts of private firms for the purposeful codification of knowledge. We incorporate costly knowledge codification in an overlapping generations framework of endogenous growth and show that the steady-state growth rate of capital being higher than that of the knowledge stock is a sufficient condition for knowledge codification. With decreasing codification costs, every overlapping generations economy will be codifying in the long run if the rate at which the costs decline is higher than or equal to the steady-state growth rate of knowledge.

Keywords: Knowledge, Human Capital, Knowledge Codification, Economic Growth.

JEL: O11, O30, O41.

*We would like to thank Clive Bell, Hans Gersbach, Benjamin Luenenbuerger, Christian Traeger and Ralph Winkler for valuable comments and suggestions. All errors remain my own.

1 Introduction

Knowledge codification is central to utilize the non-rivalry of ideas for economic growth. The usual models of economic growth treat knowledge codification as a by-product of R&D-activities and as costless. In contrast to this, one can observe great efforts by private firms for the purposeful codification of knowledge. The present paper develops a formal model of endogenous growth that incorporates knowledge codification as a means of intergenerational knowledge transfer. It identifies the circumstances under which knowledge codification takes place in the long run and studies its effects on long run economic development.

The empirical motivation for the paper are the following observations. Firstly, it is widely reported that the codified knowledge base in the world has increased rapidly over the last decades. For example, the European Union research project "Technology and Infrastructures Policy in the Knowledge-Based Economy" (TIPIK) has explicitly concentrated on "the fact that the rapid cumulative expansion of the codified knowledge-base is the salient characteristic of the development of a modern economy based on knowledge." It is argued that "an in-depth understanding of the incentives to codify,[...] [and] of the advantages and drawbacks of the codification of knowledge, is thus becoming essential for analyzing the process of innovation and growth of the economy" (European Commission, 2004, p. 3). Similarly, "OECD analysis is increasingly directed to understanding the dynamics of the knowledge-based economy and its relationship to traditional economics, as reflected in 'new growth theory'. The growing codification of knowledge and its transmission through communications and computer networks has led to the emerging 'information society'" (OECD, 1996, p. 3).

Secondly, codified knowledge is intentionally created, for example, within 'knowledge management' of private firms. Aoshima (2002) reports that Japanese automobile producers extensively use documentation on design know-how, testing results, and problematic and successful cases found in previous development activities as a means to store knowledge about past practices. Another example is Sandia National Laboratories, which conducted videotaped interviews to capture the extensive weapons design and testing expertise of their aging and retired nuclear weapons designers (Sandia National Laboratories, 1996). According to an empirical study by Edler (2003), 85% of the 497 firms from seven economic sectors which answered his questionnaire, stated that they are creating organizational memory by preparing written documentation such as lessons learned, training manuals, good work practice etc. In an online journal article, it is reported that 2.1 billion US $ have been spent in 2000 for knowledge management worldwide (Ball, 2002). At the end of the year 2000, a German consulting company estimated a market volume for "information portals" until 2002 of 6.8 billion Euro in Europe and of 1.61 billion Euro in the German market (Meta Group Deutschland GmbH, 2001).

Thirdly, one major motivation for knowledge codification within firms is to prevent the loss of knowledge of retiring employees. This was indicated by above's example

of Sandia National Laboratories conducting videotaped interviews. In Edler's study, 91% of the firms rated "to accelerate and improve the transfer of knowledge to new workers" one or two on a scale from 1 (extremely important) to 6 (not important at all) with regard to the motivations to use knowledge management. This was the most important motivation. The third most often mentioned reason for knowledge management was "to protect your firm or organization from loss of knowledge due to workers' departure" which 82% of responding firms rated one or two.
In fact numerous cases are reported where productive knowledge has been lost or is at risk of being lost. For example, Cowan et al. (2000) argue that "[...] where there are critical bodies of knowledge that are not kept in more-or-less continuous use, inadequate codification and archiving heightens the risks of 'accidental uninvention'." In a recent newspaper article, a spokeswoman of a German high tech firm expressed concerns about a third of the employees in their R&D-department being over 50 years of age and soon becoming eligible for retirement. She claims that especially in research, experience acquired over many years plays an important role as it is very costly if young researchers replicate failures that the old have had before (Astheimer, 2005). DeLong (2004) argues that due to an expected increase in retirements[1], "problems of poor documentation will become increasingly evident as more experienced employees leave behind badly flawed systems for preserving explicit knowledge about operations and the context surrounding important decisions. The implicit or tacit knowledge these veterans used to compensate for idiosyncratic documentation will be gone. What will be left will be a lot of unusable paper and electronic files" (DeLong, 2004, p. 91).

Fourthly, although some information may accrue as a by-product of research and development activities, for example, patent specifications or possibly the product itself, empirical investigations such as Levin (1986) and Mansfield et al. (1981) suggest that this is not sufficient for knowledge to flow freely. Instead substantial real resources are often required to imitate an innovation, even one entirely lacking legal protection. Levin (1986) concludes that public disclosure of a patent claim does not assure eventual diffusion of the knowledge required to make economic use of an innovation. The study of Mansfield et al. (1981) found that in a seventh of the cases imitation cost was no smaller than innovation cost. According to them, this was not due to any superiority of the imitative product over the innovation, but to the innovator's having a technological edge over its rivals in the relevant field. "Often this edge was due to superior 'know-how' - that is, better and more extensive technical information based on highly specialized experience with the development and production of related products and processes. Such know-how is not divulged in patents and is relatively inaccessible (at least for a period of time) to potential imitators" (Mansfield et al., 1981, p. 910). In line with this is Zucker et al. (1998)'s argument that particularly breakthrough inventions "may be better characterized as creating (rivalrous) human capital – intellectual human capital – characterized by natural excludability as opposed to a set of instructions

[1] According to press reports, between 2002 and 2008, 75 percent of the U.S. Defense Department's civilian workforce of 675,000 people are expected to retire and the oil and gas production industry expects to lose more than 60 percent of its employees by 2010 (Sandia National Laboratories, 1996; Sapient Corporation, 2003; Farrell, 2002).

for combining inputs and outputs which can be protected only by intellectual property rights. This natural excludability arises from the complexity or tacitness of the knowledge required to practice the innovation." Based on both extensive interviews and empirical work summarized in Zucker and Darby (1996), they believe that, "at least for the first 10 or 15 years, the innovations which underlie biotechnology are properly analyzed in terms of naturally excludable knowledge held by a small initial group of discoverers, their co-workers, and others who learned the knowledge from working at the bench-science level with those possessing the requisite know-how" (Zucker et al., 1998, p. 291).

The observations suggest that the knowledge transfer between generations is in general imperfect and that purposeful and costly knowledge codification is playing an important role in the transfer of an economy's productive knowledge.

From a theoretical viewpoint, new growth theory suggests that knowledge is non-rival. The argument is that the use of an item of knowledge – whether it is the Pythagorean theorem, a soft drink recipe or an algorithm to brew coffee – in one application makes its use by someone else no more difficult. However, although ideas themselves (e.g. the Pythagorean theorem, a soft-drink recipe or an algorithm to brew coffee) are non-rival, they possess direct economic relevance only in so far as they are embodied in either persons or physical objects. We refer to ideas embodied in persons as human capital or knowledge and to those embodied in physical objects as information[2]. Both are rival. For example, an algorithm to brew coffee can be used by many people and coffee machines simultaneously without deteriorating. It is non-rival (and disembodied). However, to really have a cup of coffee, it needs a person or a machine to use this algorithm. Hence, the idea of brewing coffee must be embodied in a person, who devotes her effort to this activity. This precludes the simultaneous use of her human capital by another activity. Similarly, a coffee machine can brew only one kind of coffee at a time at a finite quantity. The fact that ideas that are not embodied in someone or something are not directly relevant economically is immediately obvious for all ideas that have not yet been discovered or have been discovered and forgotten.[3] Consequently long run economic growth is generated by the accumulation of non-rival ideas in people and physical objects. That is, by the accumulation of human capital and information.

A person, however, has only a finite number of years that can be spent acquiring ideas. When this person dies, her human capital is lost. Any non-rival idea that this person has discovered only lives on if it is either embodied in another person or as information. The transformation of the codifiable part of knowledge via some code into information is referred to as knowledge codification.[4] For example, using a natural language to

[2]Knowing an idea means to understand it. We do not ascribe the capacity to understand to physical objects. For example, a person may be able to learn an idea, that is, understand it by attending to a scientific paper. The paper, however, does not carry knowledge, because reading a scientific paper does not automatically imply that this person can attach meaning to it, that is, knows more than before.

[3]Boldrin and Levine (1999) strongly advocate this line of thinking.

[4]There is a discussion as to what extent knowledge may be codified. On the one hand, it is a function

write down an idea in a book. As compared to human capital, information is long lasting if properly maintained and it may be more easily accessible and distributed. In this respect, it may be useful for an economic analysis to refer to an idea as codified, if every individual of a certain group (e.g. employees of a firm) can access the information independent of others. For instance, there are enough books containing the same idea, such that anyone who is interested in it is able to use a copy, or there are enough servers, such that anyone interested in an idea can download the information via the internet or intranet. In this way, information has the property of a local public good, and knowledge codification can be interpreted as creating non-rival information from rival human capital.

The standard approach in the endogenous growth theory uses the assumption that knowledge codification is an automatic by-product of research and development activities and can be treated as costless. The (implicit) argument is that when the innovative product is sold at the market, an interested individual or potential competitor could buy and reverse-engineer it. If the innovation is patented, this person could consult the patent specification and would then immediately know the idea embodied in it. However, the empirical observations suggest that the information that accrues as a by-product of research and development is, by itself, not sufficient for the transfer of knowledge between generations. Moreover, large parts of an economy's productive knowledge are not patented or embodied in a product directly sold at the market, such as for instance many process innovations. This gives rise to the observed purposeful and costly knowledge codification.

As the discourse on knowledge codification has been almost entirely on a verbal basis (see e.g. Dasgupta and David (1994) and Cowan et al. (2000)), this paper develops a formal model of endogenous growth that incorporates costly knowledge codification as a means of intergenerational knowledge transfer. We use a standard two-sector overlapping generations framework of endogenous growth in which the knowledge transfer between generations of employees in a long-lived intermediate firm is imperfect. By investing in knowledge codification this transfer can be improved. It is assumed that the capital owners, that is, the members of the old generation are the firm owners and the intermediate firm is managed on their behalf. The young generation will take over the ownership claims at the end of the period and may then be willing to compensate the retiring researchers for knowledge codification. In this way, the codification decision can be depicted as part of the general savings decision of the young generations' utility maximization problem. This decision process drives the main results.

We find that although knowledge codification positively influences an economy's long-run growth rate of output, there will initially be no knowledge codification in an economy that develops from a small level of capital. We thus examine under which conditions an economy will codify in the long run. As a main result, we show that this will be the case if the steady-state growth rate of capital is higher than that of the

of the code available, on the other, it depends on the idea itself. Highly abstract mathematical ideas seem to allow better codification than the idea "how to ride a bicycle".

knowledge stock. When both steady-state growth rates are equal, it is sufficient for long-run knowledge codification if the steady-state level of the intensive capital stock is large enough. The intuition is that under the given conditions the marginal product of another piece of knowledge preserved by knowledge codification becomes large enough such that the firm owners are willing to bear the costs of codification. By a similar reasoning we extend the analysis to include decreasing codification costs and fixed set-up costs of knowledge codification.

The focus on intergenerational knowledge transfer by costly knowledge codification has not yet been taken up by formal models of economic growth although the knowledge management literature suggests that this is an important problem.

A recent paper of Thoenig and Verdier (2004) is also concerned with the macroeconomic aspects of knowledge management. However, instead of focussing on the incentive to preserve knowledge in intergenerational knowledge transfer, they emphasize the trade-off that by knowledge codification contractual incompleteness between the firm owner and the employee can be avoided, which, however, implies the risk of information spillovers to competitors.

Further, the paper is related to the literature on technology adoption models and the literature on endogenous spillovers by own research. It possesses the same underlying motivation, in that blueprints defined as the "by-product" description of a technology are incomplete in conveying what is useful to know about the technology at hand. It needs additional human capital to fully understand and work the blueprints.[5] The focus of building up this human capital in order to apply a new technology for production is discussed in the adoption models. The present work is more related to the models which focus on the human capital necessary to understand the idea embodied in the blueprint (possibly the patent specification) in order to imitate the technology or use the idea for further research. This kind of human capital is commonly referred to as 'absorptive capacity' which is built up by own research.[6] The corresponding strand of the literature usually examines endogenous knowledge spillovers between competitors in a certain market. In such an environment the innovator of a new product is interested in conveying as little information as possible to the competitors. That is, the patent specification or other codified sources of the idea will only include the absolutely necessary descriptions. The focus in the present work is on knowledge transfer between generations of employees within a firm. In such a setting, there may be an incentive for further knowledge codification in order to reduce the "adoption costs" in terms of own research by the newly hired employees.

The proposed formal model corresponds to the standard idea-driven growth models as in Romer (1990),[7] Aghion and Howitt (1992) or Grossman and Helpman (1991). What sets it apart is that it explicitly models a human capital stock and a stock of

[5] This terminology was taken from Jovanovic (1997).

[6] See Cohen and Levinthal (1989). Griffith et al. (2003) built a model that incorporates 'absorptive capacity' into a model of economic growth.

[7] This will be discussed in the third part of the paper.

codified ideas. Ideas can only be used productively if they are learned and, hence, are part of the employee's (intellectual) human capital. The latter can be enhanced by utilizing codified ideas if previous generations have codified them. In this way, the model could be considered as what is sometimes called a hybrid version[8] between the idea driven growth models and the human capital accumulation growth models with the peculiarity of endogenous knowledge spillovers between generations by knowledge codification.

The structure of the paper is as follows. Section 2 introduces the model and describes its general dynamics. The remainder of the paper concentrates on the analysis of the economy's long run codification behavior. Section 4 concludes.

2 The Model

Consider an overlapping generations economy similar to the well-known Diamond (1965) model, in which agents live for two periods. Time is infinite in the forward direction and divided into discrete periods indexed by t. There is a continuum of individuals P_t on $[0,1] =: \mathcal{P}_t$ in each generation. There is no population growth and the size of each generation is normalized to 1. Each individual inelastically supplies one unit of labor when young and consumes its capital savings plus the capital rent when old. Hence, total labor supply in each period t is given by $L_t = 1$. The economy features two sectors. An intermediate goods sector that creates intermediate goods from physical capital and knowledge and a production sector that uses the intermediate product and labor to generate a homogenous physical good that can be used for consumption and investment. The difference to the usual models of endogenous growth is that within the intermediate sector, the individuals can influence the return on capital by investing in knowledge codification.

