Risk based inspection and maintenance planning

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Introduction

The interest in reliability and risk-based inspection planning and risk management for deteriorating structures, installations and production facilities has been steadily increasing over the last decade. It is now often a requirement by legislation, owners and operators that inspection and maintenance activities are planned to fulfil stated requirements to the maximum acceptable risk of personnel and environment and that the effect of inspection and maintenance activities is assessed in terms of life cycle costs including costs of inspections, costs of repairs and costs of failures.

Considering the strong interest in the field of reliability and risk based inspection planning and the many new developments during the last few years in different industries it was decided to try to bring together the different parties involved in the field in order to achieve an overview of the present state of the art, strong-points and short-comings in practical applications and the major challenges lying ahead.

On this basis a workshop was conducted in collaboration between the Federal Institute of Technology (ETH), Switzerland, the Technical University of Aalborg (AAU), Denmark and Bureau Veritas (BV), France. The workshop took place on December 14 – 15, 2000 at ETH Hönggerberg in Zurich, Switzerland.
In order to ensure a right balance between the issues covered, both professionals from the academia and the industry were invited. Including

- Legislative authorities
- Owner and operator organisations
- Consulting engineers

representing the fields of

- Bridges and other civil engineering structures
- Offshore structures, storage, production facilities and pipelines
- Onshore and offshore process facilities

A total of 34 professionals participated in the workshop and hereof 17 with written and/or oral contributions. Each contributor was given about 20 minutes for presentation, which was followed by about 5 minutes of discussions. In the end of the workshop a general discussion took place.

During this discussion a number of subjects were raised. In the following the main conclusions of these are first summarised and thereafter the presented papers are given.

On behalf of the organisers of the workshop I would like to express my gratitude to all participants of the workshop, which has provided us all a good overview of the current state and a basis for further progress in the area of risk based inspection planning.

Prof. Dr. Michael Havbro Faber
ETH Zürich, Institut für Baustatik und Konstruktion

Summary of Discussion

Acceptance criteria and optimality

It seems that in this area, which is normally treated, as an integral part of the various types of risk analysis there still is a need to build bridges between the fundamental theoretical concepts and the practical implementation. There seems to be significant differences of opinion in regard to how acceptable risks are formulated and even more so when it comes to the implementation of risk acceptance criteria in practice. One example, which was discussed, was the precautionary principle and whereas it was argued that this principle was appropriate for practical implementations it was argued that for theoretical reasons such a principle would be inappropriate, as it did not recognise the basic principles of the decision theory. As a conclusion of this discussion it seems fair to state that further work is required in the area. We need to bring back the principles of risk analysis and risk acceptability to the basics and having agreed on the basis we should slowly rebuild the practical implementations in the different application areas. There is a need for establishing a common understanding for the theoretical concepts and a need for establishing a homogeneous or even standardised approach for the implementation.

Inspection and maintenance planning methodology

Several presentations illustrated that these methods have found their way into practical applications in a variety of application areas. Reliability and risk based formulations for decision making in regard to inspection and maintenance activities have proven their value. However, it is generally appreciated that the methods are not yet applicable for non-professionals in the area of reliability analysis and risk analysis. And more work to this end is needed. Ongoing work points in the direction of producing generic schemes for inspection planning, thus allowing non-professionals to use pre-fabricated risk based inspection plans for their inspection scheduling. Even though this has proven possible and feasible for structures subject to fatigue degradation much more work is needed considering degradation processes for both steel and concrete structures.

Degradation of concrete structures

Inspection and maintenance planning for concrete structures as for all other types of installations requires that quantitative models are available for the description of the quality of the performed inspections as well as for the quantification of consequences of degradation. These topics are not satisfactorily covered by the state of the art. Only for very few inspection methods has work been done in regard to the quantification of the quality, e.g. half-cell potential measurements. It is needed to establish a consistent approach to the utilisation of the commonly applied inspection methods for the condition control of concrete structures in general. A further aspect is the apparently complete lack of probabilistic models for the assessment of the spatial distribution of degradation in concrete structures. A simple model for this was presented during the workshop but evidently much more work is needed in this area. Only when such models have been formulated and calibrated can the risk based inspection planning principles be fully utilised for structures and installations build of concrete.

Degradation of steel structures
The area of degradation modelling for steel structures seems to be well covered in regard to fatigue degradation. However, in the area of corrosion, the model basis is still somewhat weak. Even though a number of materials studies have been performed for the assessment of corrosion effects under various conditions corrosion models, which are adequate for implementations in engineering assessments are still very sparse. It seems that the is a need to try to bring together the corrosion material experts and the engineers who need to have a model basis for corrosion effects to support decision making in regard to structural aspects.

Consequence modelling
It seems that very little work has been performed in the area of modelling the consequences of the various states of structures seen in a maintenance perspective. Models were presented and discussed during the workshop but more work is needed in the area in order to ensure that the formulated strategies for maintenance and inspection activities and the aspects of how the individual owner organisations organisation influence these issues can be taken in to account in a way which represent the state of practice. Only when this is achieved will there be a basis for influencing and improving this.

Risk Based Inspection – the Framework

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Keywords: Risk, reliability, inspections, POD, deterioration, costs, safety, environment, acceptance criteria.

Summary

The present paper addresses the framework for Risk Based Inspection planning (RBI). Following a motivation for Risk Based Inspection planning the RBI problem complex is summarised and subsequently the individual the individual aspects of RBI are highlighted. The paper in this way addresses the theoretical framework for risk based inspection planning, consequence assessment, modelling of uncertainties, assessment of probabilities, modelling of inspections, modelling of engineering systems in terms of logical systems, modelling of deterioration processes and finally acceptance criteria for RBI.

1. Introduction

Risk management in general has become an issue subject to increased awareness by both the industry and the society during the last decades. This is to a large extent due to an urge for increased efficiency and competitiveness by the industry itself but origins also in the increasing standards regarding safety to personnel and environmental preservation, imposed by society.

Engineering systems such as offshore structures, bridges, ship hulls, pipelines and process systems are ideally designed to ensure an economical operation throughout the anticipated service life in compliance with given requirements and acceptance criteria. Such acceptance criteria are typically related to the safety of personnel and risk to environment.

Deterioration processes such as fatigue crack growth and corrosion will always be present to some degree and depending on the adapted design philosophy in terms of degradation allowance and protective measures the deterioration processes may reduce the performance of the system beyond what is acceptable. In order to ensure that the given acceptance criteria are fulfilled throughout the service life of the engineering systems it may thus be necessary to control the development of deterioration and if required to install corrective maintenance measures. In usual practical applications inspection is the most relevant and effective means of deterioration control.

Planning of inspections concerns the identification of what to inspect, how to inspect, where to inspect and how often to inspect. Even though inspections may be used as an effective means for controlling the degradation of the considered engineering system and thus imply a potential benefit they may also have considerable impact on the operation of the system and other economical consequences themselves. For this reason it is necessary to plan the inspections such that a balance is achieved between the expected benefit of the inspections and the corresponding economical consequences implied by the inspections themselves.

During the last 10 to 15 years reliability based and risk based approaches have been developed for the planning of inspections, see e.g. Skjong [19], Madsen et al. [10] and Fujita et al. [6]. These approaches have by now developed in to practical applicable procedures and are applied in various industries, see e.g. Goyet et al. [7], Sindel and Rackwitz [18] and Moan et al. [13]. These approaches take basis in the decision theory to minimize overall service life costs including direct and implied costs of failures, repairs and inspections.

2. Motivation for Risk Based Inspection Planning

Deterioration processes acting on components of engineering facilities are of a highly uncertain nature and are best described in probabilistic terms. Due to this uncertainty there will always be a certain probability that a given component of the facility fails during operation.
The consequence of component failure e.g. in terms of potential loss of lives or costs will depend on the component and its importance for the operation of the facility.
The risk associated with the component is the product of the probability of component failure and the consequence of failure. The Risk Based Inspection (RBI) approach takes basis in a quantification of risk not only on a component basis but for all components on the installation as a whole. Different inspection strategies with different inspection effort, inspection quality and costs will have different effect on the risk. By comparing the risk associated with different inspection strategies the inspection strategy implying the smallest risk can be identified.
Different deterioration processes will follow different patterns both time wise and in terms of location in the facility depending on the choice of materials, detailing of the structures and process systems, production characteristics, loading and exposure to aggressive environments. Even though design strategies may attempt to mitigate or minimise the effect of deterioration processes by choice of material or dimensions, deterioration processes will still occur due to errors or flaws during manufacturing and executions.

2.1 Influence of uncertainties

The deterioration processes are only partly understood and their evolution in time is associated with significant uncertainty see Figure 1. Statistical or probabilistic models can be formulated for the prediction of future deterioration. The probabilistic models are usually based on a mixture of physical understanding, observations and experience. Observations of the actually occurring deterioration e.g. obtained by inspection may be introduced into the models and greatly enhance the precision of their predictions.

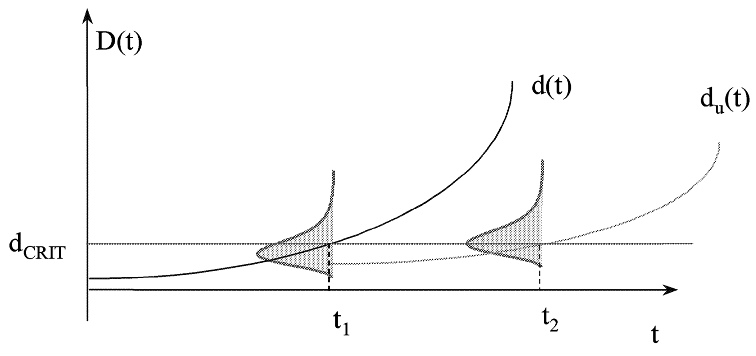


Figure 1 Illustration of predicted future deterioration.

The statistical characteristics of the future deterioration are decisive for the estimation of the future performance of a considered component and thus for the safety.

Inspections may be used as a tool to reduce the uncertainty in the predicted deterioration and/or as a means of identifying deterioration before it becomes critical.
In figure 1 is shown how the predicted future deterioration will significantly change if the observed deterioration state is used to update the probabilistic deterioration model at the time of the first

inspection t_1. If d_{crit} is assumed to be a critical "size" of the expected value of the predicted deterioration state, inspections may be planned such that this deterioration state is not exceeded.

Inspections and maintenance actions are also subject to substantial uncertainties. The quality of inspections is normally quantified in terms of their ability to detect and size the defects of consideration. Different inspection methods may thus be adequate for the inspection of different deterioration processes. Inspection strategies are specified in terms of inspection method, time intervals between inspections and coverage.

Assuming that observed deterioration states will be subject to remedial actions if they are considered serious it is easily realised that there is a strong relationship between inspection quality, inspection intervals, inspection coverage and the safety which is achieved as a result of the inspection. However, it should be emphasised that safety is only increased by remedial actions. Inspections themselves do not increase actual safety but may reduce uncertainty in the assessment of the actual condition.

Inspection planning based on the RBI approach is a rational and cost efficient decision framework for determining

- where to inspect
- what to inspect
- how to inspect
- when to inspect

and at the same time ensuring and documenting that requirements to the safety of personnel and environment are fulfilled.

Furthermore the RBI approach readily provides guidance on actions to be taken depending on the inspection results.

Traditional inspection planning approaches takes basis in prescriptive rules and leaves little possibility to adapt the inspection effort to the actual condition of the components nor the importance of the component for the operation of the installation.

The RBI approach is a condition based approach and provides a rational basis for adapting the inspection effort to the condition of the component and for prioritising inspection efforts in accordance with the importance of the individual components and the different deterioration mechanisms.

In comparison to traditional inspection planning methods only the RBI approach

- targets inspection efforts to high risk components
- quantifies the effect of inspections
- documents the safety of the engineering facility

3. The Problem Complex

Risk based inspection planning takes basis in the formulation of acceptance criteria specifying the acceptable risk in relation to safety to personnel, environmental risk and costs consequences. The inspection and maintenance activities are then planned such that the acceptance criteria are fulfilled throughout the service life of the considered engineering facility. The performance of engineering facilities i.e. the degradation over time is subject to a number of uncertainties. These include operational conditions, material characteristics and environmental exposure. The uncertainties have origin in inherent physical randomness and in uncertainties associated with the models used to assess the performance of the systems. If, furthermore, the statistical basis for the assessment of the uncertainties is limited then also statistical uncertainties may be important.

It is also important to realise that the degree of control of the engineering systems achieved by the inspections is strongly influenced by the reliability of the inspections, i.e. their ability to detect and size degradation. The reliability and thus the information achieved by inspections is strongly dependent on the quality of the planned inspections, the coverage of the planned inspections and the times where the inspections are performed. The reliability of inspections themselves may be subject to very significant uncertainty and this must be taken into account in the planning of inspections and when the results of inspections are interpreted and used to update the predicted performance of the considered engineering facility.

Given that inspections reveal a state of degradation, which is unacceptable, various methods of repairs may be implemented and the future performance of the engineering facility will thus depend on the choice of repair method as well as the quality of the implemented repair.

The problem complex is visualised in Figure 2.

Figure 2 Risk based inspection planning complex.

From figure 2 it is seen that the number of decision variables and dependencies underlying an inspection planning problem is rather large and calls for a systematic treatment.

4. Theoretical Framework

Formulating the inspection and maintenance planning problem as a problem where the overall service life costs are minimized the pre-posterior analysis from the classical decision theory see e.g. Raiffa and Schlaifer [16] and Benjamin and Cornell [1] provides a consistent and systematic framework for its solution. Here a short summary is given closely following Faber et al. [3].

The inspection decision problem may be represented as shown in Figure 3. With reference to Figure 3, the parameters defining the inspection plan may be collected in $\mathbf{i} = (\mathbf{\Delta t}, \mathbf{l}, \mathbf{r})^T$ where $\mathbf{\Delta t} = (\Delta t_1,..,\Delta t_N)^T$ are the intervals between the times of N inspections $\mathbf{t} = (t_1,..,t_N)^T$, $\mathbf{l} = (\mathbf{l}(t_1),..,\mathbf{l}(t_N))^T$ are the locations to inspect at the inspection times with $\mathbf{l}(t_i) = (l_1,..,l_{M(t_i)})^T$. Finally $\mathbf{r} = (r_1,..,r_N)^T$ defines the reliability (quality) of the planned inspections.

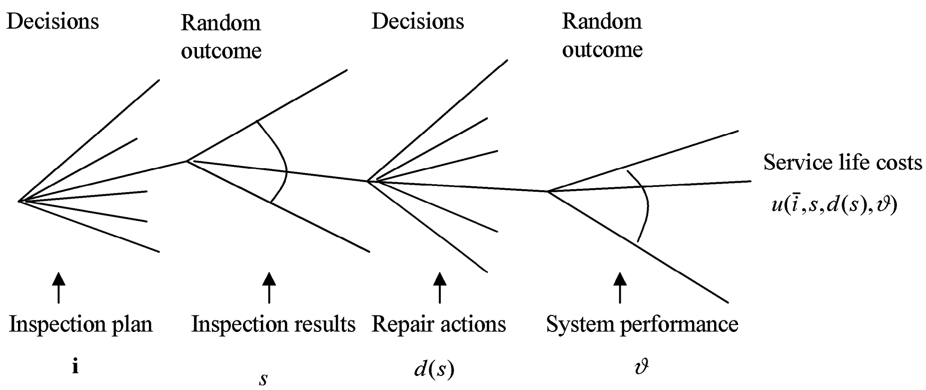


Figure 3. Inspection planning decision tree.

The inspection results are uncertain due to the fact that they depend not only on the uncertain performance of the inspection itself but also on the uncertain state of degradation. The uncertain inspection results, see Figure 3, are modeled by the random vector $\mathbf{S} = (\mathbf{S}(t_1),..,\mathbf{S}(t_N))^T$ in which the individual components refer to the results obtained from the inspections at the different locations $\mathbf{l}(t_i)$. $M(t_i)$ is the total number of inspection locations at time t_i. $d(s)$ is a decision rule defining the repair action to take depending on the inspection result. Finally ϑ is the realization of the uncertainties θ influencing the state of the system.

The utility associated with the inspection plan and the repair decision rule is denoted $u(\mathbf{i}, \mathbf{s}, d(\mathbf{s}), \vartheta)$ and the optimal inspection may be determined as the plan which maximizes the expected utility

$$u^* = \max_{\mathbf{i}} \max_{d} u(\mathbf{i}, d) \quad (1)$$

where the expected utility u is determined as

$$u(\mathbf{i}, d) = E_{\theta,\mathbf{S}|\mathbf{i}}\left[u(\mathbf{i}, \mathbf{S}, d(\mathbf{S}), \theta\right] \quad (1)$$

If inspection plans are performed simultaneously with the design of the system the design variables may easily be included in the decision problem as together with the inspection decision variables.

Usually the utility function may be readily associated with the service life costs and the optimization problem in equation (1) reformulated as a cost minimization problem as

$$\min_{\mathbf{i}} \min_{d} E_{\theta,\mathbf{S}|\mathbf{i},d}\left[C_{Total}(\mathbf{i},\mathbf{S},d(\mathbf{S}),\theta\right] \tag{2}$$

where $C_{Total}(\mathbf{i},\mathbf{S},d(\mathbf{S}),\theta)$ is the total service life costs. If the total expected costs are divided into inspection, repair and failure costs and a constraint related to the minimum level of service life reliability is added the optimisation problem is

$$\begin{aligned} &\min_{\mathbf{i},d}(C_I(\mathbf{i},d)+C_R(\mathbf{i},d)+C_F(\mathbf{i},d)) \\ &\quad s.t. \quad \beta(T,\mathbf{i},d) \geq \beta_{\min} \end{aligned} \tag{3}$$

where C_I are the expected inspection costs, C_R the expected repair costs and C_F the expected failure costs. $\beta(T)$ is the generalised safety index defined by

$$\beta(T) = -\Phi^{-1}(P_F(T)) \tag{4}$$

and $P_F(T)$ is the system failure probability in a specified reference period T such as one year or the service life.

5. Assessment of Consequences

Depending on the type of component considered the event of repair and failure may have significant consequences on material damage, production and operational loss as well as the safety for personnel.

Whereas inspections, repairs and failures have immediate consequences on costs the event of failure may in addition have consequences on the potential loss of lives.

The assessment of the economical consequences related to an inspection plan including costs of material damage and costs of production and operational loss takes basis in the event tree shown in Figure 4.

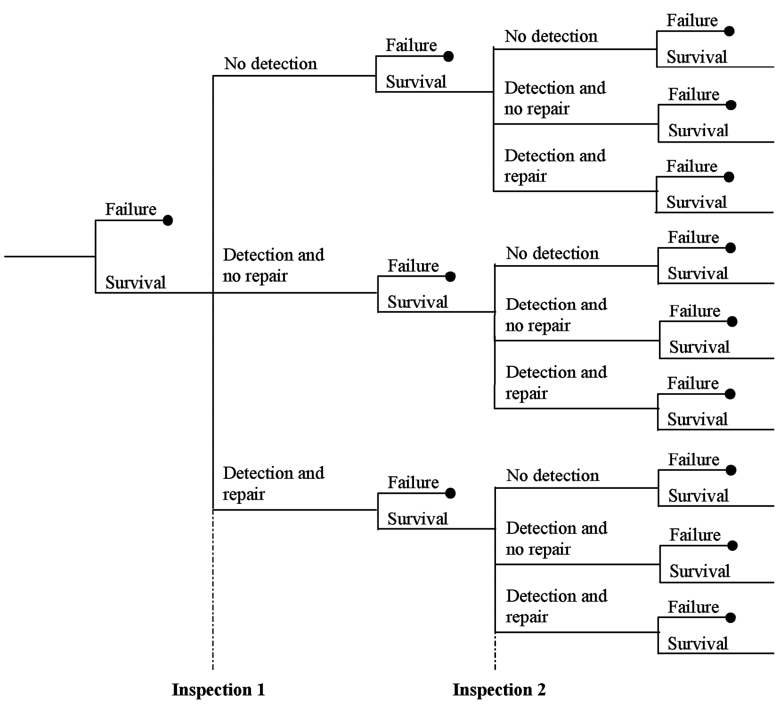


Figure 4. Illustration of an event tree for inspection and maintenance planning.

For illustration consider the detection and repair event branch in Figure 4. This event is conditional on the results of previous inspections, the event of survival until the time of the inspection, the event of detection of deterioration damages and the event that the detected damage fulfils the criteria for repair. The product between the probability for this conditional event and the cost of the required repair is denoted the expected repair cost. Similarly the probability of failure is calculated in terms of a series of conditional events leading up to the event of failure.

The capitalised expected costs of inspection can be evaluated as

$$C_I = \sum_{i=1}^{N} M(i) C_I(r_i)(1 - P_F(t_i)) \frac{1}{(1+\gamma)^{t_i}} \tag{5}$$

where it is assumed that all inspections at a given inspection time are performed with the same reliability. γ is the real rate of interest. The capitalized expected repair costs are evaluated as

$$C_R = \sum_{i=1}^{N} \sum_{j=1}^{M(i)} C_{R,i,j} P_{R,i,j} (1 - P_F(t_i)) \frac{1}{(1+\gamma)^{t_i}} \tag{6}$$

where $C_{R,i,j}$ are the costs and the probability of repair and $P_{R,i,j}$ respectively and where index *i,j* refers to the *j*'th inspected location at the *i*'th inspection time. The capitalised expected failure costs are

$$C_F \cong \sum_{i=1}^{N+1} C_F(t_i)(P_F(t_i) - P_F(t_{i-1})) \frac{1}{(1+\gamma)^{t_i}} \tag{7}$$

where $C_F(t_i)$ are the costs of system failure at time t_i.

For some types of components such as process equipment failure may be the initiating event for incidents with consequences stretching far beyond the consequences for the component itself and which may lead to loss of lives and damages to the environment. For such components the consequences of failure are assessed with the aid of quantitative risk analysis (QRA) such as failure tree analysis and the estimated consequences given component failure are assessed by event tree analysis in terms of potential loss of lives (PLL) or in terms of the fatal accident rate (FAR) and estimated frequencies of different categories of environmental damages. An example of an event tree/failure tree for such analysis is illustrated in Figure 5.

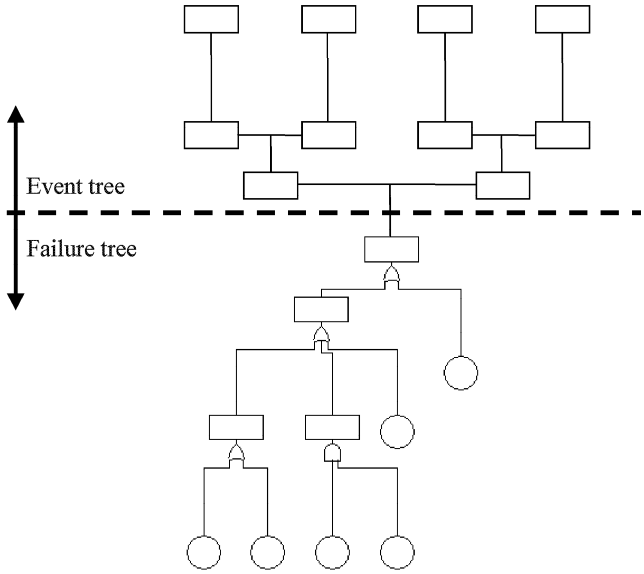


Figure 5 Event tree/failure tree for consequence assessment.

When assessing the consequences of failure the following aspects are generally important :

- Degradation mechanism – how will the component degrade
- Location of the component
- Function of the component
- System effects – how does the component interact with other components
- Repair philosophy – what is the line of action should the component fail

6. Probabilistic Modelling

The probabilistic modelling in risk based inspection and maintenance planning is a central issue as this together with the modelling of the consequences associated with the possible states of the structure defines the basis for the decision making.

The probabilistic modelling starts by the identification of a formal mathematical representation of the relevant events in terms of limit state functions and uncertain basic variables. Having identified the relevant limit state functions the problem remains to establish probabilistic models for the uncertain basic variables.

In the following first the problem of establishing limit state functions is considered. Thereafter the probabilistic modelling of the basic random variables is addressed. The reader is referred to Madsen et al. [10] and Thoft-Christensen and Baker [21] for a comprehensive introduction to the methods.

Limit state functions

In reliability and risk-based assessment and inspection planning for engineering facilities, limit state functions are used to describe any event, which is relevant for the decision making. This includes both adverse and favourable events. Adverse events are e.g. the event of failure, excessive degradation etc. Favourable events are e.g. that inspections show no degradation. Depending on the problem at hand it may thus be necessary to formulate limit state functions, which are able to represent such events.

It is convenient to distinguish between events representing adverse states of the structure and events representing new information about the performance of the system and the loading and/or environmental exposure on the system.

Adverse events $\mathbf{F}$ are here defined as

$$\mathbf{F} = \{\mathbf{x} | g(\mathbf{x}) \leq 0\} \tag{8}$$

and events defining new information about the structure such as inspection and test results $\mathbf{I}$ as

$$\mathbf{I} = \{\mathbf{x} | h(\mathbf{x}) * 0\} \tag{9}$$

where $*$ may represent $\leq, >$ or $=$ depending on the type of information.

In principle, it does not matter whether the information is quantitative or qualitative. Formal reliability assessment, however, requires a quantitative type of statement as a starting point for further processing. Qualitative statements like "the system looks fine" should therefore be translated into quantitative statements like: "no cracking of concrete", "no sign of corrosion" and so on. If one also knows, from other experiments, what the threshold values are for observing visual cracks and corrosion, these statements can be used in the formal reliability assessment.

Limit state equations may in principle be formulated at any level of approximation within the range of a purely scientific mathematical description of the physical phenomena governing the problem at hand (*micro-level*) and a purely empirical description based on observations and tests (*macro-level*). For inspection planning of engineering facilities the physical modelling is, however, generally performed at an intermediate level sometimes referred to as the *mesa-level*. Limit state equations in engineering reliability analysis will, therefore, in general be based on a physical understanding of the problem but due to various simplifications and approximations it will always to some extent be empirical. This introduces a so-called model uncertainty, which is associated with the level of approximation applied in the formulation of the limit state function. It is important that the model uncertainty is fully appreciated and taken into account in the uncertainty modelling. For process and pipe components the limit state equations describe failure modes such as burst and leak under the influence of different types of corrosion and erosion phenomena. For structural components limit state equations typically describe events relating to fatigue crack growth and or corrosion. An illustration of a limit state function for a simple corrosion wall thinning failure is given in Figure 6.

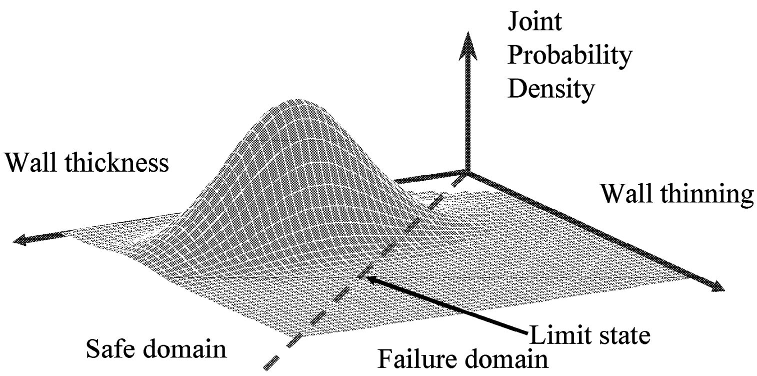


Figure 6 Illustration of limit state for wall thinning failure due to corrosion

Basic random variables

Having identified the limit state function a probabilistic model must be established for the representation of the uncertain basic variables $\mathbf{X}$ entering the limit state function.

The uncertain basic variables represent physical uncertainties, statistical uncertainties, measurement uncertainties, uncertainties related to new information and finally the model uncertainties. The physical uncertainties are typically uncertainties associated with the loading, the environmental exposure, the geometry and the material properties. The statistical uncertainties arise from incomplete statistical information e.g. due to a small number of materials tests. Finally the model uncertainties must be considered to take into account the uncertainty associated with the idealised mathematical descriptions used to approximate the actual physical behaviour of the structures.

Uncertain basic variables modelling physical quantities such as the material strength characteristics or load characteristics obviously represents physical uncertainties. An example is shown in 7 illustrating the uncertainty modelling for the corrosion rate for a pressure vessel with and without the presence of corrosion inhibitor.