2.1 Knowledge and Information

Let there be an infinite set of ideas and assign each idea an index $i \in \mathcal{I}$. Every idea that becomes economically viable is initially embodied in its inventor and – at a cost – may be expressed as information. As this paper focusses on intergenerational knowledge transfer, the following will refer to information as the purposefully created local public good mentioned in the introduction. Generally, there are two ways to transmit knowledge between generations. One is face-to-face interaction between a teacher or mentor and his scholar[9]. The other is knowledge codification by the one generation and attending to this information by a later generation, for example, by writing and reading a book. We will take both kinds of transfer as substitutes.[10]. We say a person

[8] See for example Klenow (1998).

[9] This is the focus of the article by Jovanovic and Nyarko (1995)

[10] This does not preclude the existence of an idea's tacit counterpart that cannot be conveyed via code, as is often claimed in the literature on the tacitness and codifiability of knowledge. See e.g.

born at time t, $P_t \in \mathcal{P}_t$ knows an idea i, if she is able to attach meaning to it. Or synonymously, if "i is embodied in P_t". Hence, person P_t's knowledge or human capital is defined as $\mathcal{T}_{P_t} = \{i \in \mathcal{I} \mid \text{i is embodied in } P_t\}$. The stock of information is the index set of the codified ideas $\mathcal{C}_t = \{i \in \mathcal{I} \mid \text{i is codified}\}$. For illustrative purposes, imagine a bookshelf with each book containing exactly one idea. The book's title is i indicating the idea it contains. Knowing a person's human capital, one could immediately tell which books this person can understand, that is, $\{i \in \mathcal{T}_{P_t} \cap \mathcal{C}_t\}$ and which ones she would encounter difficulties with $(\{\mathcal{I} \setminus \mathcal{T}_{P_t} \cap \mathcal{C}_t\})$.

To operationalize this concept of knowledge, we represent the knowledge stock as an interval in $\mathbb{R}$. That is, we specify $\mathcal{I} = \mathbb{R}_+$. Further, we order the ideas according to their difficulty. More precisely, if $i < j$ then idea i is easier to comprehend or more basic than idea j. For example, addition is less difficult than solving differential equations. Recalling the bookshelf example, this would mean that the books are ordered, e.g. from left to right, starting with the ones containing the rather basic ideas and becoming successively more difficult. They are indexed continuously beginning at 0. We are defining a person's measure of human capital by the index of the most difficult idea she is able to understand. More precisely, $\tau_{P_t} = \sup \mathcal{T}_{P_t}$. The central assumption of the ordering concept is:

Assumption (A1) *If i is embodied in person P_t, then $j < i$ is also embodied in person P_t.*

As we defined $\mathcal{I} = \mathbb{R}_+$, τ_{P_t} would be the Lebesgue-measure of $\mathcal{T}_{P_t}$. The definitions imply that an individual with human capital of 5 possesses the set of tacit components $[0, 5]$ and, thus, comprehends the contents of all information indexed by $i \in [0, 5]$, respectively is able to use ideas $i \in [0, 5]$ in production.[11]

Let $C_t := \sup \mathcal{C}_t$ represent the economy's stock of information. The ordering concept may involve:

Assumption (A2) *Before an idea indexed by i can be codified, all ideas with index j, $j < i$ must have been codified. This implies that if i is codified in period t, then $j < i$ is also codified in t.*

With assumption (A2), C_t is identical to the Lebesgue-measure of $\mathcal{C}_t$.

2.2 The Production Sector

Final-goods production is characterized by a continuum (on [0,1]) of identical firms which produce the homogenous good with the use of labor $L_{A,t}$ and the intermediate good x_t as inputs. Since final-goods firms earn zero profits and own no assets, they

Collins (1974), David (1998)

[11]Note that other ordering concepts, for example a chronological order, would work just as well as long as (A1) holds.

can be ignored in the specification of endowments.[12] The firms maximize profits and act competitively in the product and factor markets. For better tractability, the most convenient way to model final production is by means of a representative firm whose production and factor demands represent aggregate values. The aggregate production function is of the constant-returns-to-scale type:[13]

$$F(x_t, L_{A,t}) = x_t^{\alpha} L_{A,t}^{1-\alpha},$$

where $\alpha \in (0,1)$. Consequently, the representative firm solves the following maximization problem:

$$\max_{x_t, L_{A,t}} \pi_t^{fp} = F(x_t, L_{A,t}) - p_{x,t} x_t - w_{A,t} L_{A,t}.$$

As it is a constant-returns-to-scale firm, its input demands are defined only after the scale of operation is pinned down. However, the demand for labor and intermediate goods is characterized by the first order conditions. Labor and the intermediate goods are being compensated by their marginal product:

$$\begin{aligned} x_t^d &= \left(\frac{p_{x,t}}{\alpha}\right)^{\frac{1}{\alpha-1}} L_{A,t}, \\ L_{A,t}^d &= \left(\frac{1-\alpha}{w_{A,t}}\right)^{\frac{1}{\alpha}} x_t. \end{aligned}$$

2.3 The Intermediate Goods Sector

In general, the intermediate sector is assumed to consist of long-lived intermediate firms whose ownership is handed down from one generation to the next. More precisely, the capital stock is sold publicly at the end of a period t to the next period's old generation. This process could be interpreted as secondary public offering. Further, the young generation of t may additionally increase the capital stock of $t+1$ by saving more than the amount of capital left from the previous generation. Physical capital left at the disposal of the intermediate firm is interpreted as ownership claims. As mentioned in the introduction, we assume that the firm owners collectively decide on the values of the control variables of the intermediate firm's profit maximization problem. In order to depict the mechanics of the model as easy as possible, in this model the intermediate goods sector is characterized by a single intermediate firm.[14]

2.3.1 The Decision Process

The economic problem with respect to the transfer of knowledge by knowledge codification between two succeeding generations of employees can be translated into the

[12] For illustrative purposes one could assume that the final-goods firms are managed and owned by the members of the young generation.

[13] According to Sargent (1979), the preceding assumptions guarantee the existence of such a representative final-product firm.

[14] The model can also be extended to include oligopolistic intermediate sectors. See Schneider (2007).

overlapping generations framework by the following three stage game:

Stage 1: At the end of period t, the new capital owners may invest in knowledge codification before the employees of the intermediate firm, the researchers, retire.[15]

Stage 2: At the beginning of $t+1$, the newly hired researchers are asked to compensate the firm owners for their codification investment. (Under the assumption that the knowledge database or library is excludable[16].)

Stage 3: The capital owners decide on giving access to the firm's information stock.

We assume that allowing access to the knowledge database does not incur costs. It is clear that no newly hired researcher would make a compensatory payment independent of the decisions of her colleagues, as she knows that the capital owners will provide the information for free. By backward induction, the firm owners, knowing that they bear the full codification costs, will only invest in knowledge codification up to the profit-maximizing amount. This may imply a hold-up problem in case knowledge codification positively influences the next period's wage level.[17]

One may ask whether the investment problem within the firm is well depicted by this game structure. For example, why do the firm owners give away the information for free, although they know that the new employees might benefit from it by higher wages? The argument is that as long as the firm owners can only ask for a compensatory payment, the new employees will always deny. Suppose the game were extended to more than three stages, in which after the third stage the new employees are asked for a compensatory payment and the firm owners decide on giving access to the information stock in an alternating sequence. With finitely many stages, there must be a last stage before the production of the intermediate goods has to start. At the last but one stage, the new employees know that if they again deny a compensatory payment, the firm owners would give access to the information for free at the last stage, as otherwise they would forgo a certain amount of profits. Further, a threat by the firm owners to destroy the information would be empty because they would forgo additional profits. In this way, profit maximization of the firm owners proceeds in two stages:

Stage 1: At the end of period t, the firm owners collectively decide on how much to invest in knowledge codification before the researchers retire.

[15]Note that within the overlapping generations structure, the next period's researchers are not born, yet.

[16]With respect to excludability, two specifications would be possible. On the one hand, a codified idea could be a public good with respect to the new researchers as soon as the the firm owners give access to it. On the other hand each employee could be given exclusive access to certain ideas dependent on her compensatory payment. These specifications have not been explicitly distinguished as it does not change the outcome of the subgame perfect Nash equilibrium.

[17]For a detailed discussion on the hold-up problem with respect to knowledge codification see Schneider (2007).

Stage 2: At the beginning of $t+1$, given the amount of ideas that have been codified, the capital owners decide on the amount of intermediate goods to produce and on how many new researchers to hire.

Of course by backward induction, the second stage maximization problem of $t+1$ is solved given a certain stock of information and then the optimal codification decision is taken. The collective decision process with respect to knowledge codification takes the following form:

- ▷ Every individual capital owner can propose an amount $\zeta_{t,prop}$ to spend on knowledge codification. The costs are split up among the individuals according to their capital shares.
- ▷ Each shareholder votes for or against the proposal and the amount is approved according to the unanimity rule.
- ▷ If the proposal has been accepted, the amount is collected and the retiring researchers are paid to codify their ideas.

For simplicity we assume that this decision process does not incur transaction costs. It would also be possible to assume that a manager acts on behalf of the capital owners.

It is clear that given the capital and information stock in period $t+1$, the second stage maximization problem is a static profit optimization within the period. However, the codification decision at the first stage cannot be made independent of the households' preferences concerning the distribution of consumption between periods. Hence, the second stage problem will be solved at the end of this section and the decision on knowledge codification will be elaborated further within the households' utility maximization problem.

2.3.2 Research and Intermediate Goods Production

The intermediate good can be produced according to the following production function: $x_t = G(K_t, \tau_t) = K_t \tau_t$.[18] K_t denotes the measure of the capital stock at time t. The knowledge stock of the intermediate firm which corresponds to the knowledge stock of the economy in period t, τ_t, is defined by the index of the most difficult idea in the union of all sets of tacit components $\tau_t = \sup \cup_{P_t \in \mathcal{P}_t} \mathcal{T}_{P_t}$. It equals the highest

[18] Note that this specification implies that a technological improvement increases production of the final product in the same way as an increase in the stock of capital, which is usually referred to as Solow-neutral technological progress. The innovation process could be regarded as both, process or product innovation. The interpretation of the first would be that the higher knowledge stock allows for the production of more homogenous intermediate goods with a certain capital input. On the other hand, one could argue that the capital stock is transformed into higher quality intermediate products, which possess a productivity in final-goods production identical to x_t units of some standard intermediate good.

measure of human capital in the set $\mathcal{P}_t$ at time t.

Human capital of a person $P_t \in \mathcal{P}_t$ may originate from three sources:[19]

$$\tau_{P_t} = q\tau_{t-1} + \beta \max\{0, C_t - q\tau_{t-1}\} + \varepsilon(q\tau_{t-1})^{\Phi}.$$

First, every individual is exogenously transferred a share q of the economy's knowledge stock of the previous period, e.g. in school or by other educative means.[20] This transfer is imperfect, $q \in (0,1)$.

As a second source, the members of the new generation who are becoming researchers in the intermediate firm can enhance their human capital by attending to codified ideas if previous generations of researchers have codified their knowledge. Recall the bookshelf example: If the highest index of the books in the shelf C_t is greater than the highest index of the ideas the young generation of t has already learned, $q\tau_{t-1}$, they are interested in reading the books indexed by $(q\tau_{t-1}, C_t]$. In each period the new generation is able to build up additional human capital of a fraction $\beta \in (0,1)$ of those codified ideas.[21] As (A2) is a strong requirement, most part of the analysis will consider situations without (A2). Then, the following assumption with regard to knowledge codification holds instead.

Assumption (A2') *In a period t and for all $i, j \geq q\tau_t$: Before an idea indexed by i is codified, all ideas with index j, $j < i$ have been codified already. This implies that if i is codified in period t, then $j < i$ is also codified in t.*

Finally, a person P_t deciding to do research is able to generate a number $\varepsilon(q\tau_{t-1})^{\Phi}$ of new ideas. The term "new ideas" is supposed to reflect the subjective perspective of person P_t. That is, the ideas generated are new to P_t, but not necessarily to all other persons. We examine the case of $\Phi = 1$, which means that researchers are becoming more productive with an increasing stock of knowledge to draw upon. This corresponds to the usual 'standing on the shoulders of giants' argument of endogenous growth models.

The process of knowledge acquisition shows a sequential nature, proceeding from schooling via reading to own research. For analytical simplicity, the first two kinds of knowledge transmission happen at no time at the beginning of each period. Research and development, however, takes time, such that a person has to decide as to whether she is going to work in final-goods production or to do research. Let $L_{R,t}$ denote the number of persons who are hired to do research in period t and let them be arranged on the continuum from 0 to $L_{R,t}$. Since they have been transferred the same amount of human capital from the old generation and generate the same number of new ideas each, they all possess the same level of knowledge. More precisely, $\tau_{P_t^1} = \tau_{P_t^2}$,

[19]The dynamics of the knowledge stock have also been inspired by a working paper on technological regress by Aiyar and Dalgaard (2002).

[20]It could also be interpreted as imperfect human capital transfer within the intermediate firm.

[21]β reflects how easy ideas can be acquired from reading.

$\forall P_t^1, P_t^2 \in [0, L_{R,t}]$.[22] Further we assume spillover-effects occur within research groups which are defined as $\mathcal{G}_t \subseteq [0, L_{R,t}]$. Let $L_{\mathcal{G}_t}$ be the Lebesgue-measure of $\mathcal{G}_t$, representing the number of researchers in research group $\mathcal{G}_t$.[23] Research groups may be employed by the intermediate firm which can be interpreted as a single research joint venture owned by the old generation. Hence, having only one research group of size $L_{R,t}$ implies that $\tau_{P_t} = \tau_{R,t}$, $\forall P_t \in [0, L_{R,t}]$, where

$$\tau_{R,t} = q\tau_{t-1} + \beta \max\{0, C_t - q\tau_{t-1}\} + \varepsilon q\tau_{t-1} L_{R,t}.$$

Accordingly, the economy's knowledge stock is determined by $\tau_t = \tau_{R,t}$. It is the maximum level of knowledge that can be productively employed at time t. The dynamics of the economy's knowledge stock can then be written as follows[24]

$$\tau_t = \begin{cases} q\tau_{t-1} + \beta(C_t - q\tau_{t-1}) + \varepsilon q\tau_{t-1} L_{R,t}, & C_t > q\tau_{t-1}; \\ q\tau_{t-1} + \varepsilon q\tau_{t-1} L_{R,t} & , \quad C_t \leq q\tau_{t-1}. \end{cases}$$

2.3.3 Second Stage Profit Maximization

Within period t, the intermediate firm maximizes profits by deciding on the intermediate goods supply and the number of researchers to employ. Hence, it faces the following problem:

$$\max_{x_t, L_{R,t}} \pi_t^{int} = p_{x,t}(x_t)x_t - w_{R,t} L_{R,t}.$$

Taking the information stock and physical capital as given, the problem is one-dimensional in $L_{R,t}$. Using the demand for intermediate products of the final-goods firms, the necessary condition of the intermediate entrepreneurs' optimization problem writes

$$w_{R,t} = \alpha^2 x_t^{\alpha-1} \frac{\partial x_t}{\partial L_{R,t}} L_{A,t}^{1-\alpha}.$$

In this way, the supply of intermediate goods, x_t^s, and the factor demand for researchers, $L_{R,t}^d$, are given by[25]

$$\begin{aligned} x_t^s &= \left(\frac{\alpha^2 \frac{\partial x_t}{\partial L_{R,t}}}{w_{R,t}} \right)^{\frac{1}{1-\alpha}} L_{A,t}, \\ L_{R,t}^d &= \left(\frac{\alpha^2}{w_{R,t}} \right)^{\frac{1}{1-\alpha}} \left(K_t \frac{\partial \tau_t}{\partial L_{R,t}} \right)^{\frac{\alpha}{1-\alpha}} L_{A,t} - \frac{1}{\frac{\partial \tau_t}{\partial L_{R,t}}} (q\tau_{t-1} + \beta \max\{0, C_t - q\tau_{t-1}\}). \end{aligned}$$

[22] Analogously, the symmetry of the knowledge level applies to the workers.

[23] Of course, this definition involves a measure theoretic problem. Since single points on the interval $[0, L_{R,t}]$ possess measure 0, there exist research groups $\mathcal{G}_t$ consisting of more than one person with measure 0. We preclude such groups by assumption.

[24] The assumption of only one research group in the economy is not necessary to obtain this difference equation. However, all research groups of the economy must be of equal size.

[25] The sufficient condition for a maximum is satisfied due to the strict concavity of the objective function in $L_{R,t}$: $\frac{\partial^2 \pi_t}{\partial L_{R,t}^2} = \alpha^2(\alpha-1)\tau_t^{\alpha-2} \left(\frac{\partial \tau_t}{\partial L_{R,t}} \right)^2 K^\alpha L_{A,t}^{1-\alpha} < 0$.

Consequently, the profits of the intermediate firm accrue to

$$\pi_t^{int} = \alpha K_t^{\alpha} L_{A,t}^{1-\alpha} \tau_t^{\alpha}(1 - \alpha \tau_t^{-1} \frac{\partial \tau_t}{\partial L_{R,t}} L_{R,t}) =: K_t r_t.$$

For reasons of clarity in exposition, $L_{R,t}^d$ was not inserted into the profit function. The profits are allocated among the shareholders of the intermediate firm proportional to their shares. Hence, the return on capital investment is

$$r_t = \alpha K_t^{\alpha-1} L_{A,t}^{1-\alpha} \tau_t^{\alpha}(1 - \alpha \tau_t^{-1} \frac{\partial \tau_t}{\partial L_{R,t}} L_{R,t}). \tag{1}$$

2.4 The Costs of Knowledge Codification

Knowledge codification takes time and physical resources as carriers of information such as paper, CD-ROMs or hard disks. There is also a need for reproduction and storage devices. Additionally knowledge databases have to be maintained and administrated. Costs could even include the creation of new codes, e.g. if the existing one does not suffice for expression. Although the notion 'knowledge codification' may also comprise the creation of all kinds of artifacts, for illustrative purposes, it will be referred to as the creation of information via some natural language.[26] For simplicity, We assume that every idea can be codified at the same cost of γ_t units of the homogenous good within period t. The codification costs per idea γ_t may change over time, hence the index t in the cost function. However, the marginal costs are supposed to be constant within each period. In this way, the cost function shows a linear form within periods:

$$\Gamma_t(\triangle C_t) = \gamma_t \triangle C_t.$$

Additionally fixed entry costs f may accrue once in the first period of knowledge codification for setting up computer systems, establishing management structures or building archives or libraries.[27]

Three implicit assumptions with respect to the cost function are that, first, the costs of knowledge codification per idea do not depend on the number of new employees to access the information in the next period. According to the definition of information, it may well be that an increasing number of researchers may necessitate further costly copying and distribution activities. However, it is assumed that the cost share of these activities due to variations in the number of new researchers between periods is negligible.

Second, if the new capital owners decided on a positive codification level, those to actually codify the ideas are the researchers of that period as they possess the highest level of knowledge. However, would they codify at the marginal costs γ_t? The reasoning is in the Bertrand-fashion: As long as there is more than one researcher, they

[26]For illustrative purposes, one could think of writing a book for the previously described bookshelf.
[27]It will explicitly be mentioned in the analysis when entry costs are considered.

will agree to codify at the marginal costs because they are symmetric with respect to knowledge. The situation with only one researcher is mathematically precluded by the representation of the population on the interval $[0, 1]$, because positive research implies a research group of measure greater than zero. For a more general interpretation, it is assumed that when having only one researcher, the firm owners offer a forcing contract that contains a compensation for knowledge codification slightly above the marginal costs. The researcher will accept the contract. This is the case as the firm owners know that the researcher will be better off if she codified at slightly above the marginal costs and hence has no incentive to revise the conditions of the contract, even if the researcher would neglect the contract at first.[28]

Third, for simplicity it is assumed that information once created cannot be retransformed into the homogeneous consumption good.

2.5 The Problem of the Household

Each individual lives for two periods and maximizes the discounted sum of utilities. There is a constant discount factor $\delta > 0$.[29] The individuals inelastically supply one unit of labor when young and may choose as to whether they want to work in final-goods production or in research and development. The budget constraint when young is given by the wage w_t that can be split into consumption today $c_{1,t}$, saving $w_t s_t$ in physical capital and investment in knowledge codification $w_t \varsigma_t$. The physical capital savings plus the real return on capital r_{t+1} equal consumption when old $c_{2,t+1}$[30]. The household privately saves in physical capital which is interpreted as ownership claims on the intermediate firm and then takes part in a collective decision within the firm on knowledge codification. The problem of the household can be depicted by the following three stage process.

- Stage 1: The household decides on how much it would like to save in physical capital and how much to propose for investment in knowledge codification.
- Stage 2: The collective decision with respect to knowledge codification within the intermediate firm is taken.
- Stage 3: The household may adjust capital savings given the investment in knowledge codification.

It is assumed that the households cannot commit to or be constrained to a certain amount of physical capital saving. In particular, at the third stage, the individual can revise her decision after the amount of knowledge codification has been set. In this way,

[28]It depends on the number of the stages of the game how often the researcher may neglect the contract. However, she will accept it at the last stage.

[29]This implies a constant rate of time preference $\rho > -1$.

[30]The index indicates whether the person is young (1) or old (2) in period t.

each young individual makes a decentral decision over her capital savings.[31] As the model is entirely deterministic, the individuals know that they are all symmetric with respect to wage and preferences. Accordingly, at the first stage, she decides whether she would like to make a proposal on the codification investment by solving

$$\max_{s_t,\varsigma_t} U_t(c_{1,t}, c_{2,t+1}) = u(c_{1,t}) + \delta u(c_{2,t+1})$$

subject to

$$\begin{aligned} c_{1,t} &= w_t(1 - s_t - \varsigma_t), \\ c_{2,t+1} &= (1 + r_{t+1})s_t w_t. \end{aligned}$$

For notational convenience, let $S_t := s_t w_t$ and $\zeta_t := \varsigma_t w_t$, where $s_t, \varsigma_t \in [0,1]$. The individual's proposal on the intermediate firm's codification investment would then be $\zeta_{t,prop} = \int_0^1 \zeta_t dP_t$. Further, it is assumed that utility takes the form of a CIES-utility function:

$$u(c) = \frac{c^{1-\theta} - 1}{1-\theta},$$

where $\theta \geq 0.5$. The agent obtains the utility maximizing pair (s_t, ς_t) from the following necessary conditions:[32]

$$\begin{aligned} u'(c_{1,t}) &= \delta u'(c_{2,t+1})(1 + r_{t+1}), \\ u'(c_{1,t}) &= \delta u'(c_{2,t+1}) \frac{\partial r_{t+1}}{\partial \varsigma_t} s_t. \end{aligned}$$

With the particular functional form of utility, the conditions write

$$\begin{aligned} s_t &= \frac{1 + r_{t+1}}{\frac{\partial r_{t+1}}{\partial \varsigma_t}}, \\ \varsigma_t &= 1 - \frac{1 + r_{t+1}}{\frac{\partial r_{t+1}}{\partial \varsigma_t}} \left(\delta^{-\frac{1}{\theta}} (1 + r_{t+1})^{\frac{\theta - 1}{\theta}} + 1 \right). \end{aligned}$$

As information cannot be transformed into the consumption good, a negative codification investment suggestion is not possible. Hence, an agent will make a suggestion if her optimal level $\zeta_{t,prop}$ is greater than zero. In particular, if no proposal has been put forth or no proposal has been accepted, the optimization problem is equivalent to

[31] This is plausible because although by coordination the households could choose the monopoly capital stock, each individual has an incentive to deviate by increasing her saving rate. The coordination in the codification case is possible as by assumption they have to pay the codification costs immediately and hence cannot deviate to lower codification. Of course, every household could privately pay for additional knowledge codification, but none has an incentive to do so.

[32] Appendix A verifies that there is a unique solution of the necessary conditions which implies a maximum of the objective function.

the regular maximization problem in an overlapping generation model with production.

Since all individuals are flexible to choose their profession, the model features only one labor market, and wages of the different occupations must be equal in equilibrium. As a consequence, each household faces an identical optimization problem, such that in fact each person makes the same proposal on knowledge codification expenses which, of course, will be accepted. It is thus convenient to work with a representative household as being the young generation characterized by its date of birth t.[33] Each generation faces the first stage optimization problem and its solution is interpreted as aggregate savings S_t and investment in knowledge codification $\zeta_t \geq 0$.

2.6 Sequence of Events

This section summarizes the time-line of the model's typical events in a regular period t:

(1) At the beginning of period t, the members of the new generation are exogenously transferred a share q of the economy's knowledge stock of the previous period: $\tau_{P_t} = q\tau_{t-1}$, $P_t \in \mathcal{P}_t$.

(2) The intermediate firm hires a number $L_{R,t}$ of researchers. If the researchers of the previous periods have codified their ideas, the new generation of researchers in t would additionally build up human capital of $\beta(C_t - q\tau_{t-1})$.

(3) The intermediate firm produces intermediate good x_t with the capital saved by the now old generation $\mathcal{P}_{t-1}$ and the researchers' human capital. The researchers receive wage $w_{R,t}$. The profits π_t^{int} are split among the capital owners.

(4) The final-product firm rents the intermediate products at price $p_{x,t}$ and hires a number $L_{A,t}$ of workers at a wage $w_{A,t}$ in order to produce the final good Y_t.

(5) At the end of period t, the young generation decides how much of the wage income to consume $c_{1,t}$ and how much to transfer to the next period. On the one hand, the young generation (decentrally) saves in physical capital, which ensures ownership rights of the intermediate firm in $t+1$ involving a return r_{t+1}. On the other hand, it may invest in knowledge codification which is the result of a collective decision made at the shareholder's meeting of the intermediate firm. Knowledge codification increases the knowledge stock of $t+1$ and consequently the rent r_{t+1}.

(6) Dependent on the investment in knowledge codification, ζ_t, the researchers of period t codify an amount $\triangle C_t$ of their ideas.

[33] In fact, individuals belonging to the same generations may be heterogenous with respect to human capital. Earning the same wage, this heterogeneity does not carry over to the households' utility maximization problem.

(7) The old generation sells its capital to the young, consumes its receipts K_t plus the return on capital $r_t K_t = \pi_t^{int}$, and then dies.

As usually assumed in overlapping generations models, there is one physical good that can be consumed. Output that is not consumed can be used as capital in the following period. The literature suggests two ways of interpretation with respect to the capital stock. In both, savings of the young generation in t equal the capital stock of period $t+1$. In the first, the young generation buys the current capital stock from the old with their savings and "net-saves" the difference $S_t - K_t$, which becomes the net increase in capital of the subsequent period. The second approach assumes that capital is "eatable" or consumable, such that the old generation consumes its capital at the end of its life. In this way, the capital stock of the next period is always identical to consumption forgone of the previous period, which has become productive with a time lag.

Both interpretations could apply to our model, however, suggesting a long-lasting intermediate firm, we will go with the first.[34] In detail, we assume that the intermediate firm transforms the capital stock to the intermediate good, which is rent to final-goods production. Over one period the intermediate good depreciates in the sense that it cannot be used in final production unless it is overhauled. But it can still be used as raw capital in the next period.[35]

2.7 Sequential Markets Equilibrium

The economy comprises three markets: the labor market, the market for intermediate products and the market for the consumption good. The following analysis will focus on the sequential markets equilibrium, which is defined by the three markets to clear in each period.

Definition 1 *Given $K_1, \tau_0 > 0$, $C_1 \geq 0$, and $S_t = K_{t+1}$, $C_{t+1} = f(C_t, \zeta_t)$, a sequential markets equilibrium is allocations $c_{2,1}$, $\{c_{1,t}, c_{2,t+1}, S_t, \zeta_t, L_{A,t}, L_{R,t}, x_t\}_{t=1}^{\infty}$ and prices $\{p_{x,t}, w_t\}_{t=1}^{\infty}$, such that*

(i) they solve the utility maximization problem of the households and the profit maximization problems of the end-product firms and of the intermediate firm for all $t \geq 1$ and

(ii) in every period the economy is in temporary equilibrium, that is, for all $t \geq 1$:

[34] Adopting the first interpretation and assuming that capital is not consumable may be problematic for finite time horizons and if the capital stock is decreasing. Both problems will not occur in this paper.

[35] It is well known that overlapping generations economies may exhibit equilibria with asset bubbles. As bubbles are not the focus of the paper, we exclude them from the following analysis.

(a) *(Labor Market)*

$$L_t^s = L_{A,t}^d + L_{R,t}^d,$$

(b) *(Intermediate Goods Market)*

$$x_t^s = x_t^d m,$$

(c) *(Final Goods Market)*

$$Y_t^s = c_{1,t} + c_{2,t} + S_t + \zeta_t.$$

As the supply and demand functions are derived by the respective optimization problems, it is clear that the model satisfies (i). Further, the properties of the final-goods-production function imply that the temporary equilibrium of the labor market will exhibit a positive share of workers in this sector due to the Inada conditions. Using the respective demand functions from the first order conditions of the representative final-goods firm and the intermediate firm, the equilibrium condition writes

$$\begin{aligned} L_t^s = \left(\frac{1-\alpha}{w_t}\right)^{\frac{1}{\alpha}} K_t \tau_t & \left(1 + \left(\frac{\alpha^2}{w_t}\right)^{\frac{1}{1-\alpha}} \left(K_t \frac{\partial \tau_t}{\partial L_{R,t}}\right)^{\frac{\alpha}{1-\alpha}}\right) \\ & - \frac{1}{\frac{\partial \tau_t}{\partial L_{R,t}}} (q\tau_{t-1} + \beta \max\{0, C_t - q\tau_{t-1}\}). \end{aligned}$$

Since $L_t^s - L_{A,t}^d = L_{R,t}^d$, we receive the equilibrium wage

$$w_t = (1-\alpha)^{1-\alpha} \left(\alpha^2 K_t \frac{\partial \tau_t}{\partial L_{R,t}}\right)^{\alpha}.$$

Note that as $\frac{\partial \tau_t}{\partial L_{R,t}} = \varepsilon q \tau_{t-1}$, the wage is independent of the amount of knowledge codification of the previous period. In this way, the model characterized by a specification of the production of intermediate goods with Solow-neutral technological progress does not exhibit a hold-up problem with respect to knowledge codification. However, knowledge codification may possess positive externalities to the wage levels from the next but one period on.