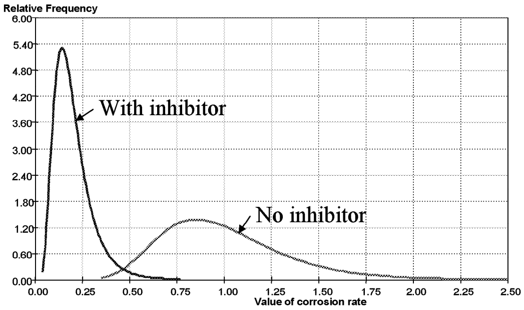


Figure 7 Illustration of the probability density function for the corrosion rate of a pressure vessel with and without the presence of corrosion inhibitor.

If test results or observations of a physical quantity are available and if the family of probability distribution is known the parameters of the distribution may be estimated using methods of statistical analysis. For some uncertainties, physical arguments can be used to identify the appropriate family of distribution function, however, in general there is some freedom in the choice of distribution function.
Preferably Bayesian statistical methods see e.g. Lindley [8] should be applied for the assessment of distribution parameters as these can accommodate for the combination of both subjective and frequentistic information.

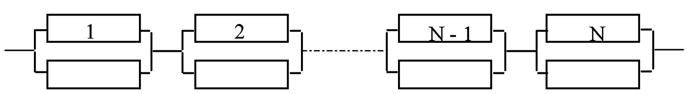
7. Assessment of Probabilities

The assessment of probabilities takes basis in the formulation of limit state equations for the relevant failure and deterioration modes and the modelling of the basic variables.

In order to define the events of failure, which are required to calculate the probabilities corresponding to the different branches in the event tree in Figure 4 it is necessary to represent the engineering systems in terms of logical systems. In the following such logical systems are given representative for the inspection-planning problem for different engineering systems.

Inspection of structures subject to fatigue deterioration
For structures subject to fatigue deterioration such as offshore jacket structures, steel bridges and ship hulls deterioration must be controlled by inspection in order to avoid that the fatigue damages are so significant that the safety of the structures in regard to extreme load situations becomes insufficient. The inspection planning problem for such structures has been addressed in numerous publications, see e.g. Goyet et al [7] and Lotsberg [9], however, only a few publication considers the system effects see e.g. Faber et al. [3].
A logical systems model, which may adequately represent the structure, is given in Figure 8.

Component degradation failure elements



Ultimate structural capacity failure elements

Figure 8. Logical system representation for inspection planning of structures subject to fatigue.

In Figure 8 a series system is given with N components where N is the number of hot spots. The series system consists of a number of two component parallel systems. The first component of the parallel system represent the event of fatigue failure of a hot spot and the other element represent the event of structural system collapse failure given the event of fatigue failure of the first element. The approximation introduced by this type of logical system is that it does not take into account the effect of the event of fatigue failure of more than one hot spot on the event of system collapse. For offshore structural system with a high level of reliability in regard to fatigue failure and for structures with a low level of redundancy this is a reasonable assumption. For structural systems with a very high degree of redundancy or a low level of reliability in regard to fatigue failure, such as ship hull structures, the approximation may be of limited use. In the latter case the probability is high that more than one fatigue failure event is involved in a structural system collapse event and this must be taken into account in the modelling. A formulation, which accommodates this, is given in Faber et al. [3].

<u>Inspection of concrete structures subject to corrosion</u>

Inspections of concrete structures subject to corrosion deterioration of reinforcement bars such as bridges in saline environments and offshore concrete installations are necessary in order to avoid that the structural capacity does not deteriorate to a degree where the safety in regard to extreme load conditions is insufficient. However, due to the fact that repair costs can be very considerable if the deterioration is allowed to progress to a level beyond what is cosmetically acceptable, the load carrying capacity of the structure is usually not important for the planning of inspections. Instead the problem is governed by the costs of repairs. Inspection planning for bridge and offshore concrete structures has been considered in e.g. Engelund et al. [2] and Faber et al. [5].

According to maintenance strategies adapted in practice repairs are initiated when a certain percentage of the surface, i.e. the critical area, of the structure is affected by corrosion e.g. exhibits signs of corrosion such as colouring, see e.g. Faber et al. [5]. If deterioration failure elements for this type of degradation are defined for sub areas within which the deterioration may be assumed to be almost fully dependent, the repair event for the structure may be defined as the event of failure of the number of failure elements, which constitutes the critical area.

A logical system for the representation for the structure is shown in Figure 9. The elements of the parallel systems in Figure 9 correspond to the event of deterioration of the individual sub areas. The number of elements M in the parallel system corresponds to the number of sub areas necessary to constitute the critical area. Finally the number of parallel systems N in the series system correspond to the number of different combinations by which M different combinations of sub elements can be sampled out of the total area considered.

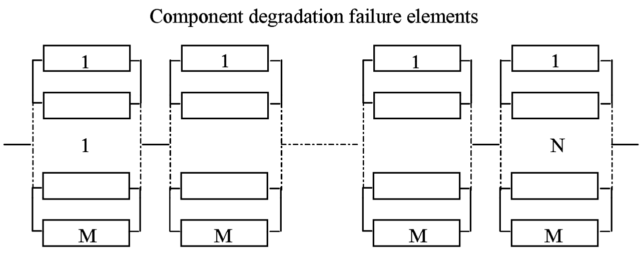


Figure 9. Logical system representation for inspection planning of concrete structures subject to corrosion of reinforcement.

<u>Inspection of pipeline segments subject to corrosion</u>
Pipeline segments subject to corrosion processes deteriorate and inspections must be performed to control that the safety in regard to through corrosion and burst failure is sufficient throughout the service life of the pipeline. Deterioration can take place either as localised corrosion or as general corrosion or a combination of the two, in accordance with the chemical composition of the media in the pipeline and on the material characteristics of the pipeline itself. Depending on the spatial dependency structure in the deterioration process the pipeline segment may be modelled as a series system where the elements correspond to sub segments in which the deterioration can be assumed to be almost fully dependent. The system is illustrated in Figure 10

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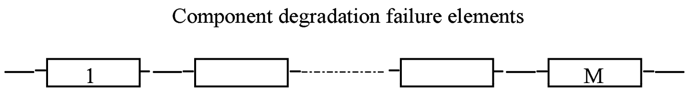

Figure 10. Logical system representing a pipeline segment in inspection planning.

The number of components in the series system corresponds to the number of sub-segments necessary to constitute the considered pipe segment.

In general the required probabilities can be written as

$$P_F(T) = \begin{cases} P\left(S(0) \cap \overline{S}(T)\right) & , \ 0 \le T < t_1 \\ P_F(t_i) + P\left(S(0) \cap \bigcup\limits_j \left\{ B_j \cap \overline{S}_j^U(T) \right\} \right) & , \ t_i \le T < t_{i+1} \end{cases} \tag{10}$$

where S refers to the survival event for the system and $\overline{S}$ to the failure event. B_j represents all combinations of inspection and repair events leading to a system configuration j after the i'th inspection $\overline{S}_j^U$ is the event of failure in the interval between the i'th and the $i+1$'th inspection. U indicates that the system characteristics may have changed due to repairs and that the event probability is updated on the basis of the previous inspection results.
The probability of repair may be found in a similar way by

$$P_R(t_{i+1}) = P(S(0) \cap \bigcup_j \{ B_j \cap R(t_{i+1}) \}) \tag{11}$$

where $R(t_{i+1})$ is the repair event at inspection time $i+1$.
The system survival events, the inspection event and the repair event in equation (10)-(11) depend very much on the configuration of the considered system as discussed in the above and it is thus not possible to write these more precisely for a general system.

8. Degradation Processes

The degradation processes acting on the considered engineering facility are usually subject to substantial uncertainty originating in the loading and exposure conditions as well as the material characteristics of the components of the facility. A probabilistic modelling of the degradation processes should not only reflect these uncertainties directly but should also be formulated such that the information collected at inspections may be used to update the predicted future evolution of the degradation process.

For steel structures the dominating deterioration processes are fatigue and corrosion and various combinations of these typically resulting in fatigue crack growth, fracture or simply excessive loss of cross-sectional area and subsequently yield failure.
For concrete structures the dominating deterioration processes are corrosion of the reinforcement due to ingress of chlorides or due to carbonation but also frost thaw cycles and alkali-silica reactions are important.
A typical situation concerning fatigue crack growth in a welded connection of an offshore steel structure is shown in figure 11.

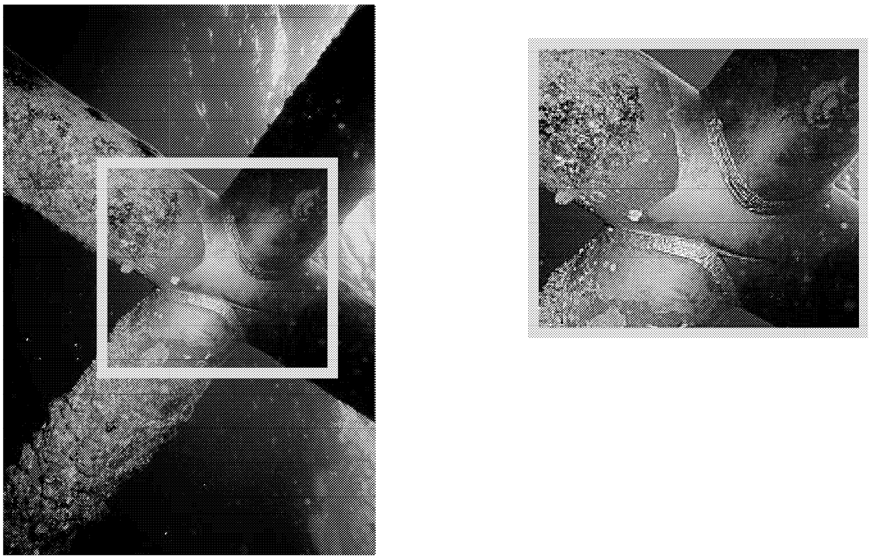

Figure 11 Fatigue crack in an offshore steel jacket.

A situation concerning the combined effect of corrosion and fatigue on stay cables is shown in figure 12.

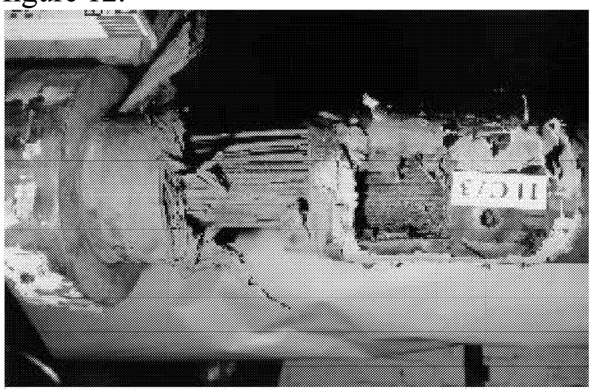

Figure 12 Combined effect of fatigue and corrosion on cable stay.
For components of process facilities such as piping and pressure vessels the major degradation processes are corrosion. These processes in turn highly depend on the material characteristics and the media they contain.
In figure 13 a segment of a corroded offshore pipeline is shown with substantial development of corrosion.

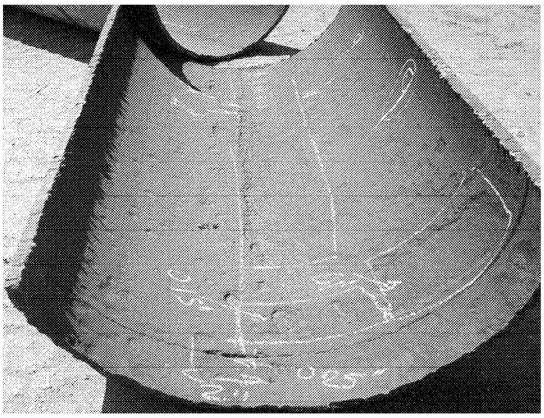

Figure 13 Pipeline segment exhibiting significant corrosion degradation.

The appropriate formulation of the probabilistic models for deterioration processes, is often rather case dependent, in order to capture the specific circumstances prevailing the considered problem. It is thus not possible to give a general description on how to formulate these. The reader is referred to Engelund et al. [2], Faber et al. [5] and Melchers [12] for examples.

9. Inspection Modelling

During the last decades most industries have been adapting the concept of probability of detection (POD) as a measure of the quality of inspections. In addition to this also the uncertainty in sizing of detected (POS) deterioration is applied see e.g. Sørensen et al. [20].
The POD represents the ability of a particular inspection method and the applied procedure to detect deterioration. Typically the POD is given as a probability distribution function for the so-called detectable defect size A_d see Figure 14.

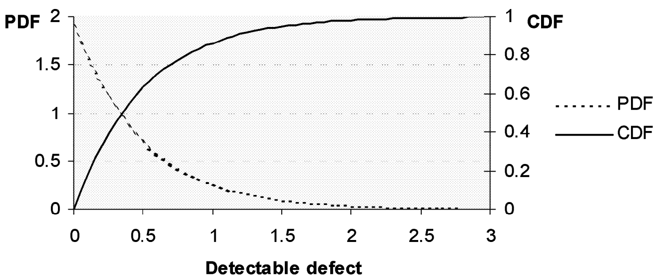

Figure 14 Exponentially distributed POD with mean value 0.5 mm.

The uncertainty associated with the sizing of detected defects is typically given as probability distribution function of the measurement uncertainty ε_{insp}. Often A_d is modelled by the exponential distribution and ε_{insp} by the Normal distribution. The distribution parameters of A_d and ε_{insp} differ from inspection method to inspection method and furthermore highly depends on the access to the locations to be inspected. Presently developed POD curves assume that the location of interest for inspection is inspected 100%, an assumption, which requires some thought in applications. These aspects are discussed in the following.
Different engineering systems have different characteristics in regard to the way deterioration processes act and degrade the system performance. Furthermore the operating conditions for the systems may also be very different. For some engineering systems such as offshore jacket structures it is possible to identify locations where the probability of e.g. fatigue deterioration is high, the so-

called hot spots. For this type of structure hot spots are typically the welded connections in the member joints. However, even though hot spots may be identified they still represent a certain weld length. It is obvious that if only one location on the length of the weld is inspected this has limited value in degradation control. On the other hand a 100 % inspection of the entire weld length may not be practical or even not possible. In practice typically only a certain percentage of the weld is inspected see e.g. Moan et al. [13] where it is reported that typically 70% of the weld in a joint is inspected. The information about the degradation in the joint as a whole then depends very much on the dependency structure of deterioration in the weld. Experience indicates that the present practice for inspection of joints in offshore structures identifies most fatigue cracks see e.g. Sharp et al. [17]. Considering the offshore structure as a system the dependence between deterioration from hot spot to hot spot has effect on the performance and the safety of the system. Furthermore inspection results obtained from inspection of one hot spot may give information about the deterioration in other hot spots. As the number of hot spots may be rather significant for an offshore structure and even higher for e.g. ship hull structures the problem arises to determine how many of these should be inspected to achieve a sufficient control of the degradation. For structural engineering systems subject to fatigue deterioration applicable models can usually be established on the basis of modern reliability methods.
For engineering systems such as process pipe systems or offshore pipelines subject to localised internal corrosion the situation is exactly opposite. For such systems it is not possible to identify locations where degradation is more probable than any other, i.e. there are no hot spots and as a consequence it is in general necessary to inspect a much larger extent of the system. The system may still be considered as a discrete number of locations or elements in a system model. The number of elements required to represent the system depends on the dependency structure between deterioration at different locations.
The situation is, however, somewhat different for engineering systems subject to corrosion. For such systems, models can sometimes be assumed on the basis of experience with specific systems. But before more general applicable approaches to the problem can be established for such systems, more fundamental research concerning the corrosion deterioration processes is required see e.g. Melchers [12].