Inserting the equilibrium wage into the labor demand functions and using $L_t^s = 1$ gives the equilibrium allocation of labor according to

$$\begin{aligned} L_{A,t} &= \min\left\{1, \frac{1-\alpha}{\alpha^2 + 1 - \alpha}\left(1 + \frac{q\tau_{t-1} + \beta \max\{0, C_t - q\tau_{t-1}\}}{\frac{d\tau_t}{dL_{R,t}}}\right)\right\}, \\ L_{R,t} &= \max\left\{0, \frac{\alpha^2}{\alpha^2 + 1 - \alpha} - \frac{1-\alpha}{\alpha^2 + 1 - \alpha} \frac{q\tau_{t-1} + \beta \max\{0, C_t - q\tau_{t-1}\}}{\frac{d\tau_t}{dL_{R,t}}}\right\}. \end{aligned}$$

The corner solution $(L_{A,t}, L_{R,t}) = (1, 0)$[36] would involve a successively decreasing knowledge stock because the ideas of the previous period cannot be recovered fully by direct transfer and knowledge codification. As shown in appendix B, the economy's equilibrium allocation of labor shows a strictly positive share of researchers whenever the following condition holds:

$$\frac{\alpha^2}{1-\alpha} > \frac{q + \beta(1-q)}{q\varepsilon}. \tag{2}$$

Otherwise, the economy would stay in a low level trap with a declining knowledge stock and declining output as a consequence. Without explicit mention otherwise, we assume that (2) holds.

As utility is non-saturated in consumption, one of the market clearing conditions is redundant by Walras law. We leave aside the consumption goods market and concentrate on the intermediate goods market equilibrium condition, which is written as

$$x_t^d = \left(\frac{\alpha}{p_{x,t}}\right)^{\frac{1}{1-\alpha}} L_{A,t} = \left(\frac{\alpha^2 \frac{\partial x_t}{\partial L_{R,t}}}{w_t}\right)^{\frac{1}{1-\alpha}} L_{A,t} = x_t^s.$$

Hence, for all $w_t > 0$ there will be a positive price $p_{x,t}$ that solves the intermediate goods market equilibrium condition. Using the equilibrium wage we have

$$p_{x,t} = \alpha^{2\alpha-1} K_t^{\alpha-1} \left(\frac{\partial \tau_t}{\partial L_{R,t}}\right)^{\alpha-1} (1-\alpha)^{1-\alpha}.$$

Let the homogeneous good be the numéraire. The sequential market equilibrium is then characterized by a price vector

$$\{p_{x,t}, w_t\}_{t=1}^{\infty} = \left\{\alpha^{2\alpha-1} K_t^{\alpha-1} \left(\frac{\partial \tau_t}{\partial L_{R,t}}\right)^{\alpha-1} (1-\alpha)^{1-\alpha}, \quad (1-\alpha)^{1-\alpha} \left(\alpha^2 K_t \frac{\partial \tau_t}{\partial L_{R,t}}\right)^{\alpha}\right\}_{t=1}^{\infty}.$$

2.8 Dynamics

This section introduces the general dynamics of the model, which are described by the difference equations of the economy's three stocks: capital, knowledge and information. The development of the capital stock is determined by each period's saving decision, $K_{t+1} = s_t w_t$. Using the equilibrium wage it transforms into

$$K_{t+1} = s_t K_t^{\alpha} (1-\alpha)^{1-\alpha} \alpha^{2\alpha} \left(\frac{d\tau_t}{dL_{R,t}}\right)^{\alpha}$$

and leads to a growth rate of capital according to

$$g_{K,t} = \frac{K_{t+1} - K_t}{K_t} = s_t K_t^{\alpha-1} (1-\alpha)^{1-\alpha} \alpha^{2\alpha} \left(\frac{d\tau_t}{dL_{R,t}}\right)^{\alpha} - 1. \tag{3}$$

[36] Note that there is no other corner solution due to the Inada-conditions for final goods production and a positive value of the knowledge stock in the first period τ_1.

The behavior of the knowledge stock, given the equilibrium number of researchers, can be written as

$$\tau_{t+1} = \nu(q\tau_t(1+\varepsilon) + \beta(\max\{0, C_{t+1} - q\tau_t\})),$$

where $\nu = \frac{\alpha^2}{\alpha^2+1-\alpha}$. From this equation, we obtain the growth rate of knowledge as

$$g_{\tau,t} = \nu q(1+\varepsilon) + \nu\beta\frac{\max\{0, C_{t+1} - q\tau_t\}}{\tau_t} - 1.$$

Finally the stock of information accumulates according to

$$C_{t+1} = C_t + \triangle C_t \quad , \quad \triangle C_t \geq 0.$$

In the two extreme cases in which the researchers are not codifying at all (indexed by "woC") and the one in which they are codifying every new idea in each period ("wC"), the maximum term in the knowledge stock's difference equation will be zero in the first case and we can set $C_t = \tau_{t-1}$ in the latter. This gives

$$\begin{aligned} g_{\tau,t}^{woC} &= \nu q(1+\varepsilon) - 1, \\ g_{\tau,t}^{wC} &= \nu(q(1+\varepsilon) + \beta(1-q)) - 1. \end{aligned}$$

As we preclude negative codification and codification of ideas that have not yet been discovered, $g_{\tau,t} \in [g_\tau^{woC}, g_\tau^{wC}]$, $\forall t$.

The common standing on the shoulders of giants specification of the research process implies long run growth. As shown in appendix D, the economy possesses two kinds of steady states distinguished by whether the economy exhibits positive or zero codification. Moreover, an economy with zero codification will approach growth at constant rates in the long run. Every steady state exhibits the same relation of the growth rates of capital and knowledge:

$$g_{K,s} = \frac{\alpha}{1-\alpha} g_{\tau,s}. \tag{4}$$

With regard to output, we can write

$$Y_t = F(K_t, L_{A,t}, \tau_t) = K_t^\alpha \tau_t^\alpha \left(\frac{1-\alpha}{\alpha^2\varepsilon q}\right)^{1-\alpha} (1 + g_{\tau,t-1})^{1-\alpha}.$$

Log-differentiating and using (4) verifies that the steady-state growth rate of output equals that of capital

$$g_{Y,s} = g_{K,s} = \frac{\alpha}{1-\alpha} g_{\tau,s}.$$

The dynamics show that with knowledge codification, the economy would reach a higher steady-state growth rate of output. The magnitude crucially depends on the transfer rate q and the reading capacity β. Hence, knowledge codification may account for a large proportion of an economy's growth rate if other knowledge transfer capabilities are bad. When bearing no cost, the researchers would always codify their knowledge to the full extent at the end of the period.

3 Codification Behavior of the Economy

This is the main section of the paper, in which we examine under what conditions an economy will realize the higher growth rates of output with knowledge codification. For this purpose, it will first be necessary to revisit the household problem before establishing three lemmata which unfold the structure of the problem. Thereafter, we will present the results.

Recall the representative agent's utility-maximization problem as introduced in section 2.5. The new firm owners may have an incentive to invest in knowledge codification in the amount of ζ_t. Since one cannot codify something that is not known, and precluding double codification, $\triangle C_t \in [0, \tau_t - C_{t-1}]$. With γ_t being constant within periods and $S_t = K_{t+1}$, the household's utility maximization problem can equivalently be solved via the control variables $K_{t+1}, \triangle C_t$. For convenience, we will use this notation in the following.

The usual procedure is to calculate the derivative of the rent with respect to the amount of codification in t and solve the first order condition for $\triangle C_t$. This is what we are going to do, however, $\frac{dr_{t+1}}{d\triangle C_t}$ exhibits a discontinuity which necessitates some preliminary considerations.
Recall the difference equation of knowledge given that the economy will be in temporary equilibrium in each period:

$$\tau_{t+1} = \nu q \tau_t (1 + \varepsilon) + \nu\beta(\max\{0, C_{t+1} - q\tau_t\}). \tag{5}$$

The maximum term indicates that the codification of one more idea in t contributes to the knowledge stock in $t+1$ only if the young generation in $t+1$ has not been transferred the respective human capital. That is, codification in t of ideas that the young generation of $t+1$ will know without reading the information (because they will have learned it in school), possesses no value in $t+1$. Therefore, codification of an additional idea given the information stock C_t - this idea would be indexed by $C_t + \eta$, $\eta \to 0$ - would enhance the knowledge stock of time $t+1$ by

$$\frac{d\tau_{t+1}}{d\triangle C_t} = \begin{cases} \nu\beta & , \quad \text{if} \quad C_t \geq q\tau_t; \\ 0 & , \quad \text{if} \quad C_t < q\tau_t. \end{cases}$$

Accordingly, we can distinguish the ideas to be codified in t by their marginal value in the subsequent period $t+1$. We denote the number of ideas that are of zero marginal value by $\triangle C_{ie,t} \in [0, \max\{q\tau_t - C_t, 0\}]$ and those with positive marginal value by $\triangle C_{e,t} \in [0, \max\{0, min\{\tau_t(1-q), \tau_t - C_t\}\}]$. It follows that $C_{t+1} = C_t + \triangle C_{ie,t} + \triangle C_{e,t}$. Consequently, we can rewrite (5) as

$$\tau_{t+1} = \nu q \tau_t (1 + \varepsilon) + \nu\beta(\triangle C_{e,t} + \max\{0, C_t - q\tau_t\}). \tag{6}$$

The middle term expresses that the young generation can build up knowledge from reading information codified in the last period $\triangle C_{e,t}$ and, in case it has been transferred a measure of human capital less than what has already been codified in the periods before t, it can additionally attend to "the older books" labelled by indices $i \in [q\tau_t, C_t]$. The latter would imply that all additionally generated information in t is of positive value in the next period, that is, $\triangle C_t = \triangle C_{e,t}$. On the other hand, if $C_t < q\tau_t$, $\triangle C_{ie,t}$ is positive. This implies that the representative agent may also have to pay for information that will not generate any benefit in the next period. Hence, the codification decision of the young generation is affected by assumption (A2). It may involve an additional 'entry cost' for knowledge codification. Without (A2), the representative agent could codify ideas with positive marginal value in the next period only. In this case, C_t cannot be interpreted as the measure of the stock of information. It is just the highest index of all codified ideas. However, with assumption (A2') the term $C_{t+1} - q\tau_t$ is well defined as the (Lebesque)-measure of efficient information. Hence, the dynamics of the knowledge stock are also well defined with (A2') instead of (A2).

Rather than asking how many ideas in total, that is $\triangle C_t$, the young of t will codify, the discontinuity can be eliminated by reformulating the question to: How many ideas of positive marginal value $\triangle C_{e,t}$ is the young generation willing to codify in period t, given the stock of information C_t, that is, given that it may have to codify some ideas of zero marginal value in $t+1$. This structurally corresponds to the household's situation with (A2') and fixed entry costs f. Altogether, the first order conditions of the utility maximization problem then write[37]

$$\begin{aligned} u'(c_{1,t}) &= \delta u'(c_{2,t+1})(1 + r_{t+1}), \\ \gamma_t u'(c_{1,t}) &= \delta u'(c_{2,t+1}) \frac{\partial r_{t+1}}{\partial \triangle C_{e,t}} K_{t+1}. \end{aligned}$$

Inserting optimal saving into the necessary condition with regard to codification then yields

$$\gamma_t(1 + r_{t+1}) = \frac{\partial r_{t+1}}{\partial \triangle C_{e,t}} K_{t+1}. \tag{7}$$

This equation gives all pairs $(K_{t+1}, \triangle C_{e,t})$ that are candidates of the solution to the household's maximization problem. Or, in other words, it defines a function $\triangle C_{e,t}(K_{t+1})$[38], that determines the optimal choice of $\triangle C_{e,t}$ for each optimal K_{t+1}. As we preclude negative codification, the solution to the household's problem must be a point on the graph of this function or involve the corner solution $\triangle C_{e,t} = 0$.

[37]Using $\zeta_t = \gamma_t \triangle C_t$ and $S_t = K_{t+1}$, the household's problem can be transformed to

$$\begin{aligned} \max_{K_{t+1}, \triangle C_{e,t}} \quad U_t(c_{1,t}, c_{2,t+1}) &= u(c_{1,t}) + \delta u(c_{2,t+1}), \\ \text{subject to} \qquad c_{1,t} &= w_t - K_{t+1} - \gamma_t \triangle C_t, \\ c_{2,t+1} &= (1 + r_{t+1}) K_{t+1}. \end{aligned}$$

[38]Using the implicit function theorem, let $M(K_{t+1}, \triangle C_{e,t}) = \frac{\partial r_{t+1}}{\partial \triangle C_{e,t}} K_{t+1} - \gamma_t(1 + r_{t+1}) = 0$. We receive $\frac{\partial M}{\partial \triangle C_{e,t}} = \frac{\partial^2 r_{t+1}}{\partial \triangle C_{e,t}^2} K_{t+1} - \gamma \frac{\partial r_{t+1}}{\partial \triangle C_{e,t}} < 0, \quad \forall K_{t+1} \geq 0$.