10. Acceptance Criteria for Inspection Planning

Following the recent work of Rackwitz [15] and in accordance with the principles of the decision theory the acceptable probability of failure for an engineering system or any other activity for that matter should be established on the basis of an optimisation where the consequences of failure are assessed in terms of preferences expressed e.g. in monetary terms. This approach is highly facilitated by the fundamental work by Nathwani [14] addressing the value of the individual to society by means of the Life Quality Index. However, the implementation of such approaches in practice still lies ahead and approaches resting on the (less optimal) judgmental power of the individual decision-maker must be pursued in the meanwhile.

The individual decision-maker may have very different preferences depending on personal factors like carrier stage and role in society. Consequently different decision-makers may have quite different views on the consequences of failure.

In order to avoid structures which are unacceptable to society even though optimal for a particular decision maker it is in order to preserve values to the society and to protect the individual in society necessary to introduce requirements on the maximum acceptable probability of failure – which in accordance with Rackwitz [15] should be represented in terms of a failure rate.

The acceptable failure rates and probabilities are related to the type of facility considered. Hereunder its exposure to and use by the public as well as its role and value for the society.

In order to implement an integrated RBI approach considering e.g. an offshore production facility as a whole, into a practical applicable procedure it is necessary first to formulate the overall acceptance criteria for the various functions, such as process equipment, pipelines, sub-structures, etc. Thereafter the relation between the acceptance criteria and the performance of the different components of the installation is established. This problem may readily be approached by taking basis in the risk analysis performed as a part of the concept studies and design verification (FMECA, RAM, QRA) as well as the various relevant design reports.
On the basis of these studies the next step is to establish the relationship between the objectives (personal risks, regulative requirements and design assumptions, environmental risks, production down-time, expected costs) and the inspection strategies for all structures and process component considered in the RBI analysis. The principle is illustrated in Figure 15.

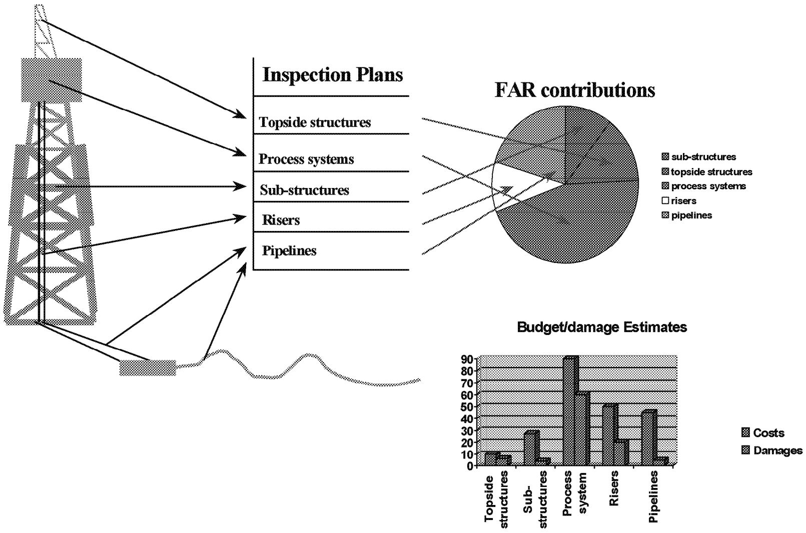

Figure 15 Illustration of the integrated structure/process system RBI analysis.

The overall acceptance criteria, which are usually considered as objectives for the inspection planning, are

- Personal risk
  - risk of fatalities
  - risk of impairing escape and evacuation
  - occupational risk
- Environmental risk
- Economical risk

The inspection activities are planned such that minimum requirements to safety are fulfilled and such that costs are minimised. Assessing the effect of inspections on both safety and costs also facilitates the As Low As Reasonable Practicable (ALARP) principle. The ALARP principle implies that increased inspection efforts shall be considered as long as the costs of the additional inspection efforts are proportional to the decrease in risk.

For each of the acceptance criteria, the contributions from the individual functions of the considered engineering facility such as structural components, process equipment, pipelines, etc. are identified.

In this way the allocation of contributions is identified and may be used as a target for the operational phase of the installation. The maximum acceptable contribution from each of the different functions may be established by scaling by the ratio of what is overall acceptable to what has been achieved through the design.

The acceptance criteria for each function may be "broken down" into acceptance criteria for the performance for the individual components comprising the function. This requires that risk analyses are performed relating the event of component failure to the consequences in terms of e.g. loss of lives and monetary losses. The risk analysis performed for the verification of the design of the installation may be utilised for this purpose.

For some engineering facilities and/or individual components hereof the performance requirements given in terms of the acceptance criteria have no or only insignificant implications for the performance and thus no impact on the required inspection effort. For such facilities/components other legislative requirements to the inspection effort should be considered, as they may become decisive for setting the inspection requirements. Typical legislative requirements are specified in codes of practice for the design and operation of e.g. structures, pipe works and pressure vessels. For e.g. welded connections in bridges and offshore structures the implicit code requirement to the safety of the structure in regard to total collapse may be assessed through the requirement (for connections with "substantial" failure consequences) to the ratio between the design fatigue life of the considered connection to the design service life of the structure (the Fatigue Design Factor - FDF). The annual probability of fatigue failure (in the last year in service) P_{FAT_j} for such a welded connection (i.e. design fatigue factor 10) corresponds to the acceptable probability of structural collapse i.e. P_{AC}. A typical maximal allowed annual probability of collapse failure is in the order of 10^{-5}. On this basis it is possible to establish approximate connection specific acceptance criteria in regard to fatigue failure. For each joint j the conditional probabilities of structural collapse given failure of the considered joint $P_{COL|FAT_j}$ are determined and the individual joint acceptance criteria for the annual probability of joint fatigue failure are found as $P_{AC_j} = \dfrac{P_{AC}}{P_{COL|FAT_j}}$.

The inspection plans must then satisfy that $P_{FAT_j} \leq P_{AC_j}$ for all years during the operational life of the structure. The annual probability of joint fatigue failure P_{FAT_j} may in principle be determined on the basis of either a simplified probabilistic SN approach or a probabilistic fracture mechanics approach provided the fracture mechanical model has been calibrated to the appropriate SN model. Here it is suggested to use a fracture mechanics approach, as this also forms the basis for the inspection planning.

11. Conclusions and Prospects

The present paper outlines the problem complex of inspection planning, summarises the theoretical basis for its systematic treatment within the framework of the Bayesian decision theory and highlights the aspects of consequence assessment, modelling of uncertainties, assessment of probabilities, modelling of inspections, modelling of engineering systems in terms of logical systems, modelling of deterioration processes and finally acceptance criteria for RBI.

The developments during the last two decades in reliability and risk based inspection planning have been substantial and fruitful. Risk Based Inspection planning techniques have by now reached a matured level allowing for efficient implementation into practice.

Future developments are still needed for enhancing the use of Risk Based Inspection planning in practice, however, simplifying numerical operations and aiming for the application of the methodologies by non-expert users also.

12. References

1 Benjamin, J.R. and Cornell, C.A. 1970, Probability, statistics and decision for civil engineers, McGraw-Hill, NY.

2 Engelund, S. Sørensen, J.D. and Sørensen, B. 1998, Evaluation of repair and maintenance strategies for concrete coastal bridges on a probabilistic basis, ACI Materials Journal.

3 Faber, M.H., Sørensen, J.D. and Kroon, I.B., 1992, Optimal inspection strategies for offshore structural systems. In Proc. OMAE92, Calgary, Canada, pp. 145-151.

4 Faber, M.H., Kroon, I.B. and Sørensen, J.D., 1996, Sensitivities in structural maintenance planning, Journal of reliability engineering and systems safety, 51, Elsevier Science Limited, pp. 317-329.

5 Faber, M.H., Berge, H.E., Tiller, I. Hall, M.E.,1999, Reliability based assessment of offshore concrete structures subject to chloride ingress. To be printed in Proc. OMAE99, Newfoundland, Canada.

6 Fujita, M., Schall, G. Rackwitz, R., 1989, Adaptive reliability based inspection strategies for structures subject to fatigue. In Proc. ICOSSAR89, San Francisco, USA, pp. 1619-1626.

7 Goyet, J. Faber, M.H., Paygnard, J.-C. and Maroni, A., 1994, Optimal inspection and repair planning: Case studies using IMREL software, In Proc. OMAE94, Houston, USA, pp. 325-333.

8 Lindley, D.V., Introduction to Probability and Statistics from a Bayesian Viewpoint (Vol. 1+2), Cambridge: Cambridge University Press. 1976.

9 Lotsberg, I. and Sigurdsson, G., Wold, P.T., 1999, Probabilistic inspection planning of the Åsgaard A FPSO hull structure with respect to fatigue. To be printed in Proc. OMAE99, New Foundland, Canada.

10 Madsen, H.O., Krenk, S. and Lind, N.C., Methods of Structural Safety, Prentice Hall, 1986.

11 Madsen, H.O., Sørensen, J.D. Olesen, R., 1989, Optimal inspection planning for fatigue damage of offshore structures, In Proc. ICOSSAR 89, San Francisco, USA, pp. 2099-2106.

12 Melchers, R., 1995, Probabilistic modeling of seawater corrosion of steel structures, In. Proc. ICASP 7, Paris, France, Vol. 1. pp. 265-270.

13 Moan, T. Vårdal, O.T., Hellevig, N.C. and Skjoldli, K. 1997, In-service observations of cracks in North Sea jackets: a study on initial crack depth and POD values, In Proc. OMAE97, pp. 209-218.

14 Nathwani, J.S., Lind, N.C., Pandey, M.D., Affordable Safety by Choice: the Life Quality Method,Institute for Risk Research, University of Waterloo, Waterloo, Canada 1997.

15 Rackwitz, R., Optimization – the basis for code-making and reliability verification, Journal of Structural Safety, Elsevier Science, Vol. 22, (2000), pp 27-60.

16 Raiffa, H. & Schlaifer, 1961, R. Applied Statistical Decision Theory. Harward University Press, Cambridge University Press, Cambridge, Mass..

17 Sharp, J.V., Stacey, A. and Birkinshaw, M., 1995, Review of data for structural damage to offshore installations, In Proc. 4'th international conference "offshore structures – hazards, safety and engineering, London, UK.

18 Sindel, R. and Rackwitz R., 1996, Calculation of the failure probability and inspection optimization for cracks in ships. Technische Universität Munchen, Institut für Tragwerksbau.

19 Skjong, R., 1985, Reliability based optimization of inspection strategies. In Proc. ICOSSAR 85

Vol. III, Kobe, Japan, pp. 614-618.

20 Sørensen, J.D., Faber, M.H., Rackwitz, R. and Thoft-Christensen, P.T. 1991, Modeling in optimal inspection and repair. In Proc. OMAE91, Stavanger, Norway, pp.281-288.

21 Thoft-Christen, P. and Baker, M.J., Structural Reliability and Its Applications, Springer-Verlag, Berlin-Heidelberg-New York, 1982.

The DuraCrete Approach

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Keywords: Service life, durability, stochastic, design, new structures; existing structures, concrete.

Summary

In the past years much efforts have been spent on a new design approach for the durability of existing and new concrete structures. The results of these efforts can be seen in recent work of several international scientific committees of Rilem, CEB, CIB and FIP. These have finally resulted in the Brite/Euram project DuraCrete where a design manual has been developed for a service life design of concrete structures. The new design method is based on performances (both serviceability and ultimate limit states), reliability and a distinct design service life. This is in contrast with the conventional design approach, that is based on deem-to-satisfy rules, mainly to prevent degradation of the concrete.

DuraCrete offers the possibility to make tailor made designs based on performance characteristics of the materials involved and focussing on the specific environment, use and maintenance strategy for the structure. The method has a high potential for use in practice as it can directly be connected to the modern structural design approach.

The project DuraCrete was completed in 1999. Since then new developments have occurred, indicating the new possibilities and advantages of the new service life design method. One of the examples is the design of the Western Scheldt Tunnel, the European thematic network DuraNet and further steps in the process of the development of codes. The paper will highlight these examples and delve into the significance for the concrete industry and the society.

1. Introduction

DuraCrete is a design method for the durability of concrete. The conventional design method is based on deem-to-satisfy rules. In contrast with this method, DuraCrete is explicitly based on performances, reliability and a design service life. These are also the most important key words for the modern structural design codes. The DuraCrete methodology can therefore seamlessly be connected to the structural design code and even be integrated with it. DuraCrete can be applied to both new and existing structures.

DuraCrete is the result of a development that started in the eighties by recognizing that the service life of structures is a stochastic quantity. This means that the service life can only be described in terms of probability. Soon after that TNO has demonstrated [1] for various materials, including concrete, that proper design decisions could be made by taking the stochastic nature into account. The further development of the method was mainly directed to concrete structures. The reasons for this are:

- the concrete industry had already played a leading role in the development of the performance and reliability based structural design method; DuraCrete is a logic extension of this development [2]
- the availability of proper degradation models and data for these models
- the insight that the flexibility in making a concrete mix and in choosing the composing materials could benefit from this method; all kind of new types of cements and other binders could not benefit from their special properties as the conventional codes based the acceptance of new binders on their composition and not on the performance related properties.

Rilem [3] and CEB [4] (at this moment called ***fib***) have adopted the new design method. CIB and Rilem have taken the initiative to extend it to other structural and building materials [5].

Twelve partners have started in 1996 within the European research program Brite/EuRam a research project 'DuraCrete', where they have developed the design methodology to a practical design tool. The project was finished by publishing a design manual for new and existing concrete structures [6]. This manual is not the end of the development. The manual should be seen as the start of a further and broader development.

With respect to research on cement and concrete, there is a strong tendency to base that on 'compliance tests'. This notion has been introduced in DuraCrete. The meaning of such tests is, that basic material properties are determined in the laboratory under defined conditions. The results of such tests can be used in the service life design formulas, after correction to the specific conditions of the concrete structure (curing, age, environment etc.). The big difference between these tests and conventional ones is, that the former are directed to performance related properties and no longer to the composition of the material.

2. Concept of DuraCrete

2.1 Basis for design

In principle the design strategy for durability is to select an optimal material composition, construction method and structural detailing to reliably resist, for a specified period of use, the degradation threatening the structure. In the case of redesign there is however less freedom available, as we have to accept (partly) the choices that have been made during the original design. As the redesign is based on the same principles as the design we will concentrate further on the design of new structures.

Sometimes it will be clear that a design is very reliable. The structure is, for example, protected to the aggressive environment by tanking, membranes, coatings etc., or non-reactive materials like stainless steel reinforcement have been used, another possibility is to inhibit corrosion by cathodic protection. In that case the calculation of the reliability may be omitted.

The durability design guide follows the same principles (reliability and performances) as a structural design code. This means that the durability design will be based upon:

- realistic and sufficiently accurate definitions of environmental actions (different micro-environmental aggressiveness classes) depending on the resulting type of degradation
- material parameters for concrete and reinforcement
- mathematical models for degradation processes
- performances expressed as limit states
- reliability.

2.2 Structural design

In modern codes, like the Eurocode, the limit state function expresses the basis of the conventional design procedure for the safety and the serviceability of structures. This limit state defines the border between an adverse state (such as collapse, buckling, deflection, vibration) and the desired state. The limit state can in principle be formulated as:

$$R - S = R(X_1, X_2, ..., X_n) - S(X_{n+1}, X_{n+2}, ..., X_m) = 0 \tag{1}$$

In which:
R - a function that describes the load bearing capacity of the structure
S - a function that describes the influence of the load on the structure
X_i - a basic variable for the functions R or S.

The structural design procedure is elaborated in such a way that the failure probability is restricted:

$$P\{\text{failure}\} = P_f = P\{R - S < 0\} < P_{\text{target}} = \Phi(-\beta) \tag{2}$$

In which:
$P\{\text{failure}\}$ or P_f - the probability of failure of the structure
P_{target} - the accepted maximum value of the failure probability

Φ - standard normal distribution function (mean = 0 and standard deviation = 1)
β - reliability index (parameter normally used instead of the failure probability)

With the aid of probabilistic techniques this failure probability can be calculated. In practice the design has however been simplified to a semi-probabilistic level with characteristic values and partial factors γ, that are calibrated in such a way that the target reliability will be achieved:

$$R_c / \gamma_R - S_c . \gamma_S = R_d - S_d > 0 \tag{3}$$

In which:
R_c - load bearing capacity of the structure based on characteristic values
γ_R - material factor
S_c - characteristic value of the influence of the loading
γ_S - load factor.
R_d - design value of the load bearing capacity
S_d - design value of the load.

2.3 Calculation example

A relative simple calculation example of such a limit state function can be derived from Figure 1. The beam has two supports and a rectangular cross section. The beam is loaded in the middle section by a concentrated load F. The span is l, the width b and the height h.

The maximum bending moments M_{middle} occurs in the middle section:

$$S = M_{midden} = F/2 \quad l/2 = \tfrac{1}{4} Fl \tag{5}$$

The capacity M_{max} of the middle section is, in the case of linear elastic material behaviour:

$$R = M_{max} = W f = 1/6\ bh^2\ f \tag{6}$$

In which:
W – the section modulus
f - the material strength (in this example either tensile or compressive strength).

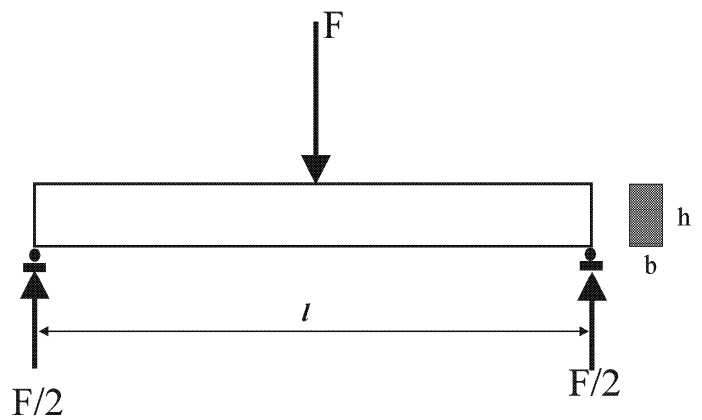

Figure 1: Beam on two supports

Equilibrium is possible as long as relationship (7) applies:

$$R - S > 0 \quad \text{or} \quad 1/6\ bh^2\ f - \tfrac{1}{4} Fl > 0 \tag{7}$$

An overview of the parameters, including their stochastic characteristics, is given as an example in Table 1. On basis of this information a reliability index $\beta = 4.0$ can be calculated. The Eurocode 1 requires for an ultimate limit state a reliability index of at least $\beta = 3.8$. The conclusion is therefore that this design is safe.

Table 1: Overview of the parameters of the calculation example

Basis variable	description	distribution	mean	standard deviation
b	beam width	deterministic	150 mm	-
h	beam height	deterministic	400 mm	-
F	concentrated load	normal	250 kN	75 kN
l	span	deterministic	5000 mm	-
f	material strength	normal	200 N/mm^2	20 N/mm^2

2.4 Time dependent design

In the performance based structural design both the resistance R and the load S are considered to be time independent. In many situations this is not realistic. The load can be time dependent or the capacity can change in time due to degradation. Relationship (1) should than be rewritten as a time dependent limit state function [2], taking such effects into consideration:

$$R(t) - S(t) > 0 \tag{8}$$

A special case for this limit state function occurs if either R or S is not time dependent. These relationships do in principle not differ from (2). Relationship (8) applies for all values of t in the time interval (0,T). T is the intended service period (i.e. reference period). From a mathematical point of view it can be stated that relationship (8) can be used for durability design. The service life concept can be expressed in a design formula, similar to (2):

$$P_f(T) = P\{R - S < 0\}_T < P_{target} = \Phi(-\beta) \tag{9}$$

In which:

$P_f(T)$ - the probability of failure of the structure within T

T - intended service period.

Probably it will be possible to simplify relationship (9) in a later stage to one similar with that for the conventional design procedure (3).

The mathematical model for describing the event "failure", i.e. passing a durability limit state, comprises a load variable S and a resistance variable R, see Fig. 2. Failure occurs if the resistance is smaller than the load. The probability of failure within the period of time [0;T]. $P_f(T)$ is defined as the probability that the load does not exceed the resistance within the given period T.

$$P_f(T) = 1 - P\left\{R(t) > S(t) \quad \forall t \in [0;T]\right\} \tag{10}$$

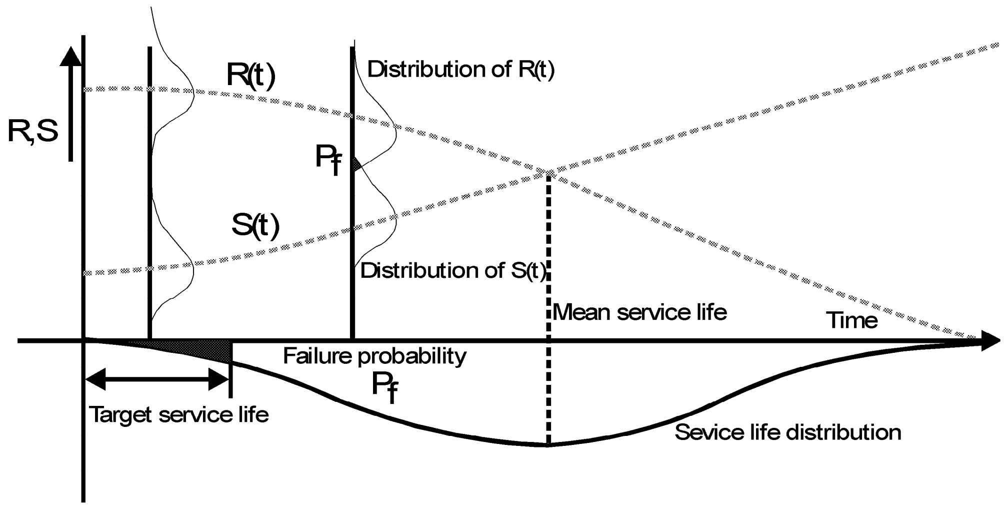

Figure 2: Failure probability and target service life (illustrative presentation)

2.5 Limit states and design service life

The first step of this design will be the definition of the desired/required performance(s) of the structure. The client or the owner of the structure are asked to define their requirements for quality and target service life. Further requirements can be given in building codes. The definition of performance criteria will be related to a limit state criterion.

Figure 3 shows an example of the performance (damage function) of a concrete structure with respect to reinforcement corrosion and related limit states. This example is taken from [7] where a description is given of the Western Scheldt Tunnel in The Netherlands. The tunnel lining is loaded by salt (sea)water from the outside and from salt leakage water and de-icing salts from the inside. Due to the ingress of chloride ions depassivation of the reinforcement will occur and corrosion can start. If enough corrosion products have been formed, the concrete cover will crack. Finally spalling of the concrete cover will occur. In the last stage the reduction f the cross section of the involved reinforcing bars will end in the collapse of the structure. Depassivation, and cracking represent in principle serviceability limit states related to durability. Collapse represents an ultimate limit state, whereas spalling can involve both serviceability and safety related to durability.

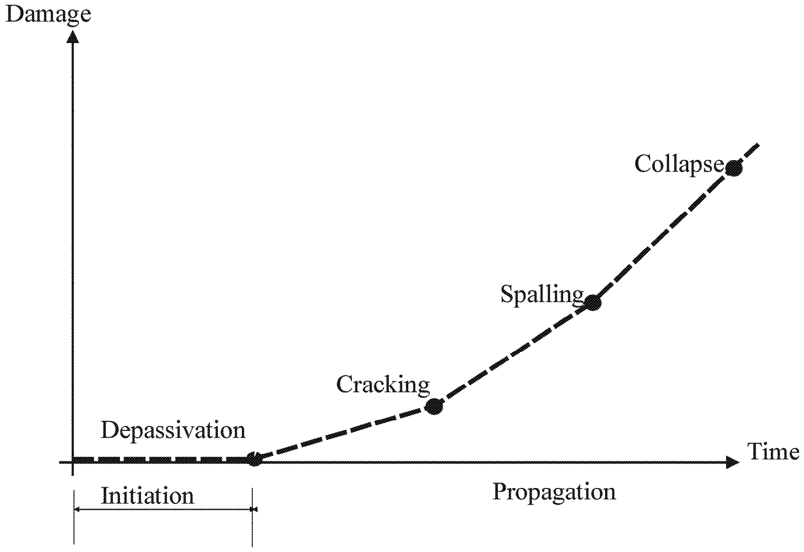

Fig 3: Determination of service life and limit states with respect to reinforcement corrosion

2.6 Calculation example including a degradation model

The second step of the durability design is to analyse the environmental actions and to identify the relevant degradation mechanisms. Mathematical models describing the time dependent degradation processes and the material resistance against it are needed. The big step forward to performance related durability design is that these models enable the designer to evaluate the time-related changes in materials and structures depending on the specific material and environmental conditions. In this calculation example we will simplify the approach by assuming only degradation of the material.

Further to the example given in Figure 1 we assume that the dimensions of the cross section reduce with x mm per year (See Figure 3). This means that after a period t the width is reduced to (b-xt) and the height to (h-xt). The original relationship (7) changes due to this degradation to:

$$R(t) - S > 0 \quad \text{or} \quad 1/6\,(b-xt)\,(h-xt)^2\, f - \tfrac{1}{4} F l > 0 \qquad (11)$$

In Table 2 a new set of parameters is defined, based on the parameters as in Table 1 and including the new parameter x. A probabilistic calculation based on these parameters results in the reliability indexes given in Figure 4. For t = 0 we find back the original result β = 4.0. This value reduces in the course of time. After about 3.3 year the value β = 3.8 is exceeded. After about 65 year the remaining reliability β = 0.0; this means a probability of failure of 50 %.