Equation (7) illuminates very nicely the household's tradeoff between saving, that is capital investment, and knowledge codification. On the left hand side is the amount of consumption the young would receive when old, if they used the marginal codification costs for saving, which in optimum must be equal to the marginal benefits of codification. Using the equilibrium allocation of labor, the rent given by (1) is written as

$$\begin{aligned} r_t &= K_t^{\alpha-1} L_{A,t}^{-\alpha} \tau_t^{\alpha} (L_{A,t} - 1 + \alpha) \\ &= K_t^{\alpha-1} \left(\tfrac{1-\alpha}{\alpha^2}\right)^{-\alpha} \left(\tfrac{d\tau_t}{dL_{R,t}}\right)^{\alpha} \left(\tau_t \left(\tfrac{1-\alpha}{\alpha^2}\right) \left(\tfrac{d\tau_t}{dL_{R,t}}\right)^{-1} - 1 + \alpha\right). \end{aligned} \tag{8}$$

Calculating the derivative of r_{t+1} with respect to $\triangle C_{e,t}$, inserting it into the marginal condition (7) and using the difference equation of the knowledge stock as in (6), we can solve for $\triangle C_{e,t}$:

$$\begin{aligned} \triangle C_{e,t} = \tfrac{K_{t+1}}{\gamma_t} - \tfrac{K_{t+1}^{1-\alpha}}{\frac{d\tau_{t+1}}{d\triangle C_{e,t}}} \left(\tfrac{1-\alpha}{\alpha^2}\right)^{\alpha-1} \left(\tfrac{d\tau_{t+1}}{dL_{R,t+1}}\right)^{1-\alpha} + \tfrac{\alpha^2-\nu}{\frac{d\tau_{t+1}}{d\triangle C_{e,t}}} \tfrac{d\tau_{t+1}}{dL_{R,t+1}} \\ - \frac{\nu q \tau_t}{\frac{d\tau_{t+1}}{d\triangle C_{e,t}}} - \max\{0, C_t - q\tau_t\}. \end{aligned} \tag{9}$$

Note that these marginal considerations imply that "entry costs" are sunk, in the sense that they have to be paid in any case, or there are no fixed costs for codification. With assumption (A2), this may be because the previous generation has created enough information such that $\triangle C_{ie,t} = 0$. Of course, without entry costs for codification, the young's willingness to codify as reflected in equation (9) is optimal. With (A2) or (A2') and $f > 0$, it is necessary to verify that the representative agent's life-time utility with codification, given entry costs ($\gamma_t \triangle C_{ie,t}$ and/or f) is higher than her utility when only investing in capital and hence not incurring fixed costs. Assumption (A2) certainly represents an extreme, possibly rather hypothetical case where all previously invented ideas have to be codified before being able to codify an idea i. The reasoning behind it may be that if new ideas draw upon previous ideas, the codification of the first would have to refer to the latter. However, if those are not codified, it is not obvious whether this can be easily done, for example, if vocabulary of precisely defined and commonly understood terms is lacking. Cowan et al. (2000) refer to this situation as the lack of a 'code-book'. This case however will not be the main focus of the analysis. Hence, we will proceed with marginal considerations and thereafter address the case with entry costs f that have to be paid once when setting up the structures for knowledge codification. In each period, the case with assumption (A2) can then be interpreted as the previous one with the peculiarity that the entry costs are growing as long as the economy does not codify. Let the case with (A2') and $f = 0$ be the standard situation. This means agents are able to just codify ideas with positive marginal value in $t+1$ without any entry costs. We will make explicit notion when fading in f or (A2).

We are now interested in the codification behavior over time:

$$d\triangle C_{e,t} = \left(\frac{\partial \triangle C_{e,t}}{\partial \tau_t} + \frac{\partial \triangle C_{e,t}}{\partial K_{t+1}} \frac{\partial K_{t+1}}{\partial \tau_t}\right) d\tau_t + \left(\frac{\partial \triangle C_{e,t}}{\partial C_t} + \frac{\partial \triangle C_{e,t}}{\partial K_{t+1}} \frac{\partial K_{t+1}}{\partial C_t}\right) dC_t + \frac{\partial \triangle C_{e,t}}{\partial K_{t+1}} \frac{\partial K_{t+1}}{\partial K_t} dK_t$$

$$= \frac{\partial \triangle C_{e,t}}{\partial \tau_t} d\tau_t + \frac{\partial \triangle C_{e,t}}{\partial C_t} dC_t + \frac{\partial \triangle C_{e,t}}{\partial K_{t+1}} \underbrace{\left(\frac{\partial K_{t+1}}{\partial \tau_t} d\tau_t + \frac{\partial K_{t+1}}{\partial C_t} dC_t + \frac{\partial K_{t+1}}{\partial K_t} dK_t \right)}_{dK_{t+1}}.$$

Changing stocks of knowledge and information possess a direct and an indirect effect via K_{t+1} on $\triangle C_{e,t}$. For convenience, we subsume the indirect effect in dK_{t+1} and will refer to the direct effect when speaking of influences of the knowledge stock or the stock of information on knowledge codification in period t. From (9) it is obvious that the direct effect of a marginal increase in C_t is either zero, if the maximum term is zero[39], or negative. Taking the partial derivative with respect to τ_t yields

$$\frac{\partial \triangle C_{e,t}}{\partial \tau_t} = \begin{cases} \frac{K_{t+1}^{1-\alpha}}{\nu\beta} \left(\frac{1-\alpha}{\alpha^2 \varepsilon q} \right)^{\alpha-1} (1-\alpha)\tau_t^{-\alpha} q + \frac{(\alpha^2-\nu)\varepsilon q - q}{\nu\beta} < 0 & , C_t \leq \tau_t; \\ \frac{K_{t+1}^{1-\alpha}}{\nu\beta} \left(\frac{1-\alpha}{\alpha^2 \varepsilon q} \right)^{\alpha-1} (1-\alpha)\tau_t^{-\alpha} q + \frac{(\alpha^2-\nu)\varepsilon q - q}{\nu\beta} + q < 0 & , C_t > \tau_t. \end{cases}$$

The direct effect of knowledge growth on codification is negative because it increases the rent which, in turn, increases opportunity costs for codification. In contrast, an increasing knowledge stock has no direct effect on the codification benefit, because the rent in $t+1$ is linear in τ_{t+1}[40]. Choosing a marginally higher level of capital saving would influence the representative household's optimal amount of knowledge codification according to

$$\frac{\partial \triangle C_{e,t}}{\partial K_{t+1}} = \frac{1}{\gamma_t} - \frac{(1-\alpha)K_{t+1}^{-\alpha}}{\nu\beta} \left(\frac{1-\alpha}{\alpha^2 \varepsilon q} \right)^{\alpha-1} \tau_t^{1-\alpha}.$$

The partial derivative is negative for small K_{t+1}. More precisely, if

$$\frac{K_{t+1}}{\gamma_t} < \frac{(1-\alpha)K_{t+1}^{1-\alpha}}{\nu\beta} \left(\frac{1-\alpha}{\alpha^2 \varepsilon q} \right)^{\alpha-1} \tau_t^{1-\alpha}.$$

Inserting this condition into (9), we can estimate from above the values of $\triangle C_{e,t}$ for which K_{t+1} capital saving exerts a negative influence on the benefits of knowledge codification:

$$\begin{aligned} \triangle C_{e,t} < -\alpha \frac{K_{t+1}^{1-\alpha}}{\nu\beta} \left(\frac{1-\alpha}{\alpha^2 \varepsilon q} \right)^{\alpha-1} \tau_t^{1-\alpha} + \frac{\alpha^2 \varepsilon - \nu(1+\varepsilon)}{\nu\beta} q\tau_t \\ - \max\{0, C_t - q\tau_t\} < 0. \end{aligned}$$

It shows that a negative partial derivative with respect to K_{t+1} would imply $\triangle C_{e,t} < 0$, which we precluded by assumption. Further, we can formulate

[39] Note that in this case the indirect effect via K_{t+1} is also 0. Of course the clause above holds an inaccuracy since if $C_t = q\tau_t$, the maximum term would be zero but the partial derivative of $\triangle C_{e,t}$ with respect to C_t would be negative.

[40] For this reason $\frac{\partial \frac{\partial r_{t+1}}{\partial \triangle C_{e,t}} K_{t+1}}{\partial \tau_t} = 0$. This is due to the model showing Solow neutral technological progress. With other specifications of technological progress an increasing knowledge stock directly decreases the marginal codification benefit.

Lemma 1 *There exists a single-valued function* $K_{t+1,crit} : \mathbb{R}_+ \times \mathbb{R}_+ \to \mathbb{R}_+$*, which gives the amount of the capital stock in* $t+1$*,* $K_{t+1,crit}(\tau_t, C_t)$*, such that* $\triangle C_{e,t} = 0$*.*

Proof. Consider the derivative of $\triangle C_{e,t}$ with respect to K_{t+1} at $\triangle C_{e,t} = 0$. $\triangle C_{e,t} = 0$ requires

$$\frac{K_{t+1}^{-\alpha}}{\nu\beta}\left(\frac{1-\alpha}{\alpha^2\varepsilon q}\right)^{\alpha-1}\tau_t^{1-\alpha} = \frac{1}{\gamma_t} + \frac{[\alpha^2\varepsilon-\nu(1+\varepsilon)]q\tau_t-\nu\beta\max\{0,C_t-q\tau_t\}}{\nu\beta K_{t+1}},$$

and thence[41]

$$\left.\frac{\partial\triangle C_{e,t}}{\partial K_{t+1}}\right|_{\triangle C_{e,t}=0} = \frac{\alpha}{\gamma_t} - (1-\alpha)\frac{[\alpha^2\varepsilon-\nu(1+\varepsilon)]q\tau_t-\nu\beta\max\{0,C_t-q\tau_t\}}{\nu\beta K_{t+1}} > 0.$$

Using the implicit function theorem proves lemma 1. □

Lemma 2 *Let* $\mathcal{U} \subset \mathbb{R}_+^3$ *be the set* $\{(\tau_t, C_t, K_{t+1}) | \triangle C_{e,t}(\tau_t, C_t, K_{t+1}) \geq 0\}$*.* $\triangle C_{e,t}$ *is a strictly increasing function of* K_{t+1} *on* $\mathcal{U}$*.*

Proof. $\triangle C_{e,t} \geq 0$ implies

$$\frac{K_{t+1}^{-\alpha}}{\nu\beta}\left(\frac{1-\alpha}{\alpha^2\varepsilon q}\right)^{\alpha-1}\tau_t^{1-\alpha} \leq \frac{1}{\gamma_t} + \frac{[\alpha^2\varepsilon-\nu(1+\varepsilon)]q\tau_t-\nu\beta\max\{0,C_t-q\tau_t\}}{\nu\beta K_{t+1}},$$

and therefore

$$\left.\frac{\partial\triangle C_{e,t}}{\partial K_{t+1}}\right|_{\triangle C_{e,t}\geq 0} \geq \frac{\alpha}{\gamma_t} - (1-\alpha)\frac{[\alpha^2\varepsilon-\nu(1+\varepsilon)]q\tau_t-\nu\beta\max\{0,C_t-q\tau_t\}}{\nu\beta K_{t+1}} > 0.$$

From lemma 1, we know that $K_{t+1,crit}(\tau_t, C_t)$ is continuous, and since the partial derivative of $\triangle C_{e,t}$ with respect to K_{t+1} can only be negative for $\triangle C_{e,t} < 0$, $\mathcal{U}$ must be a connected subspace of $\mathbb{R}_+^3$. □

The previous lemmata apply to the marginal consideration leaving out possible entry costs of knowledge codification f and/or $\gamma_t \triangle C_{ie,t}$. In order to address this case, it is necessary to distinguish the two situations which the agent compares in her decision process. As mentioned previously, from a representative household perspective, the decision on knowledge codification does not depend on whether the fixed entry costs originate from having to create information of zero marginal productivity in the next period due to assumption (A2) or from set up costs f or both. This is because in the situation with (A2) the household does not take into account that if it did not codify, the next generation would have to pay even higher entry costs. Hence, in the following lemma, total fixed costs are just denoted by $\tilde{f}$, which may originate from both, (A2) and f, that is, in general $\tilde{f}_t = f + \gamma_t \triangle C_{ie,t}$.[42]

For a given budget w_t:

[41] Note that $\alpha^2 - \nu < 0$ for $\alpha \in (0, 1)$.

[42] Of course, if in a previous period the entry costs f have already been paid, $\tilde{f}_t$ reduces to $\gamma_t \triangle C_{ie,t}$.

(S1) The representative agent does not pay the entry costs of knowledge codification and, hence, invests in physical capital only. Her optimal choice of physical capital saving in a period t is denoted by $\bar{K}_{t+1}$. The generation of period t will then realize life time utility of

$$u[w_t - \bar{K}_{t+1}] + \delta u[(1 + r_{t+1}(\triangle C_{e,t} = 0))\bar{K}_{t+1}] =: U_t^{woC}.$$

(S2) The household pays entry costs $\tilde{f}_t$ and chooses the optimal pair $(\hat{K}_{t+1}, \triangle\hat{C}_{e,t})$. Lifetime utility in this case can be written as

$$u[w_t - \tilde{f}_t - \hat{K}_{t+1} - \gamma_t \triangle\hat{C}_{e,t}] + \delta u[(1 + r_{t+1}(\triangle\hat{C}_{e,t}))\hat{K}_{t+1}] =: U_t^C,$$

for $w_t \geq \tilde{f}_t$, and let $U_t^C(w_t < \tilde{f}_t) := U_t^C(w_t = 0)$.

Note that the household's optimization problem possesses unique solutions in both situations (see Appendix A).

Lemma 3 *For every $(\tau_t, C_t, \tilde{f}_t) \in \mathbb{R}_+^3$ exists a unique $(\hat{K}_{t+1,crit}, \bar{K}_{t+1,crit})$ such that $U_t^C = U_t^{woC}$. For all $\hat{K}_{t+1} > \hat{K}_{t+1,crit}(\tau_t, C_t, \tilde{f})$, $U_t^C > U_t^{woC}$.*

The detailed proof can be found in appendix C.

As a direct consequence of the lemmata, we can formulate the following proposition.

Proposition 1 *An overlapping generations economy with $C_1 \leq q\tau_1$, where $\tau_1 > 0$, and an initial stock of capital K_1 close enough to zero will not be codifying at the beginning of its development.*

Proof. In other words, one can always find a K_1 such that for all τ_1 there exists a time interval $I = \{t | 1 \leq t \leq T\}$ in which $\triangle C_{e,t} = 0$. It is sufficient to show that there is no incentive to codify in the first period from a marginal perspective, leaving out entry costs. Lemma 1 and Lemma 2 imply that if $K_{t+1} < K_{t+1,crit}(\tau_t, C_t)$, the economy realizes the corner solution $\triangle C_{e,t} = 0$ ($\triangle C_{e,t}$ would be negative which is precluded by assumption). It follows from Lemma 1 and $\frac{\partial \triangle C_{e,t}}{\partial \tau_t} < 0$ that $K_{t+1,crit}(\tau_t, C_t)$ is an increasing function of τ_t. Hence, for any $\tau_1 \in \mathbb{R}_{++}$, the economy will not be codifying in the first period if $K_2 \leq K_{t+1,crit}(\tau_1, C_1)$. Equation (9) implies that $K_{t+1,crit}(\tau_1, C_1) > 0$. Consequently there exists a positive $K_2 \leq K_{t+1,crit}(\tau_1, C_1)$. Since $K_{t+1} = s_t w_t$, s_t bound from above and w_t for given τ_t a continuous function of K_t where $w_t(K_t = 0) = 0$, one can always find an initial value K_1 close enough to zero such that $K_2 \leq K_{t+1,crit}(\tau_1, C_1)$ and hence $\triangle C_{e,1} = 0$. □

The intuition of the proof is the same as for lemma 1. That is, the Inada conditions hold for physical capital saving, but not for knowledge codification as the members of the succeeding generation are exogenously transferred a positive share of the previous period's knowledge stock and hence, the marginal benefit of the first idea to be codified is finite.