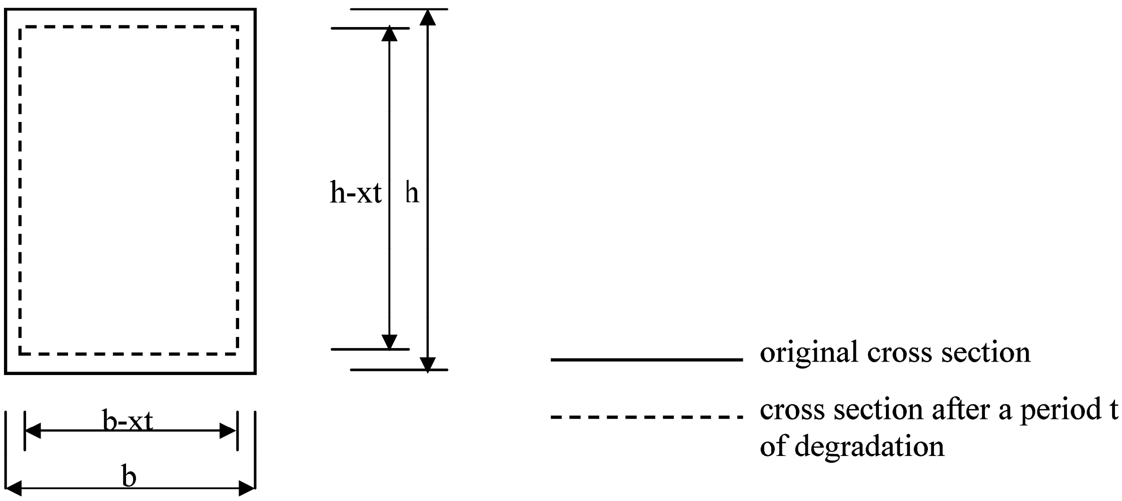

Figure 3: Calculation example with a degraded cross section

Table 2: Overview of the parameters of the calculation example including the degradation

Basis variable	description	distribution	mean	standard deviation
b	beam width	deterministic	150 mm	-
h	beam height	deterministic	400 mm	-
F	concentrated load	normal	250 kN	75 kN
l	span	deterministic	5000 mm	-
f	material strength	normal	200 N/mm^2	20 N/mm^2
x	rate of	lognormal	1 mm/year	0.05 mm/year

	degradation			

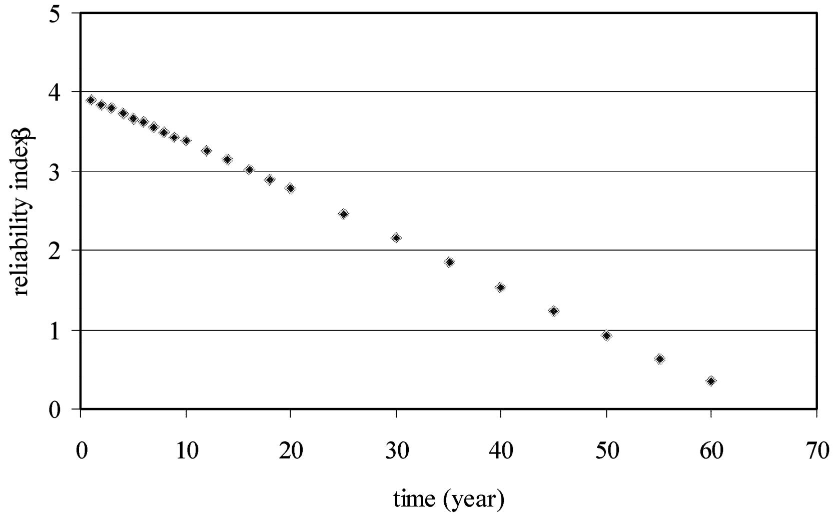

Figure 4: Reliability index in the course of time

3. Applications

3.1 Western Scheldt Bored Tunnel

DuraCrete has been applied for the service life design of the Western Scheldt Tunnel in The Netherlands [7]. The tunnel will be bored and consists of two tubes with a diameter of 10.3 m and a length of 6.5 km. In Figure 3 two sections and an overview of one of the lining segments are represented. The construction of the tunnel started in 1999 and in 2003 it will be open for traffic. The contract for this tunnel is a so called ‘design and construct’ contract.

The design requirement with respect to durability was a service life of at least 100 year. The contract gave however no design method, no specifications of the performances and no requirements for the minimum reliability. After deliberation between the owner of the tunnel and the contractor it was decided to make the design on basis of the design methodology DuraCrete. The limit states to consider were the structural limit states mentioned in the Dutch Building Decree, but in combination with the effects of carbon dioxide, chloride and corrosion. Further a limit state for reducing the probability of repair was added. This was done by defining propagation of corrosion should be prevented. According to the Dutch Building Decree the reliability index for the serviceability limit state was defined as $\beta = 1.8$ and for the ultimate limit state as $\beta = 3.6$. These values are almost equivalent to the requirements of the Eurocode. The limit state for reducing the probability of repair was considered as a serviceability limit state. This occasion was the first complete service life design of a realistic concrete structure. The design could be completed as partners that were involved in the project DuraCrete assisted both the owner, and the contractor. Most of the other new bored tunnels in The Netherlands will be designed for a service life of at least 100 year.

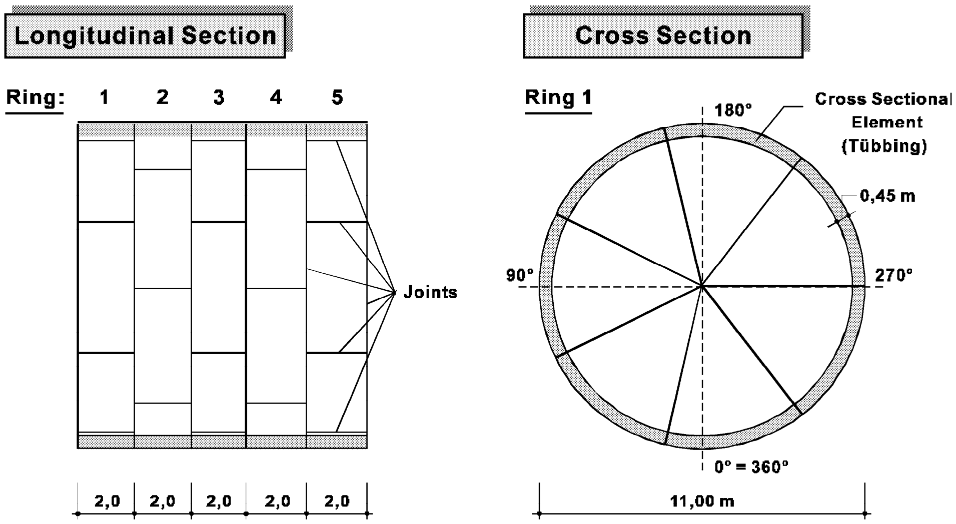

Figure 3: Longitudinal and cross section of the bored tunnel

The different models used for this durability design consist of design parameters such as structural dimensions, environmental parameters and material properties that correspond to the load and resistance variables of the structural design procedure. In the case of chloride induced corrosion the following model describing the initiation of corrosion has been identified:

$$x(t) = 2 \cdot C_{(Crit)} \cdot \sqrt{k_t \cdot D_{RCM,0} \cdot k_e \cdot k_c \cdot \left(\frac{t_o}{t}\right)^n \cdot t} \qquad (12)$$

In which:

$$k_t \cdot D_{RCM,0} = D_0 \qquad (13)$$

$$C_{(Crit)} = erf^{-1}\left(1 - \frac{C_{Crit}}{C_{SN}}\right) \qquad (14)$$

With:

erf^{1}: inverse error function

t: exposure time [year],

The other variables and their parameters are explained in Table 3.

Taking the example of onset of chloride induced corrosion the durability inputs are as follows:

- The limit state is given by the requirement that the chloride concentration at the surface of the reinforcement may not reach the critical chloride concentration.
- The resistance R is given by the critical chloride concentration and the quality and thickness of the concrete cover.
- The load S is represented by the actual chloride concentration at the reinforcement level. This depends on material parameters (chloride diffusion coefficient) and environmental effects.

Table 3: Overview of the basis variables:

Variable	Parameter	Description	Dimension	μ	σ	Distribution
1	x_c	Concrete cover	[mm]	37	2	Exponential.
2	$D_{RCM,0}$	Chloride migration coefficient	[$10^{-12}m^2/s$]	4.75	0.71	Normal
3	C_{Crit}	Critical chloride content	[m/m-%/]	0.70	0.10	Normal
4	n	Age exponent	[-]	0.60	0.07	Normal
5	k_t	Test model factor	[-]	0.85	0.20	Normal
6	k_e	Climate factor	[-]	1.00	0.10	Normal
7	k_c	Curing constant	[-]	1.00	0.10	Normal
8	C_{SN}	Surface chloride content	[m/m %]	4.00	0.50	Normal
9	t_0	Basic testing time for $D_{RCM,0}$	[year]	0.0767	-	Deterministic

On the basis of this information it is possible to make a probabilistic calculation. This has been done with the software package Strurel. For every successive year the failure probability has been calculated and added to the failure probabilities of the previous years. Further this value has been transferred to the reliability index β. The result is presented in Figure 4.

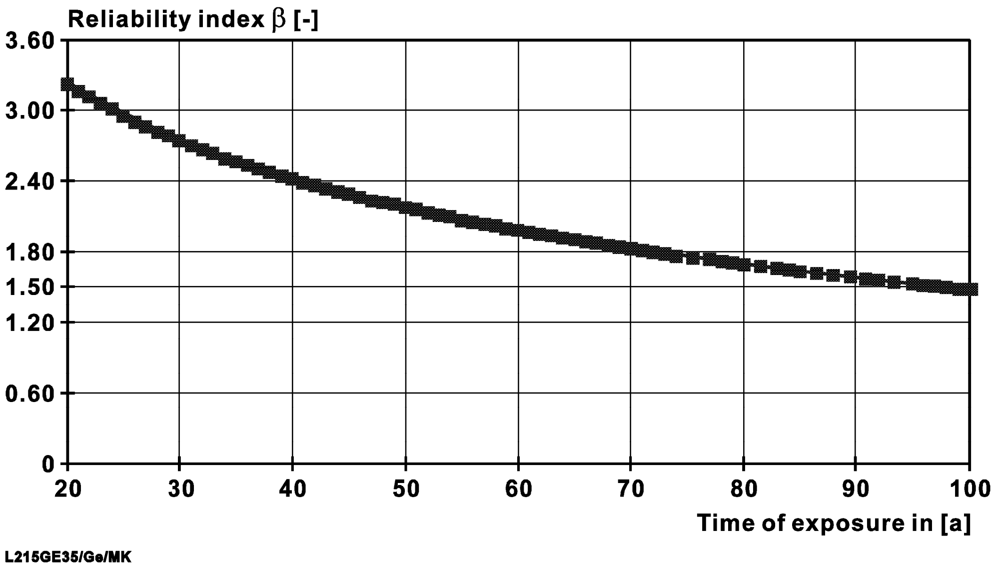

Figure 4: Reliability Index versus Time of Exposure

The figure shows that the reliability index $\beta = 1.5$ after a period of 100 year. The design requirement was however $\beta = 1.8$. This means that additional durability measures had to be applied.

3.2 High-Speed Railway Link between Amsterdam and Brussels

The Dutch part of the high-speed railway link between Amsterdam and Brussels will be built. Several bridges and tunnels will be built. The service life requirements in the design-and-construct contracts for these structures are explicitly formulated. The contractors involved have been briefed about the new durability approach before the contracts were signed.

These explicit requirements have resulted in a situation that potential suppliers of binders, aggregates, admixtures, and products like piles have to contribute with information and test data to the service life calculations of the concerning structures.

3.3 DuraNet

“DuraNet” is an international, engineering professionals network which aims to promote the adoption and wider use of a performance and reliability based service life design approach for reinforced concrete structures. The EU funded network brings together 19 partners from across Europe who are committed to improving the durability design, assessment and repair of concrete structures in Europe. A full list of partners is included below. The network aims to promote the use of service life design of concrete structures based on a probabilistic design method. The full title of DuraNet is: "Network for supporting the development and application of performance based durability design and assessment of concrete structures".

The project began in November 1998 and runs for three years. It has been developed out of the already described DuraCrete project (1996-99). The main formal activities of the network will be a series of 3 annual (invited) workshops. The first workshop [7] was a co-operation with CEN TC104 and aimed to discuss the possibilities of performance based service life design for the testing of material properties. The workshop in 2000 has been directed to the state-of-the-art with respect to service life predictions for repaired concrete structures. The final workshop will be held in 2001. In that workshop applications of DuraCrete will be demonstrated and discussed with potential users and owners of structures.

In addition “DuraNet News” (DNN) will be published periodically as the newsletter of the Network. Through the DuraNet web site (www.duranetwork.com) further information is available. The network will also provide opportunities for the members to exchange information informally through meeting other engineers at the workshops, by responding to issues in DNN and by email or correspondence. Although the life of the project is limited, the network can live on afterwards.

4. Future developments

In the coming years the DuraCrete methodology needs to be developed further. Degradation and environmental models have to be improved and extended. The assessment of existing structures needs more attention. The influence of repairs and protection systems on the service life and the reliability must be taken into account. Last but not least it is necessary to improve the practicality of the method. Initiative has been taken in the mean time to start these activities both on national and international levels. It may be expected that the end report of DuraNet will identify the need for this research and the following standardisation.

The CIB W80/Rilem PSL 172 committee ‘Prediction of the Service Life’ is currently active to study the probabilistic and performance based service life design methods. The intention is to answer the question whether this method can also be applied to other structural materials or to building materials and building products. Further they study the possibilities of simplified design methods, where a basis service life is corrected by factors, which depend on the special circumstances for that particular structure or building. The Japanese Institute of Architects has developed such a factorial method. Part of the work of the joint CIB/Rilem committee aims to relate this method to the probabilistic method.

The before mentioned activities are known in public. Other activities are initiated in the mean time and it may be expected that we will hear from them soon.

5. References

1. Siemes A., Vrouwenvelder A., and Van den Beukel A., “Durability of Concrete - A Probabilistic Approach”, *Proceedings of the Rilem Seminar on the Durability of Concrete Structures under Normal Outdoor Exposure*, Hanover, Germany, 1984
2. Siemes T., and Rostam S., “Durable safety and serviceability - a performance based design format", *IABSE report 74: Proceedings IABSE colloquium 'Basis of Design and Actions on Structures - Background and Application of Eurocode 1*, Delft, The Netherlands, 1996, pp. 41-50, 1996
3. Andrade C., Sarja A., Siemes A.J.M., and Vesikari E., “:Durability Design of Concrete Structures”, *Rilem Report Series 14 (ed. A. Sarja en E. Vesikari)*, Published by E & FN Spon, London, UK, pp. 180, 1996
4. Hergenröder M., Künzel H.M., Möller J., Nilsson J-O, Schieβl P., and Siemes T., “New Approach to Durability Design - An example for carbonation induced corrosion*”, CEB Bulletin 238*, Uitg. CEB Laussane CH, May 1997, pp.138
5. Siemes T., and Edvardsen C., “Duracrete Service Life Design for Concrete Structures A Basis for Durability of Other Building Materials and Components”, *Proceedings 8DBMC*, 1999
6. Brite/EuRAm project BE95-1347, “DuraCrete Probabilistic and Performace Based Design Manual for New and Existing Concrete Structures”, *DuraCrete Report R17,* Gouda The Netherlands, 1999
7. Breitenbücher R., Gehlen Chr., Schiessl P., Van den Hoonaard J., and Siemes T., “Service Life Design for the Western Scheldt Tunnel”, *Proceedings 8DBMC*, 1999-03-15
8. Meyer H-G., and Siemes T. (ed.*), “*Design of durability of concrete”, *Proceedings of the CEN TC 104 and DuraNet Workshop*, DIN, Berlin Germany, 15-16 June 1999, pp. 103

Risk-Based Optimisation For RC Repair Strategies: What is The Appropriate Limit State - Safety Or Serviceability?

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Keywords: Life-cycle costs; durability, structural reliability; corrosion; optimisation.