Knowing that there will be no knowledge codification at the beginning of its development, we are now interested on whether the overlapping generations economy will be codifying in the long run. "Codifying in the long run" or "codifying from some point in time on" means that there does not exist a period t_0, such that for all $t > t_0, \triangle C_{e,t} = 0$.

Proposition 2 *With constant codification costs, an overlapping generations economy will be codifying in the long run if either of the following conditions is satisfied:*

(i) The steady-state growth rate of capital is higher than that of the knowledge stock (or equivalently $\alpha > 0.5$).

(ii) The steady-state growth rate of capital is equal to that of the knowledge stock (or equivalently $\alpha = 0.5$) and

$$\frac{\left(\frac{1-\alpha}{\alpha^2}\right)^{\alpha-1}(\varepsilon q)^{1-\alpha}}{\nu\beta(k_s^{woC})^\alpha(1+g_\tau^{woC})^{\frac{\alpha^2}{1-\alpha}}} < \frac{1}{\gamma} + \frac{\varepsilon q(\alpha^2-\nu)-\nu q}{\nu\beta k_s^{woC}(1+g_\tau^{woC})^{\frac{\alpha}{1-\alpha}}}, \tag{10}$$

where k_s^{woC} is the steady-state level of $k_t = \frac{K_t}{\tau_t}$ without codification.

Proof. We will prove this result by contradiction. The intuition is the following. No codification implies that the economy approaches steady-state growth. It also requires that $\forall t > t_0$, $(\tau_t, C_t, K_{t+1}) \notin \mathcal{U}$. If (i), steady-state growth causes K_{t+1} to grow more than $K_{t+1,crit}$, $\forall t$. If (ii), the steady state level of k involves $K_{t+1} > K_{t+1,crit}$. Hence, for (i) and (ii), steady state growth and no codification are contradictory implying that the economy will be codifying in the long run.

With regard to (i), suppose an economy characterized by $\alpha > 0.5$ will not codify in the long run. That is, $\exists t_0$, such that $\forall t > t_0$, $\triangle C_{e,t} = 0$. Consequently, the overlapping generations economy must approach steady-state growth where $g_{K,s} = \frac{\alpha}{1-\alpha} g_{\tau,s}$. $\alpha > 0.5$ implies that $g_{K,s} > g_{\tau,s}$.
Further let $\triangle C_{e,t_0} = 0$. No codification in the long run means that it is optimal for each subsequent generation to invest in capital only. Hence, $\forall t > t_0$,

$$\triangle C_{e,t} = \frac{K_{t+1}}{\gamma} - AK_{t+1}^{1-\alpha}\tau_t^{1-\alpha} - B\tau_t - \max\{0, C_t - q\tau_t\} \leq 0, \tag{11}$$

where $A = \left(\frac{1-\alpha}{\alpha^2}\right)^{\alpha-1}\frac{(\varepsilon q)^{1-\alpha}}{\nu\beta}$ and $B = \frac{\nu q-(\alpha^2-\nu)\varepsilon q}{\nu\beta} > 0$. Since by assumption the economy does not codify, we can drop the maximum term without loss of generality[43] and rewrite

[43]From $\triangle C_{e,t} = 0$ by assumption and τ_t growing at a constant rate, it follows that the maximum term must be 0 from some point in time on. However, since $\max\{0, C_t - q\tau_t\} \leq (1-q)\tau_t$, we could also estimate $B\tau_t + \max\{0, C_t - q\tau_t\}$ from above by $D\tau_t$ with an appropriate $D > 0$ without affecting the results.

the condition as

$$\frac{K_{t+1}}{\gamma} \leq AK_{t+1}^{1-\alpha}\tau_t^{1-\alpha} + B\tau_t.$$

For $t \to \infty$ both sides of the equation grow without bound. As the economy is growing with constant rates in the long run, (11) will hold for all t if the left hand side is growing at a rate lower than or equal to that of the right hand side. Dividing both sides by τ_t and log-differentiating gives

$$g_{K,t+1} - g_{\tau,t} \leq \frac{AK_{t+1}^{1-\alpha}\tau_t^{-\alpha}((1-\alpha)g_{K,t+1} - \alpha g_{\tau,t})}{AK_{t+1}^{1-\alpha}\tau_t^{-\alpha} + B}.$$

Inserting the relation of the steady-state growth rates of capital and knowledge ($g_{K,s} = \frac{\alpha}{1-\alpha}g_{\tau,s}$) transforms the above inequality into

$$g_{K,s} \leq g_{\tau,s} \quad \text{or} \quad \alpha \leq 1-\alpha.$$

This contradicts the presumption of $g_{K,s} > g_{\tau,s}$ or $\alpha > 0.5$, respectively.

Consider (ii) where $g_{K,s} = g_{\tau,s}$. Again suppose the economy does not codify but satisfies (10). Without codification the economy approaches steady-state behavior in the long run, which implies $k_t = constant$. Note that with CIES-utility the steady-state equilibrium without knowledge codification is unique. In accordance with the household's utility maximization, no codification in the long run involves that $\forall t > t_0$, equation (11) is satisfied. Inserting $K_{t+1} = k_s^{woC}\tau_t^{\frac{\alpha}{1-\alpha}}(1+g_\tau^{woC})^{\frac{\alpha}{1-\alpha}}$ yields

$$\frac{\left(\frac{1-\alpha}{\alpha^2}\right)^{\alpha-1}(\varepsilon q)^{1-\alpha}}{\nu\beta(k_s^{woC})^\alpha(1+g_\tau^{woC})^{\frac{\alpha^2}{1-\alpha}}} \geq \frac{1}{\gamma}\tau_t^{\frac{2\alpha-1}{1-\alpha}} + \frac{\varepsilon q(\alpha^2-\nu)-\nu q}{\nu\beta k_s^{woC}(1+g_\tau^{woC})^{\frac{\alpha}{1-\alpha}}}.$$

By assumption $\alpha = 0.5$ and hence

$$\frac{\left(\frac{1-\alpha}{\alpha^2}\right)^{\alpha-1}(\varepsilon q)^{1-\alpha}}{\nu\beta(k_s^{woC})^\alpha(1+g_\tau^{woC})^{\frac{\alpha^2}{1-\alpha}}} \geq \frac{1}{\gamma_t} + \frac{\varepsilon q(\alpha^2-\nu)-\nu q}{\nu\beta k_s^{woC}(1+g_\tau^{woC})^{\frac{\alpha}{1-\alpha}}}.$$

This contradicts (10). As a consequence, the conditions of proposition 2 preclude that there is a t_0 such that $\forall t > t_0$, $\triangle C_{e,t} = 0$. Hence, the economy will codify in the long run. □

Intuitively speaking, a higher growth rate of capital as compared to that of knowledge implies that the marginal product of capital declines faster than the marginal benefit of knowledge codification. As a consequence, at some point the representative agent uses some of her capital savings to invest in knowledge codification. However, when assuming that $\alpha \approx 0.3$ as often suggested in empirical work, the present result could not explain the observed efforts for knowledge codification. But instead of being costant, it seems plausible that the costs of knowledge codification decreased over the last decades. In the analysis, we thus want to allow for codification costs that decline monotonically over time at a constant rate g_γ. We can then state the following result.

Proposition 3 *An overlapping generations economy will be codifying in the long run if the steady-state growth rate of knowledge exceeds that of capital by less than the rate at which the codification costs decline.*

Proof. The proof uses a similar reasoning as that of proposition 2. Suppose the economy does not codify in the long run and $g_{\tau,s} - g_{K,s} < -g_\gamma$. Let $\triangle C_{e,t_0} = 0$. Further, (11) must hold $\forall t > t_0$. Neglecting the maximum term in (11) and log-differentiating yields

$$g_{K,t+1} - g_{\tau,t} - g_\gamma \leq \frac{AK_{t+1}^{1-\alpha}\tau_t^{-\alpha}((1-\alpha)g_{K,t+1} - \alpha g_{\tau,t})}{AK_{t+1}^{1-\alpha}\tau_t^{-\alpha} + B}.$$

Without codification the economy approaches steady state behavior. Inserting the relation of the steady state growth rates, the inequality above can be written as

$$g_{\tau,s} - g_{K,s} \geq -g_\gamma.$$

This contradicts $g_{\tau,s} - g_{K,s} < -g_\gamma$. □

It further follows:

Corollary 1 *Every overlapping generations economy will be codifying from some point in time on if the rate at which the codification costs decline is greater than or equal to the steady-state growth rate of the knowledge stock.*

Proof. This is a direct result of the proof of proposition 3 as $g_{\tau,s} - g_{K,s} \geq -g_\gamma$ can be transformed into

$$\alpha \leq \frac{1+\frac{g_\gamma}{g_{\tau,s}}}{2+\frac{g_\gamma}{g_{\tau,s}}} \quad , \quad -g_\gamma < 2g_{\tau,s};$$
$$\alpha \geq \frac{1+\frac{g_\gamma}{g_{\tau,s}}}{2+\frac{g_\gamma}{g_{\tau,s}}} \quad , \quad -g_\gamma > 2g_{\tau,s}.$$

$g_{\tau,s} \leq -g_\gamma < 2g_{\tau,s}$ implies that for the economy to not codify in the long run, α must be smaller than or equal to 0. If $-g_\gamma > 2g_{\tau,s}$, α must exceed 1 for zero codification. This contradicts that for all overlapping generations economies $\alpha \in (0,1)$. □

It can be seen from the proof of corollary 1 that an $\alpha < 0.5$ is also consistent with knowledge codification in the long run as long as the codification costs are declining fast enough. Having discussed under what conditions the economy will start to codify sometime, the following proposition focuses on full codification, that is, at the end of each period t, $C_{t+1} = \tau_t$. For simplicity of the argument, let $-g_\gamma < 2g_{\tau,s}$ for the remainder of the paper without loss of generality.

Proposition 4 *An overlapping generations economy in steady state equilibrium that satisfies the following condition*

$$\alpha > \frac{1 + \frac{g_\gamma}{g_{\tau,s}}}{2 + \frac{g_\gamma}{g_{\tau,s}}} \tag{12}$$

is codifying fully.

Proof. First, as shown in the proof of proposition 3, an overlapping generations economy that satisfies the condition given in proposition 4 will start to codify at some point. The number of effective ideas to be codified $\triangle C_{e,t}$ is chosen according to (9), which can be written as

$$\frac{\triangle C_{e,t} + \max\{0, C_t - q\tau_t\}}{\tau_t} = \frac{K_{t+1}}{\tau_t \gamma_t} - AK_{t+1}^{1-\alpha}\tau_t^{-\alpha} - B. \tag{13}$$

Since the economy is assumed to be in steady state, $\triangle C_{e,t}$ must be greater than 0 (because (12) precludes a steady state where $\triangle C_{e,t} = 0$). Hence, above's equality must hold as long as $\triangle C_{e,t}$ possesses an interior solution. Being in steady state, both sides of (13) must grow at equal rates. Log-differentiating yields

$$g_{\triangle C_e + \max\{0, C - q\tau\},t} - g_{\tau,t} = \frac{\frac{K_{t+1}}{\tau_t\gamma_t}(g_{K,t+1} - g_{\tau,t} - g_\gamma) - \frac{AK_{t+1}^{1-\alpha}}{\tau_t^{\alpha}}[(1-\alpha)g_{K,t+1} - \alpha g_{\tau,t}]}{\frac{K_{t+1}}{\tau_t\gamma_t} - AK_{t+1}^{1-\alpha}\tau_t^{-\alpha} - B}.$$

Inserting the steady state relation of the growth rates of knowledge and capital transforms the above equation into

$$g_{\triangle C_e + \max\{0, C - q\tau\},s} - g_{\tau,s} = \frac{\frac{K_{t+1}}{\tau_t\gamma_t}(g_{K,s} - g_{\tau,s} - g_\gamma)}{\frac{K_{t+1}}{\tau_t\gamma_t} - AK_{t+1}^{1-\alpha}\tau_t^{-\alpha} - B}.$$

As the denominator of the right hand side is positive (because $\triangle C_{e,t} > 0$), the entire fraction will be positive whenever $g_{K,s} - g_{\tau,s} - g_\gamma > 0$. The latter can be directly transformed into the condition given in the proposition. The right side being positive implies that the amount of ideas to be codified grows faster than the knowledge stock. This violates the steady state condition that for $\triangle C_{e,t} > 0$, $g_{\triangle C_e + \max\{0, C - q\tau\},s} = g_{\tau,s}$. The only possibility to satisfy this condition is for $\triangle C_{e,t} + \max\{0, C_t - q\tau_t\}$ to realize its upper bound at $(1-q)\tau_t$ as a corner solution. Consequently, an economy in steady state characterized by $\alpha > \frac{1 + \frac{g_\gamma}{g_{\tau,s}}}{2 + \frac{g_\gamma}{g_{\tau,s}}}$ must be codifying fully.
Note also that in the case of fixed codification costs an economy in steady state is codifying fully if $\alpha > 0.5$, that is, if $g_{K,s} > g_{\tau,s}$. □

We can then summarize:

Corollary 2 *If $g_{\tau,s} - g_{K,s} \neq g_\gamma$, an overlapping generations economy in steady state exhibits either full or zero codification in the long run.*

Proof. This corollary is an immediate consequence of propositions 3 and 4. If $g_{\tau,s} - g_{K,s} > g_\gamma$, full codification directly follows from proposition 4.
With regard to $g_{\tau,s} - g_{K,s} < g_\gamma$, the proof of proposition 3 implies that this condition is contradictory to $\triangle C_{e,t} > 0$, $\forall t$. Hence, the economy cannot realize steady-state growth with positive codification. From proposition 3's proof further follows that if an economy shows steady state behavior $\forall t > t_0$ and $\triangle C_{e,t_0} = 0$, then $\triangle C_{e,t} = 0$, $\forall t > t_0$. As a consequence, only steady state growth without codification is consistent with the condition $g_{\tau,s} - g_{K,s} < g_\gamma$.

□

The section precluded possible entry costs to knowledge codification until now. The following proposition considers fixed entry costs that have to be paid once at the beginning of knowledge codification as a kind of set-up costs.