Summary

A structural deterioration life-cycle reliability model was used to calculate probabilities of cracking and spalling for typical reinforced concrete bridge decks. A life-cycle cost analysis was then used to optimise cover, water-cement ratio and replacement strategy, for a typical RC bridge deck subject to chloride contamination from de-icing salts. This illustrative example considered various discount rates and service lives, and found for instance, that reducing the cover from that currently specified for design results in a significant increase in life-cycle costs for highly aggressive environments. A comparison is made between expected costs of failure caused by strength (collapse) and serviceability limit states to help determine the appropriate limit state for the optimisation of RC inspection and repair strategies.

1. Introduction

Corrosion-initiated longitudinal cracking and associated spalling of the concrete cover are particularly common problems in reinforced concrete (RC) structures. Corrosion is initiated mainly by chloride contamination, often in conjunction with inadequate cover or poor quality concrete and the cost of maintaining, repairing or replacing degraded existing structures is immense. Clearly, there is a strong financial incentive for the optimal allocation of resources, not only for repair and rehabilitation strategies, but also for initial design, construction and proposed maintenance of future structures in order to optimise life-cycle performance. In the present paper, life-cycle performance is measured by time-dependent probabilities of cracking and spalling and life-cycle costs.

A structural deterioration life-cycle reliability model is developed herein to calculate probabilities of cracking and spalling for typical RC structures. The model includes the random variability of chloride diffusion, critical threshold chloride concentration, corrosion rates, concrete material properties, element dimensions and reinforcement placement. The source of deterioration is de-icing salts and corrosion products (rust) are expansive, leading to the formation of tensile stresses and subsequent longitudinal cracking and spalling. In the present paper a RC bridge deck is considered for illustrative purposes. It is assumed that spalling will be widespread across the structure and so after spalling the bridge deck will be replaced. Time-dependent probabilities of spalling are calculated for annual increments over the lifetime of the structure (120 years) and the probability that multiple deck replacements will be needed during the life of the structure is then calculated. Life-cycle costs considering initial construction costs, costs of improved durability and replacement costs (including user delay costs) can then be estimated. Life-cycle costs for different durability design specifications and replacement strategies (eg. quality of replacement deck) will be compared and optimised. Finally, a comparison will be made between expected costs of failure caused by strength (collapse) and serviceability limit states to help determine the appropriate limit state for the optimisation of RC inspection and repair strategies.

2. Longitudinal Cracking And Spalling

Most structural engineering research has focused on ultimate strength limit states when developing codes of practice or assessing life-cycle performance. However, serviceability and other functional failures greatly outnumber catastrophic failures (i.e., collapse) and so it is reasonable to assume that serviceability failures constitute the largest single source of economic loss [1]. For RC structures the occurrence of longitudinal cracking (coincident cracks following the line of the reinforcement caused by reinforcement corrosion), delamination and spalling of the concrete cover is referred to herein as a serviceability failure. Serviceability failures result in the need for repairs, replacement or more frequent inspections to monitor further deterioration. Clearly, this contributes considerably to the financial resources that need to be allocated to ameliorate these effects of deterioration.

Liu and Weyers [2] have developed a model to predict the time to first cracking (hairline crack of width less than 0.05mm) - referred to herein as T_{cr1}. In this model time to cracking is influenced by a large range of time-dependent and strength dependent variables; namely, bar diameter, bar spacing, cover, concrete tensile strength (related to water-cement ratio), time to corrosion initiation and corrosion rate. However, it is generally accepted that the service life of a structure is reduced considerably only if crack widths exceeding 0.3-0.5mm are not repaired. Preliminary results from accelerated corrosion testing of typical RC slabs at The University of Newcastle suggest that crack widths in the range 0.3-0.5mm occur at approximately 20-30T_{cr1} for time-variant corrosion rates (or 6-10T_{cr1} for time-invariant corrosion rates).

Corrosion initiation and propagation models developed by Vu and Stewart [3] are used in the present study. Recent improvements include:

- the effect of durability specifications on chloride diffusion and corrosion rates; and
- time-invariant corrosion rates (influenced by rust formation).

3. Probability Of Longitudinal Cracking And Spalling

The probability that longitudinal cracking and spalling (F_S) will occur at least once during the time interval T is defined herein as

$$F_S(T) = \Pr(T > T_i + T_{cr}) = \Pr(T > T_i + 30T_{cr1}) \tag{1}$$

where T_i is the time to corrosion initiation and T_{cr1} is the time to first cracking. This is a first passage probability which can be conveniently obtained from Monte-Carlo simulation analysis. Statistical parameters for surface chloride concentration, diffusion coefficient, critical threshold chloride concentration, time-invariant corrosion rates, model errors, and concrete material and dimensional variables are given in Table 1. Probabilistic analyses using these statistical parameters were conducted to assess the influence of cover and water-cement ratio on spalling probabilities, see Stewart [4] for further details. It is assumed that cover, water-cement ratio, surface chloride concentration, diffusion coefficient and other random variables are homogenous over the deck area. Figure 1 shows the probability of spalling for a RC bridge deck (top deck and soffit) subject to repeated applications of de-icing salts.

Table 1. Statistical Parameters for Corrosion, Dimensional and Material Variables [3].

Parameter	Mean	COV	Distribution
Model Error (D)	1.0	0.2	Normal
C_o	3.5kg/m^3	0.5	Lognormal
Threshold Cl	0.9kg/m^3	0.19	Uniform [0.6-1.2]
i_{corr}	$i_{corr} = \frac{37.8(1 - w/c)^{-1.64}}{cover}\ (\mu A/cm^2)$	–	–
$i_{corr}(t)$	$0.846 i_{corr} t_P^{-0.29}$ t_P=time since corrosion initiation	–	–
Model Error (i_{corr})	1.0	0.2	Uniform
C_{top} (top cover)	C_{nom}+19.8mm	σ=16.5mm	Normal
C_{bottom}	C_{nom}+8.6mm	σ=14.7mm	Normal
f'_{cyl}	F'_c+7.5MPa	σ=6MPa	Lognormal
k_w (workmanship)	0.87	0.06	Normal
f'_{ct} ($f'_c=k_w f'_{cyl}$)	$0.53\sqrt{f'_c}$	0.20	Normal
E_c	$4600\sqrt{f'_c}$	0.12	Normal

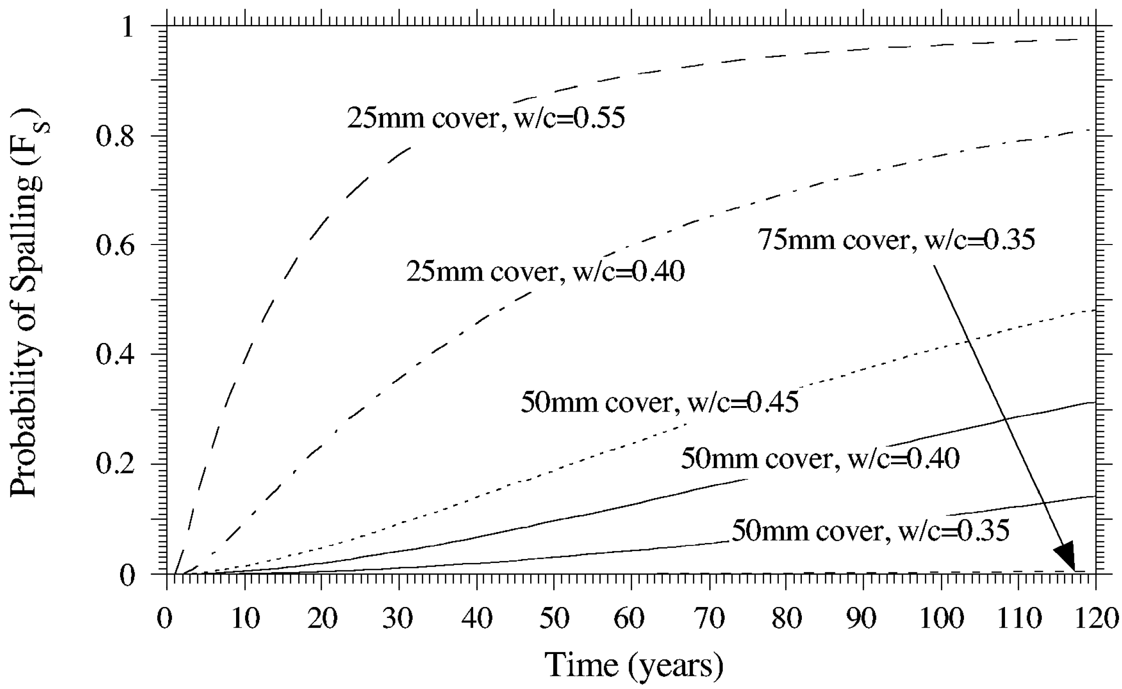

Fig. 1. Cumulative Probability of Spalling for De-Icing Salts - $F_S(T)$.

4. Life-Cycle Costs

If all attributes and consequences of a decision can be quantified in monetary units then an optimal management decision may be defined as the strategy that minimises life-cycle costs. Such a decision analysis can be formulated in a number of ways and consider various costs or benefits. In general, if the benefits of each alternative are the same then life-cycle costs up to time T may be expressed as

$$LCC(T) = C_D + C_C + C_{QA} + C_{IN}(T) + C_{MR}(T) + \sum_{i=1}^{M} p_{f_i}(T)C_{f_i} \tag{2}$$

where C_D is the design cost; C_C is the construction cost (materials and labour); C_{QA} is cost of quality assurance/control; $C_{IN}(T)$ is the expected cost of inspections; $C_{MR}(T)$ is the expected maintenance or repair costs; M is the number of independent failure limit states (flexure, shear, spalling); $p_{fi}(T)$ is the cumulative probability of failure for each limit state (i.e., probability that failure will occur anytime up to time T) and C_{fi} is the failure cost (damages, loss of life, injury, user delay, etc.) associated with the occurrence of each limit state. Costs and benefits may occur at different times so it is necessary for all costs and benefits (income) to be discounted to a present value. Note that a high discount rate favours a short service life.

At the first sign of spalling the associated loss of structural capacity (and safety) for many RC structures will not be significant since the corrosion-induced loss of diameter is only 0.1-0.5mm [e.g., 5]. As long as corrective action (repairs, replacement) is made to the structure after spalling then the probability of failure for strength limit states (flexure, shear) will essentially remain unchanged and so is not unduly influenced by durability design specifications or repair/replacement strategies. However, for other structures the effect of localised or pitting corrosion may be considerable and may not be preceded by cracking or spalling. This is beyond the scope of the present study. Hence, the following discussion is limited to spalling as being the most influential mode of failure for the estimation of life-cycle costs. Two repair (or replacement) strategies are considered (see Figure 2):

I. Spalling is immediately repaired but may re-occur during the remaining life of the structure (i.e., repair provides no improvement in durability performance of concrete cover - multiple repairs may be needed); and

II. Spalling is immediately repaired with a repair/replacement system of superior durability so that that once repaired spalling will not re-occur during the remaining life of the structure.

It is assumed that "damage" is detected when inspected and then repaired immediately. Repairs may include patching of damaged areas, or if spalling is widespread then complete structural replacement may be the preferred rehabilitation procedure [e.g., 6].

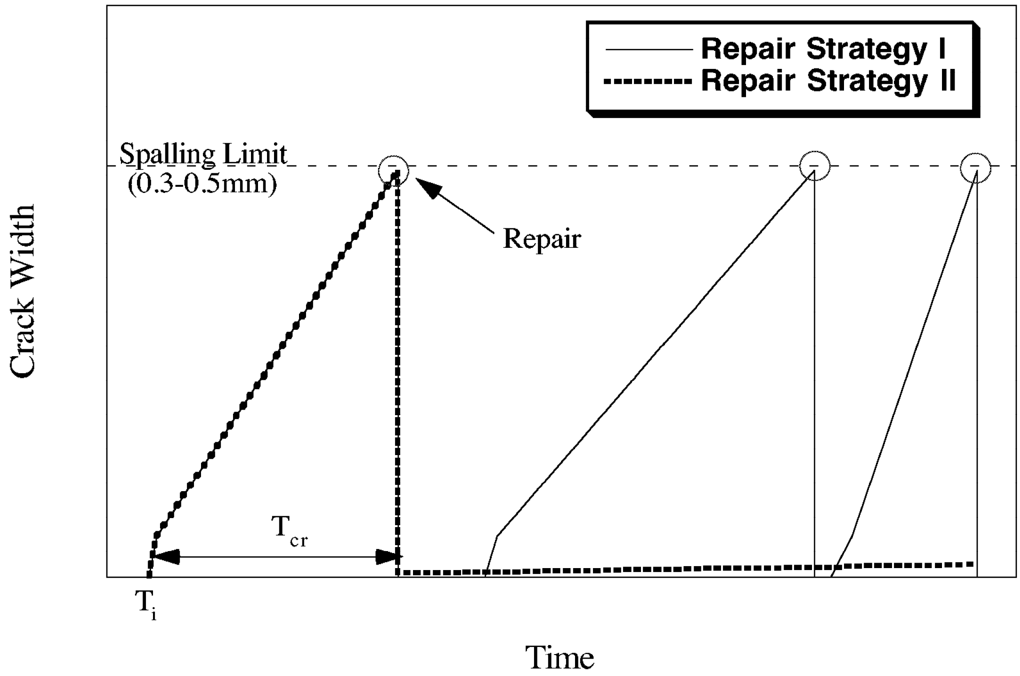

Fig. 2. Two Repair Strategies (T_i= time to corrosion initiation)

The life-cycle cost given by Eqn. (2) may be re-expressed as

$$LCC(T) = C_D + C_C + C_{QA} + C_{IN}(T) + C_{MR}(T) + E_{SF}(T) \tag{3}$$

where $E_{SF}(T)$ is the expected cost of spalling during service life (T) and C_D, C_C, C_{IN}, C_{MR} and C_{QA} are all present value costs. The expected cost of spalling up to time T is

$$E_{SF}(T) = \sum_{n=1}^{n=T} \int_{2}^{T} \frac{p_S(n,t)C_{SF}}{(1+r)^t} dt \tag{4}$$

where $p_s(n,t)$ is the probability that spalling will occur for the n^{th} time at time t, C_{SF} is the present value cost of spalling (repair, traffic delays, etc.) and r is the discount rate. See Stewart [4] for further details.