Proposition 5 *An overlapping generations economy that started to codify without fixed entry costs to knowledge codification will also do so with fixed entry costs.*

Proof. The question to be answered is whether, given the conditions under which an overlapping generations economy without fixed entry costs codified in the long run, there is a period in which the representative household receives a higher life time utility by paying the fixed entry costs to knowledge codification and choosing its optimal amount of capital saving and codification investment than by just saving capital. This is effectively a comparison of (S1) and (S2). (S2) can only yield higher life time utility if there is positive knowledge codification, otherwise the fixed costs were just wasted. Hence, the first question is: would some period's representative agent be willing to codify if she had to pay the fixed costs. That is, would it be optimal in (S2) to choose a positive amount of knowledge codification after having paid the fixed costs. If so, lemma 3 gives that there must be a $w^*(f)$ such that $U_t^C = U_t^{woC}$ and $U_t^C > U_t^{woC}$ for all $w_t > w^*(f)$. As $w^*(f)$ is finite and w_t will grow to infinity, this second condition will be given sometime if the first will hold.

Hence, consider an economy in (S2) where the representative agent has to pay fixed costs f when young and thereafter decides on the optimal amount of saving and codification. The constraints of her utility maximization problem can then be transformed into

$$\begin{aligned} c_{1,t} &= w_{net,t}(1 - s_t - \varsigma_t), \\ c_{2,t+1} &= (1 + r_{t+1}) s_t w_{net,t}, \end{aligned}$$

where $w_{net,t} = w_t - f$. Suppose the economy will not codify for all t. Then the saving rate is a function of r_{t+1} only. As $g_{\tau,t} = g_\tau^{woC} =$ const. and $g_{C,t} = 0$, the economy's dynamics are reflected by

$$K_{t+1} = s_t w_{net,t} = s_t(K_t^\alpha \tau_t^\alpha F - f).$$

With $k_t = \frac{K_t}{\tau_t^{\frac{\alpha}{1-\alpha}}}$, it writes

$$k_{t+1} = s_t w_{net,t} = s_t k_t^\alpha (1+g_\tau^{woC})^{-\frac{\alpha}{1-\alpha}} F - s_t f \tau_t^{-\frac{\alpha}{1-\alpha}} (1+g_\tau^{woC})^{-\frac{\alpha}{1-\alpha}}.$$

This implies that $f\tau_t^{-\frac{\alpha}{1-\alpha}}(1+g_\tau^{woC})^{-\frac{\alpha}{1-\alpha}}$ is decreasing at a constant rate. As s_t is bound on the interval $[s_{low}, 1)$, we can estimate it from above by 1. Hence, the last term of k_{t+1} in the above equation becomes arbitrarily small for large enough t. Consequently the difference equation approaches that of an economy without fixed costs in the limit. Being interested in long term behavior, this last term can be neglected for an appropriately large t and the previous propositions apply for positive codification in (S2) in the long run. □

Fixed entry costs that have to be paid once only delay an economy's knowledge codification but do not prevent it. This may be different in the hypothetical case with assumption (A2). This assumption implies that the codification decision may depend on the economy's codification history as previous generations that did not codify accumulated entry costs for later generations.

Proposition 6 *An overlapping generations economy that satisfies*

$$\alpha > \frac{1 + \frac{g_\gamma}{g_\tau^{woC}}}{2 + \frac{g_\gamma}{g_\tau^{woC}}}$$

will be codifying in the long run and independent of its codification history if either of the following conditions holds

(i) $\lim_{t\to\infty} \left| \frac{u'(c_{1,t}^{woC})}{u'(c_{1,t}^{woC}) - u'(c_{1,t}^{C})} \right| = 0;$

(ii) $\lim_{t\to\infty} \left| \frac{u'(c_{1,t}^{woC})}{u'(c_{1,t}^{woC}) - u'(c_{1,t}^{C})} \right| = M < \infty$ *and*
$\frac{\alpha}{1-\alpha} F > M \left(\frac{\alpha(1+g_{K,s})^\alpha E}{1+\alpha(k_{s,max}^{woC})^{\alpha-1} E (1+g_{K,s})^{\alpha-1}} \right);$

where $E = \left(\frac{1-\alpha}{\alpha^2}\right)^{-\alpha} (\varepsilon q)^\alpha \left(\frac{1-\alpha}{\alpha^2 \varepsilon q}(1+g_\tau^{woC}) - 1 + \alpha \right) > 0,$
and $F = (1-\alpha)^{1-\alpha} \alpha^{2\alpha} (\varepsilon q)^\alpha (1+g_\tau^{woC})^{-\alpha},$

Proof. The proof's reasoning is similar to that of proposition 5. No codification in the long run implies that the representative agent must prefer (S1) to (S2) for all $t > t_0$, where t_0 large enough. In (S1) the economy approaches steady state behavior without codification. Again steady-state growth and no codification are contradictory for

$\alpha > \frac{1+\frac{g_\gamma}{g_\tau^{woC}}}{2+\frac{g_\gamma}{g_\tau^{woC}}}$ if either (i) or (ii) holds. As compared to the proof of proposition 5, an additional difficulty is that the codification history enters the representative agents decision problem. In this respect, independence of the codification history means that the economy would start to codify in some period t even if it possessed no information at all, $C_t = 0$. This involves the maximum entry costs in period t of $\gamma_t q \tau_t$. Consequently, if the economy starts to codify under this condition, it must do so with lower entry costs, as well.

The formal argument proceeds as follows. In a first step, we again verify that even if the maximum entry costs had to be paid in (S2), the economy would like to codify eventually. Second, we show that under the conditions given in the proposition and given the economy would not codify, the hypothetical utility if the representative agent chose (S2) instead of (S1) would satisfy $dU_t^C > dU_t^{woC}$ from some point in time t_1 on. Hence, there must be a t_2, where $U_{t_2}^C > U_{t_2}^{woC}$, contradicting the assumption that the economy does not codify in the long run.
The detailed proof is provided upon request.

4 Conclusions

This paper has developed a formal model of endogenous growth that incorporated costly knowledge codification as a means of intergenerational knowledge transfer. The motivation was that in contrast to the usual models of endogenous growth, which treat knowledge codification as a by-product of research and development activities, great efforts of private firms for the purposeful codification of knowledge can be observed.

We find that although knowledge codification positively influences an economy's long-run growth rate of output, initially there will be no knowledge codification in an economy that develops from a small level of capital. This is due to the Inada conditions applying to physical capital saving, but not to investment in knowledge codification. We have consequently examined under which conditions knowledge codification takes place in the long run. It has been shown that an overlapping generations economy with constant codification costs will be codifying from some point in time on if the steady-state growth rate of capital is higher than that of the knowledge stock, or equivalently, if the exponent of the capital stock in the aggregate production function, α, is greater than 0.5. The intuition is that with the capital stock increasing at a higher rate than the knowledge stock, the marginal productivity of knowledge increases relative to that of capital and it will become worthwhile to forgo consumption for preserving ideas by knowledge codification. For equal steady-state growth rates of capital and knowledge, the economy will codify if the steady state relation of capital and knowledge is large enough. Assuming an empirically often suggested $\alpha \approx 0.3$, this result could not explain the recently observed efforts for knowledge codification. However, allowing for monotonically decreasing codification costs over time, it is sufficient for long-run knowledge codification that the steady-state growth rate of knowledge exceeds that of capital by less than the rate at which the codification costs decline. In this case, an $\alpha < 0.5$ would be consistent with knowledge codification in the long run. In fact, if the rate at which the codification costs decline is even higher than the steady-state growth rate of the knowledge stock, every overlapping generations economy will sooner or later engage in costly knowledge codification.

These results do not change when additionally considering fixed entry costs to knowledge codification. The entry costs will only delay but not prevent the codification investment. This may be different in the rather hypothetical case with assumption (A2), in which for an idea to be codified the more basic ideas it builds upon have to be codified first. This implies that the entry costs of knowledge codification increase as long as the economy does not codify. Whether the above conditions for knowledge codification are still sufficient depends on the specific form of the representative household's utility function.

The analysis implies several issues for future research. One would certainly be to further elaborate on the inter-temporal externalities knowledge codification gives rise to. For example, later generations may be able to utilize information that has been created by generations which already passed away, such that no direct compensation is possible. Hence, a normative analysis from a benevolent planner's perspective should be one of the desiderata. Another interesting aspect would be to consider different market structures in the intermediate sector and study their implications for knowledge codification and long-run economic growth.

APPENDIX

A Sufficient Conditions for the Household's Optimization Problem

A sufficient condition for a unique maximum of the household's optimization problem is that the Hessian matrix at the critical points be negative definite. The Hessian matrix writes

$$H = \begin{pmatrix} u''(c_{1,t})+\delta u''(c_{2,t+1})(1+r_{t+1})^2 & \gamma_t u''(c_{1,t})+\delta u''(c_{2,t+1})\frac{\partial r_{t+1}}{\partial \triangle C_{e,t}}K_{t+1}(1+r_{t+1}) \\ & +\delta u'(c_{2,t+1})\frac{\partial r_{t+1}}{\partial \triangle C_{e,t}} \\ \gamma_t u''(c_{1,t})+\delta u''(c_{2,t+1})\frac{\partial r_{t+1}}{\partial \triangle C_{e,t}}K_{t+1}(1+r_{t+1}) & \gamma_t^2 u''(c_{1,t})+\delta u''(c_{2,t+1})\left(\frac{\partial r_{t+1}}{\partial \triangle C_{e,t}}K_{t+1}\right)^2 \\ +\delta u'(c_{2,t+1})\frac{\partial r_{t+1}}{\partial \triangle C_{e,t}} & +\delta u'(c_{2,t+1})\left(\frac{\partial^2 r_{t+1}}{\partial \triangle C_{e,t}^2}K_{t+1}\right) \end{pmatrix}.$$

The eigenvalues of H will be negative at the critical points given by

$$\begin{aligned} M_1 &= -u'(c_{1,t}) + \delta u'(c_{2,t+1})(1 + r_{t+1}) = 0, \\ M_2 &= -\gamma_t u'(c_{1,t}) + \delta u'(c_{2,t+1})\frac{\partial r_{t+1}}{\partial \triangle C_{e,t}}K_{t+1} = 0, \end{aligned}$$

which together yield

$$M = -\gamma_t(1 + r_{t+1}) + \frac{\partial r_{t+1}}{\partial \triangle C_{e,t}} K_{t+1} = 0,$$

if the first principal minor is negative and the second principal minor is positive. The first principal minor is negative due to the concavity of U_t in K_{t+1}:

$$\frac{\partial^2 U_t}{\partial K_{t+1}^2} = u''(c_{1,t}) + \delta u''(c_{2,t+1})(1 + r_{t+1})^2 < 0.$$

This implies the unique maximum in (S1), that is without knowledge codification. The second principal minor will be positive, if and only if

$$\frac{\partial^2 U_t}{\partial K_{t+1}^2} \frac{\partial^2 U_t}{\partial \triangle C_{e,t}^2} - \left(\frac{\partial^2 U_t}{\partial K_{t+1} \partial \triangle C_{e,t}} \right)^2 > 0. \tag{14}$$

Using the first order conditions, we have

$$\frac{\partial^2 U_t}{\partial \triangle C_{e,t}^2} = \gamma_t^2 \frac{\partial^2 U_t}{\partial K_{t+1}^2} + \delta u'(c_{2,t+1}) \frac{\partial^2 r_{t+1}}{\partial \triangle C_{e,t}^2} K_{t+1},$$

and

$$\frac{\partial^2 U_t}{\partial K_{t+1} \partial \triangle C_{e,t}} = \gamma_t \frac{\partial^2 U_t}{\partial K_{t+1}^2} + \delta u'(c_{2,t+1}) \frac{\partial r_{t+1}}{\partial \triangle C_{e,t}}.$$

Inserting and simplifying transforms (14) into

$$\frac{\partial^2 U_t}{\partial K_{t+1}^2} \left(\frac{\partial^2 r_{t+1}}{\partial \triangle C_{e,t}^2} K_{t+1} - 2\gamma_t \frac{\partial r_{t+1}}{\partial \triangle C_{e,t}} \right) > \delta u'(c_{2,t+1}) \left(\frac{\partial r_{t+1}}{\partial \triangle C_{e,t}} \right)^2.$$

r_{t+1} linearly depends on τ_{t+1} and so does τ_{t+1} on $\triangle C_{e,t}$. Hence, $\frac{\partial^2 r_{t+1}}{\partial \triangle C_{e,t}^2} = 0$, and

$$\frac{\partial^2 U_t}{\partial K_{t+1}^2} \left(-2\gamma_t \frac{\partial r_{t+1}}{\partial \triangle C_{e,t}} \right) > \delta u'(c_{2,t+1}) \left(\frac{\partial r_{t+1}}{\partial \triangle C_{e,t}} \right)^2.$$

Cancelling $\frac{\partial r_{t+1}}{\partial \triangle C_{e,t}}$ and using M gives

$$-u''(c_{1,t}) - \delta u''(c_{2,t+1})(1 + r_{t+1})^2 > \frac{\delta}{2} u'(c_{2,t+1}) \frac{1 + r_{t+1}}{K_{t+1}}.$$

Further transformations yield

$$\frac{u''(c_{2,t+1})}{u'(c_{2,t+1})} + \frac{1}{2c_{2,t+1}} < -\frac{u''(c_{1,t})}{u'(c_{1,t})} \frac{1}{1 + r_{t+1}}.$$

Choosing a CIES-utility function, this inequality defines the lower bound of θ by $\theta > \frac{1}{2}\frac{1-s_t-\varsigma_t}{1-\varsigma_t}$. It is always satisfied for $\theta \geq 0.5$. □

B Allocation of Labor in Temporary Equilibrium

This section examines under which conditions the equilibrium allocation of labor possesses an inner solution with positive research. Generally, the question is under which conditions the following equation possesses a fixed point in (0,1):

$$L_{A,t} = \frac{1-\alpha}{\alpha^2}\frac{\tau_t}{\frac{\partial \tau_t}{\partial L_{R,t}}}, \tag{15}$$

where τ_t is a function of $L_{R,t}(= 1 - L_{A,t})$.
Due to the Inada conditions of the aggregate production function, $L_{A,t} > 0$.
To examine whether $L_{A,t} < 1$, the intuition is that since the right hand side of equation (15) is monotonically increasing in τ_t, there must be a τ_{crit} such that $L_{A,t} = 1$. Consequently, the labor market realizes corner solutions for all periods t where $\tau_t > \tau_{crit}$ and positive research in all periods t where $\tau_t < \tau_{crit}$. Therefore, we ask what is the highest level of knowledge that can be reached in each period t and then examine the allocation in the labor market. If the respective allocation shows positive research, we can be sure to also have an inner solution for lower levels of the knowledge stock. As shown in section 2.8, the highest knowledge stock in each period t is realized with full codification. Hence, the fixed point problem takes the form

$$L_{A,t} = \frac{1-\alpha}{\alpha^2}\frac{\tau_{t-1}(q+\beta(1-q)+\varepsilon q(1-L_{A,t}))}{q\varepsilon\tau_{t-1}}.$$

Consequently, $L_{A,t}$ will be smaller than 1 if the following condition holds:

$$\frac{\alpha^2}{1-\alpha} > \frac{q+\beta(1-q)}{q\varepsilon}.$$

C Proof of Lemma 3

The proof proceeds as follows. First, we ascertain that for planned capital saving $K_{t+1} > K_{t+1,crit}$[44], the representative agent would be willing to pay entry costs for codification up to a certain amount. In a second step, we identify the uniqueness of a w_t^* for every amount of fixed costs which leaves the agent just indifferent between paying entry costs for codification or investing in capital only. The uniqueness of the solutions $(\hat{K}_{t+1}, \triangle\hat{C}_{e,t}), \bar{K}_{t+1}$ to the household's optimization problem for every w_t yields the lemma's contention.

If $\tilde{f}_t = 0$, lemma 1 implies $\hat{K}_{t+1,crit} = \bar{K}_{t+1,crit} = K_{t+1,crit}$. Further, from lemma 2 we know that if $\hat{K}_{t+1} > \hat{K}_{t+1,crit}$, $U_t^C > U_t^{woC}$. That is,

$$\begin{aligned} &u[w_t - \hat{K}_{t+1} - \gamma_t\triangle\hat{C}_{e,t}] + \delta u[(1+r_{t+1}(\triangle\hat{C}_{e,t}))\hat{K}_{t+1}] \\ &\quad > u[w_t - \bar{K}_{t+1}] + \delta u[(1+r_{t+1}(\triangle C_{e,t}=0))\bar{K}_{t+1}]. \end{aligned}$$

[44] $K_{t+1,crit}$ represents the critical value of saving without entry costs as introduced by the previous lemmata.

The representative household's budget is its wage w_t. Fixed costs would reduce the budget the agent can allocate for consumption, physical capital saving and knowledge codification. A reduction of the resources to allocate must reduce utility. The reason is that utility is assumed to strictly increase in consumption. Hence reducing the budget by one unit of the homogeneous good, the agent must abstain from one unit of consumption when young, if she wants to realize the same allocation $(\hat{K}_{t+1}, \triangle\hat{C}_{e,t})$ as before or she chooses another optimal pair $(\hat{K'}_{t+1}, \triangle\hat{C'}_{e,t})$, which must lead to less lifetime utility, because otherwise it is not possible that her previous choice $(\hat{K}_{t+1}, \triangle\hat{C}_{e,t})$ was optimal. The reason is that she could always replicate $(\hat{K'}_{t+1}, \triangle\hat{C'}_{e,t})$ with the higher budget. Hence,

$$\frac{dU_t}{dw_t} > 0.$$

Since being able to choose a pair $(\hat{K}_{t+1}, \triangle\hat{C}_{e,t})$, when $\hat{K}_{t+1} > \hat{K}_{t+1,crit}$, instead of $\bar{K}_{t+1}$ leads to higher lifetime utility, utility is continuous in the budget constraint w_t and $U(w_t = 0) \leq 0$, there must be a unique $w_{net,t}$ such that

$$\begin{aligned} u[w_{net,t} - \hat{K}_{t+1} - \gamma_t \triangle\hat{C}_{e,t}] + \delta u[(1 + r_{t+1}(\triangle\hat{C}_{e,t}))\hat{K}_{t+1}] \\ = u[w_t - \bar{K}_{t+1}] + \delta u[(1 + r_{t+1}(\triangle C_{e,t} = 0))\bar{K}_{t+1}]. \end{aligned}$$

Hence, the representative household is willing to pay a maximum entry cost of $w_t - w_{net,t} =: \tilde{f}^{max}$. With this result, we define:

$$\begin{aligned} \triangle U_t(w_t, \tilde{f}_t) &:= u[w_t - \tilde{f}_t - \hat{K}_{t+1} - \gamma_t \triangle\hat{C}_{e,t}] + \delta u[(1 + r_{t+1}(\triangle\hat{C}_{e,t}))\hat{K}_{t+1}] \\ &\quad -u[w_t - \bar{K}_{t+1}] - \delta u[(1 + r_{t+1}(\triangle C_{e,t} = 0))\bar{K}_{t+1}] \\ &= U_t^C(w_t, \tilde{f}_t) - U_t^{woC}(w_t) \end{aligned}$$

and

$$\triangle U_t(w_t, \tilde{f}^{max}) = 0.$$

Let $\Sigma'_t = S_t + \zeta_t$ be the total amount of investment in t without fixed costs.[45] In the situation where the household does not codify $\bar{\Sigma'}_t = S_t = \bar{K}_{t+1}$ and in (S2) $\hat{\Sigma'}_t = \hat{K}_{t+1} + \gamma_t \triangle\hat{C}_{e,t}$. Due to the strict concavity of utility in consumption, we must have

$$\frac{d\Sigma'_t}{dw_t} > 0.$$

The argument is that the representative household chooses Σ'_t such that $u'(c_{1,t}) = \delta u'(c_{2,t+1})\frac{dc_{2,t+1}}{d\Sigma'_t}$. Relaxing the budget constraint by one unit would decrease $u'(c_{1,t})$ when keeping Σ'_t constant. If Σ'_t is unchanged, it follows that $u'(c_{1,t}) < \delta u'(c_{2,t+1})\frac{dc_{2,t+1}}{d\Sigma'_t}$. Hence, equalling out marginal utility, the agent must enhance total investment. We further know from Lemma 2 that for $\hat{K}_{t+1} > K_{t+1,crit}$, it is optimal to invest in codification. Hence, when relaxing the budget constraint by one unit, the increase in

[45] By this, $\zeta_t = \gamma_t \triangle C_{e,t}$.

lifetime utility with knowledge codification must be greater than without. That is, if $\hat{K}_{t+1} > K_{t+1,crit}(\tau_t, C_t)$,

$$\frac{dU_t^C}{dw_t} > \frac{dU_t^{woC}}{dw_t}.$$

It follows that for any level of fixed entry costs to knowledge codification and $\hat{K}_{t+1} > K_{t+1,crit}(\tau_t, C_t)$,

$$\frac{d\triangle U_t(w_t, \tilde{f}_t)}{dw_t} = \frac{dU_t^C(w_t, \tilde{f}_t)}{dw_t} - \frac{dU_t^{woC}(w_t)}{dw_t} > 0.$$

Consequently, $\triangle U_t(w_t, \tilde{f}_t) = 0$ implicitly defines a function $w_t^* : \mathbb{R}_{++} \to \mathbb{R}_{++}$ which gives the wage for every fixed cost level $\tilde{f}_t$ such that the representative household enjoys the same utility in situations (S1) and (S2). As

$$\frac{dw_t^*}{d\tilde{f}_t} = -\frac{\frac{\partial \triangle U_t(w_t, \tilde{f}_t)}{\partial \tilde{f}_t}}{\frac{\partial \triangle U_t(w_t, \tilde{f}_t)}{\partial w_t}} > 0,$$

$w_t^*(\tilde{f}_t)$ is strictly increasing in $\tilde{f}_t$.
In the case with (A2), the knowledge stock τ_t and the stock of information C_t determine entry costs $\gamma_t \triangle C_{ie,t}$. Hence, at $w_t^*(\gamma_t \triangle C_{ie,t})$, $\triangle U_t = 0$. The fact that there is a unique choice of $(\hat{K}_{t+1}, \triangle\hat{C}_{e,t})$ and $\bar{K}_{t+1}$ for every w_t completes the proof.
The claim that $\triangle U_t > 0$ for $\hat{K}_{t+1} > \hat{K}_{t+1,crit}(\tau_t, C_t, \tilde{f}_t)$ follows directly from $\frac{dU_t^C}{dw_t} > \frac{dU_t^{woC}}{dw_t}$, $\frac{d\Sigma_t'}{dw_t} > 0$ and Lemma 2. □

D Existence of Non-Trivial Steady States

In this section of the appendix, we show that different steady states with positive growth rates exist, and that the overlapping generations economy must exhibit steady state behavior in the long run in the case of zero codification. By the attribute "non-trivial", we intend to preclude $K_1 = 0$ which would imply $K_t = 0$, $\forall t$.
The overlapping generations economy is characterized by the following system of difference equations:

$$\begin{aligned} K_{t+1} &= s_t K_t^{\alpha}(1-\alpha)^{1-\alpha}\alpha^{2\alpha}(\varepsilon q)^{\alpha}\tau_{t-1}^{\alpha}, \\ \tau_{t+1} &= \nu q \tau_t + \nu\beta(\triangle C_{e,t} + \max\{0, C_t - q\tau_t\}) + \nu\varepsilon q \tau_t, \\ C_{t+1} &= C_t + \triangle C_{ie,t} + \triangle C_{e,t}. \end{aligned}$$

The respective growth rates are

$$g_{K,t} = s_t K_t^{\alpha-1}(1-\alpha)^{1-\alpha}\alpha^{2\alpha}(\varepsilon q)^{\alpha}\tau_{t-1}^{\alpha} - 1,$$

$$
\begin{aligned}
g_{\tau,t} &= \nu q + \nu\beta \frac{\triangle C_{e,t} + \max\{0, C_t - q\tau_t\}}{\tau_t} + \nu\varepsilon q - 1, \\
g_{C,t} &= \frac{\triangle C_{ie,t} + \triangle C_{e,t}}{C_t}.
\end{aligned}
$$

The growth rates change from one period to the next according to

$$
\begin{aligned}
dg_{K,t} &= s_t K_t^{\alpha-1}(1-\alpha)^{1-\alpha}\alpha^{2\alpha}(\varepsilon q)^{\alpha}\tau_{t-1}^{\alpha}(g_{s,t} - (1-\alpha)g_{K,t} + \alpha g_{\tau,t-1}), \\
dg_{\tau,t} &= \nu\beta \frac{\triangle C_{e,t} + \max\{0, C_t - q\tau_t\}}{\tau_t}(g_{\triangle C_e + \max\{0, C - q\tau\},t} - g_{\tau,t}), \\
dg_{C,t} &= \frac{\triangle C_{ie,t} + \triangle C_{e,t}}{C_t}(g_{\triangle C_{ie} + \triangle C_e,t} - g_{C,t}).
\end{aligned}
$$

Being defined by constant growth rates, steady states imply $(dg_{K,t}, dg_{\tau,t}, dg_{C,t}) = (0,0,0)$, $\forall t$. Consequently, there are two kinds of non-trivial steady states.

1. Steady state without codification
   If the economy does not codify, $\triangle C_{e,t} + \max\{0, C_t - q\tau_t\} = 0$ and $\triangle C_{ie,t} + \triangle C_{e,t} = 0$. This implies $g_{C,s} = 0$ and $g_{\tau,s} = g_\tau^{woC}$.
   As s_t is bound on $[s_{low}, 1)$, the saving rate cannot grow at a constant rate other than 0. Consequently, $g_{K,s} = \frac{\alpha}{1-\alpha} g_{\tau,s}$. Therefore, a steady state without codification is characterized by

$$
\begin{aligned}
g_{K,s} &= \frac{\alpha}{1-\alpha} g_{\tau,s}, \\
g_{\tau,s} &= g_\tau^{woC} = \nu q(1+\varepsilon) - 1, \\
g_{C,s} &= 0.
\end{aligned}
$$

2. Steady state with codification
   Positive codification implies $\triangle C_{e,t} > 0$. Therefore, the overlapping generations economy can only realize steady state behavior if $g_{C,s} = g_{\triangle C_{ie} + \triangle C_e,s}$ and $g_{\tau,s} = g_{\triangle C_e + \max\{0, C - q\tau\},s}$. By the same argument with respect to the saving rate as in the case with zero codification, the relation of the growth rates of capital and knowledge will be $g_{K,s} = \frac{\alpha}{1-\alpha} g_{\tau,s}$. Consequently, steady states with positive codification imply

$$
\begin{aligned}
g_{K,s} &= \frac{\alpha}{1-\alpha} g_{\tau,s}, \\
g_{\tau,s} &= g_{\triangle C_e + \max\{0, C - q\tau\},s}, \\
g_{C,s} &= g_{\triangle C,s},
\end{aligned}
$$

   where $\triangle C_t = \triangle C_{ie,t} + \triangle C_{e,t}$. Note that a steady state with full codification is a special case where $g_{\tau,s} = g_\tau^{wC} = g_{\triangle C_e + \max\{0, C - q\tau\},s} = g_{C,s} = g_{\triangle C,s} = \nu(q(1+\varepsilon) + \beta(1-q)) - 1$.

We will now show that without codification, the overlapping generations economy will approach a unique non-trivial steady state.[46] Let $k_t := \frac{K_t}{\tau_t^{\frac{\alpha}{1-\alpha}}}$. Knowing that in steady state $g_{K,s} = \frac{\alpha}{1-\alpha} g_{\tau,s}$, $k_t = const.$, $\forall t$, is a necessary condition for the economy to be in steady state. Assuming no codification implies $g_{C,t} = 0$ and $g_{\tau,t} = g_\tau^{woC} = constant$. In this case, $k_t = constant$ is also sufficient for steady state behavior of the economy. Hence, we can summarize the economy's dynamics by the following first order difference equation:

$$\phi(k_t) = k_{t+1} = \frac{K_{t+1}}{\tau_{t+1}^{\frac{\alpha}{1-\alpha}}} = s_t k_t^\alpha \tilde{Q},$$

where $\tilde{Q} = (1 + g_\tau^{woC})^{-\frac{\alpha(2-\alpha)}{1-\alpha}} (1-\alpha)^{1-\alpha} \alpha^{2\alpha} (\varepsilon q)^\alpha$.

With CIES-utility, the representative household's saving rate without knowledge codification can be written as

$$s_t = [\delta^{-\frac{1}{\theta}} (1 + k_{t+1}^{\alpha-1} \tilde{E})^{\frac{\theta-1}{\theta}} + 1]^{-1},$$

where $\tilde{E} = \left(\frac{1-\alpha}{\alpha^2}\right)^{-\alpha} (\varepsilon q)^\alpha (1 + g_\tau^{woC})^{-\alpha} \left(\frac{1-\alpha}{\alpha^2 \varepsilon q}(1 + g_\tau^{woC}) - 1 + \alpha\right) > 0$. Consequently, a steady state implies

$$\delta^{-\frac{1}{\theta}} (1 + (k_s^{woC})^{\alpha-1} \tilde{E})^{\frac{\theta-1}{\theta}} + 1) = (k_s^{woC})^{\alpha-1} \tilde{Q}.$$

This equation possesses a unique solution for k_s^{woC}. It can further be easily verified that in the case of zero knowledge codification the aggregate production function satisfies the sufficient conditions to not approach an equilibrium of global contraction for initial values $k_1 > 0$ (for such conditions see e.g. Galor and Ryder (1989)).

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