5. Illustrative Example: RC Bridge Deck Exposed To De-Icing Salts

The structural configuration used for this example is a three span 42m long highway bridge [7]. Transverse reinforcement has the lowest cover so this cover and its top and bottom reinforcement (Y16@225mm) are used to determine spalling probabilities. Since in this example the corrosion, dimensional and material variables are homogenous over the deck area then spalling damage will also be initiated homogeneously over the deck area; resulting in widespread damage over the deck area. U.S. highway agencies recommend deck replacement if the extent of visible spalls is over 25-40% [6]; hence it assumed herein that deck replacement is the preferred repair/replacement strategy after spalling damage is initiated. Replacement Strategies I and II are considered, see previous section.

5.1 Cost Data

The cost of deck construction (C_c) is the sum of material and labour costs. Costs of durability design specifications (or "quality" - C_{QA}) are estimated from the literature [e.g., 8,9] and are given in terms of the initial construction cost for the baseline case, see Table 2. The baseline case for costs is 50mm cover and w/c=0.40 which is set to C_C=1.0 (or US$225,000 - [7]) and costs of quality are then normalised in terms of C_C. Inspection and maintenance costs (C_{IN}, C_{MR}) are ignored in this analysis and design costs are constant and so are not needed for this comparative analysis.

Table 2. Additional or Reduced (-) Cost of Durability Design Specifications (C_{QA}).

Cover	w/c=0.35	w/c=0.40	w/c=0.45	w/c=0.50	w/c=0.55
25mm	-0.047C_C	-0.069C_C	-0.088C_C	-0.097C_C	-0.103C_C
50mm	0.022C_C	0	-0.019C_C	-0.028C_C	-0.034C_C
75mm	0.091C_C	0.069C_C	0.050C_C	0.041C_C	0.034C_C

The cost of deck replacement and associated traffic delay costs, etc. (C_{SF}) are considerable and user delay costs are often much greater than direct (agency) costs. Repair, replacement and user delay costs are structure and site specific and so it is difficult to make generalisations about these costs. Nonetheless, an analysis of cost data reported by Ehlen [10] suggest that agency disposal costs for the highway bridge considered herein are equal to the initial construction cost. Hence, the agency replacement costs are 2.00C_c+C_{QA} (disposal cost + initial construction cost) and 2.09C_c (improved durability performance - 75mm cover, w/c=0.35) for replacement strategies I and II respectively.

5.2 Results

For illustrative purposes, mean life-cycle costs based on a 4% discount rate and a service life of 120 years are shown in Figure 3. Inspections are conducted every two years. In this figure life-cycle costs represent direct costs to the bridge agency or authority. Including user costs in the life-cycle costing analysis does not influence the optimal durability design specifications described below [4].

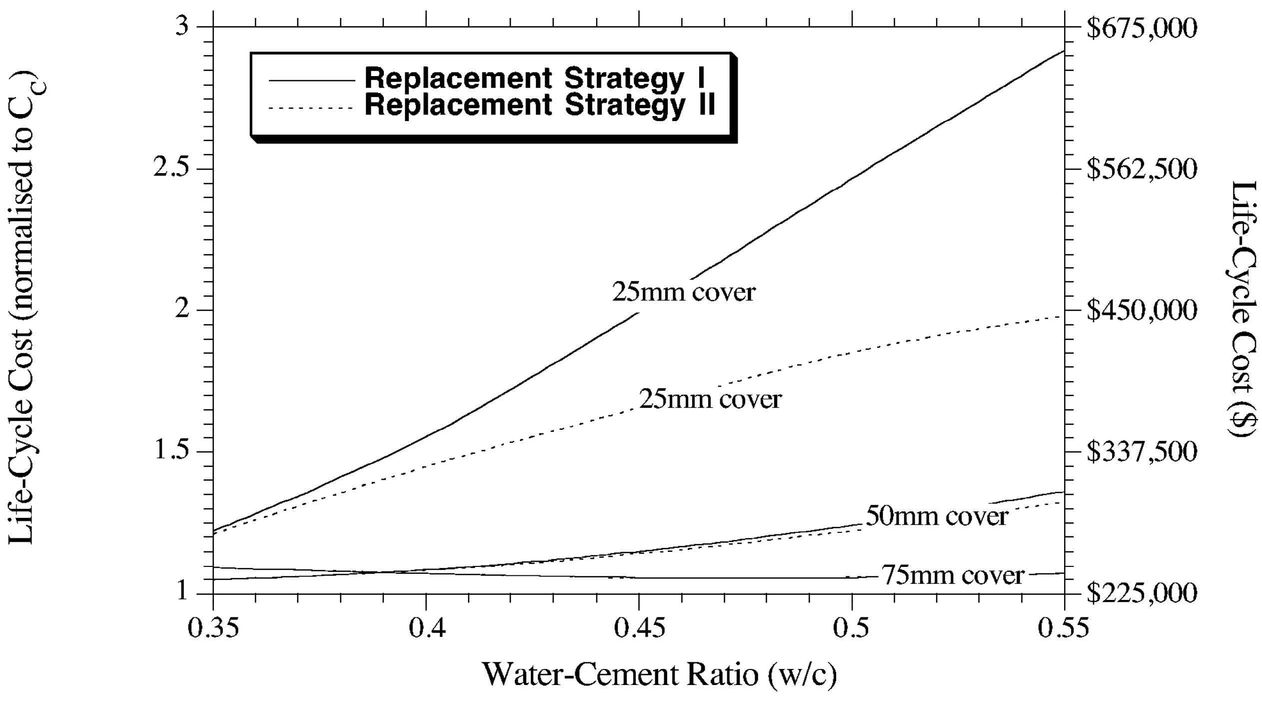

Fig. 3. Mean Life-Cycle Direct Costs for De-Icing Salts and 4% Discount Rate.

It is evident from Figure 3 that the lowest life-cycle costs occur for a 50mm cover and w/c=0.35 since the expected cost of spalling (E_{SF}) is low relative to the costs of quality. A 75mm cover with a high water-cement ratio also appears to produce near minimum life-cycle costs for both replacement strategies. This observation reflects the greater relative importance of cover than water-cement ratio on chloride penetration (diffusion) and spalling processes and the lower proportional cost of using low water-cement ratio concrete. In all cases Replacement Strategy II (single deck replacement with improved durability performance) resulted in lower life-cycle costs. Figure 3 shows that a bridge with 25mm cover and high water-cement ratio can result in additional costs of up to $450,000 over the service life of the structure - this is a significant cost differential.

Monte-Carlo simulation analysis may be used to propagate uncertainties in spalling probabilities in the estimation of life-cycle costs - this provides a probabilistic description of life-cycle cost uncertainty and may be useful when expected life-cycle costs are similar for different management strategies. For example, life-cycle costs for Repair Strategy I and 75mm cover are similar for water-cement ratios of 0.40 and 0.55. Figure 4 shows histograms of life-cycle costs obtained for each realisation of the Monte-Carlo simulation analysis, for Replacement Strategy I.

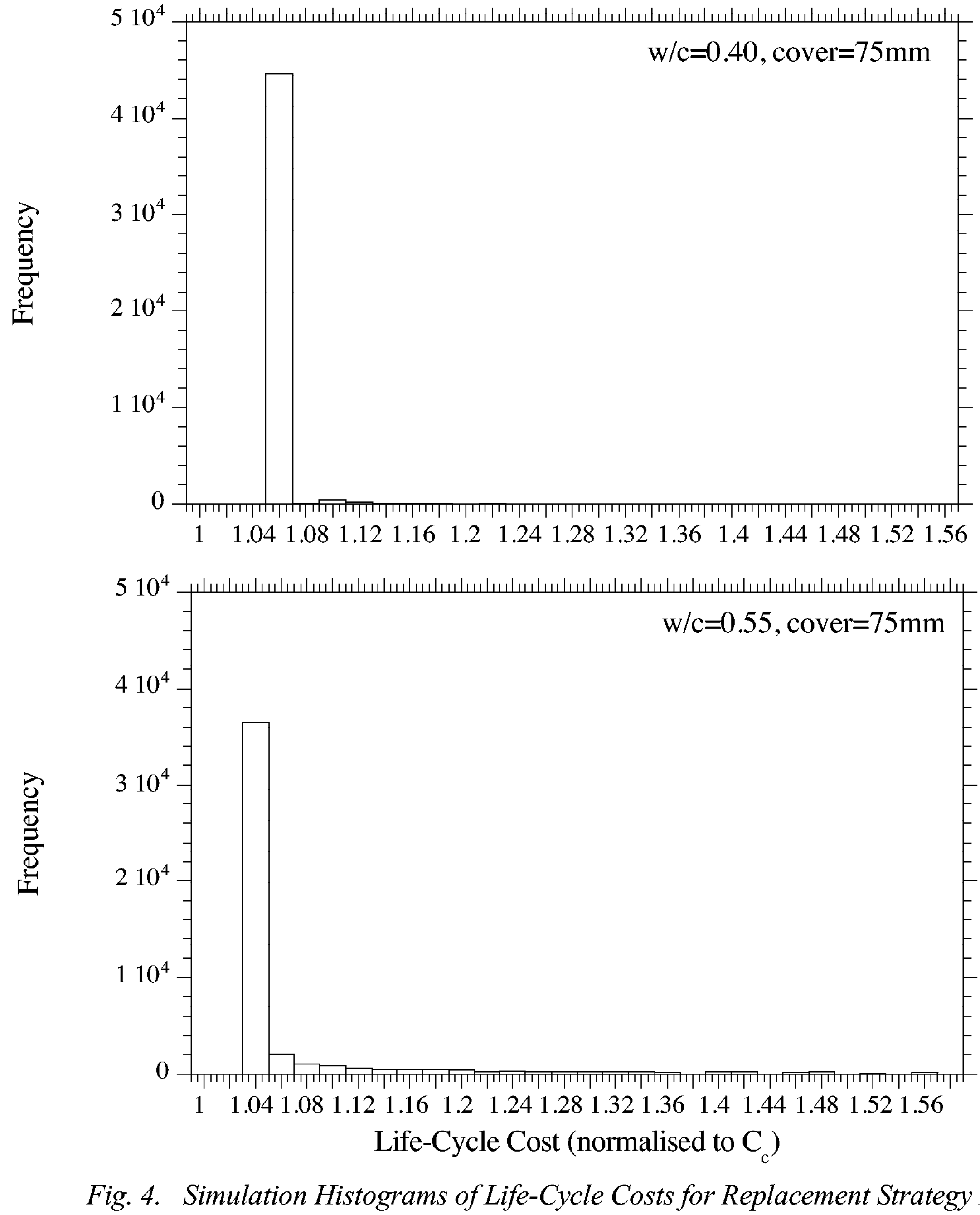

Fig. 4. Simulation Histograms of Life-Cycle Costs for Replacement Strategy I

With reference to Figure 4, the uncertainty in these life-cycle costs is higher for a higher water-cement ratio because of the higher variability of poor spalling performance. Uncertainties due to costs of failure, and repair, maintenance and inspection costs could also be included in the analysis. On the other hand, uncertainties in life-cycle costs may be reduced significantly by minimising model and parameter variabilities.

The influence of service life and discount rates on life-cycle costs are discussed in more detail by Stewart [4], as is the effect of sea-spray on spalling probabilities and life-cycle costs.

6. Comparison of Strength and Serviceability Limit States

This section will compare the effect of limit state selection (strength vs. serviceability) on life-cycle costs which may then help determine whether safety or functionality (or both) are important criteria when optimising bridge life-cycle performance and costs. The structural element under consideration is the same RC bridge deck considered in the previous section.

This preliminary comparison of life-cycle costs will consider a 50-year reference period, a 4% discount rate and expected failure costs only. The target reliability index for the strength limit state (for design) specified by Eurocode 1 is 3.8 based on a 50-year reference period. On the assumption of independence of annual failure events the annual probability of failure (p_{fA}) can then be calculated as 1.4×10^{-6}. The expected cost of failure is then

$$E_{SF}(T) = \sum_{t=1}^{50} \frac{p_{fA}\,C_{SF}}{(1+r)^t} \tag{5}$$

Expected costs of spalling are obtained from Eqn. (4) for Repair Strategy 1 (i.e., the "worst-case" of multiple repairs). Expected costs of failure for violation of strength and spalling limit states are shown in Table 3 for a range of durability specifications. Naturally, the consequences of exceeding a strength limit state (collapse) will be much higher than exceeding a serviceability limit state due to loss of life, injury and other direct and indirect costs. It has been estimated that the consequential cost factor (ϕ_{tc}) for structural collapse is 30-75 [11] where $C_{SF}=\phi_{tc}C_c$.

Table 3. Expected Costs of Failure (E_{SF}).

Limit State	Conditions	E_{SF} (ϕ_{tc}=1)	E_{SF} (ϕ_{tc}=30)	E_{SF} (ϕ_{tc}=75)
Strength:	$p_{fA}=1.0\times10^{-3}$	$0.021C_c$	$0.63C_c$	$1.56C_c$
	$p_{fA}=1.4\times10^{-6}$	$3.1\times10^{-5}C_c$	$9.3\times10^{-4}C_c$	$0.002C_c$
Spalling:	w/c=0.40, 50mm cover	$0.062C_c$	–	–
	w/c=0.50, 50mm cover	$0.222C_c$	–	–
	w/c=0.40, 25mm cover	$0.544C_c$	–	–

Table 3 shows that the expected costs of failure are higher for spalling even if the consequential cost factor is much greater then 75 or if the annual probability of structural collapse is much higher than the target reliability level. Hence, at least for this particular case, life-cycle costs will be influenced by serviceability considerations more than strength considerations. However, this conclusion may change for other structural configurations or other cost assumptions, and it is possible to include both (or more) limit states in a life-cycle cost analysis. Work is continuing to develop and compare life-cycle cost analyses considering multiple limit states.

7. Further Work

These life-cycle cost calculations are based on relatively simple costing data and so are meant to help illustrate the potential of life-cycle cost analyses only. More detailed, realistic and accurate analyses may be conducted that consider inspection, maintenance and repair costs and more accurate agency-specific costing data. For example, if spatial variability of surface chloride concentration and cover are included in the analysis then the proportion of the structural element damaged by spalling can be estimated. The efficiency and cost of concrete patching and other repair and replacement strategies and costs may then be considered for a range of structural elements and causes of corrosion. The effect of localised (pitting) corrosion on cracking and spalling is also an important consideration. These are areas of ongoing research at The University of Newcastle.

Finally, it might be noted that the consideration of cracking and spalling limit states removes the need for load/resistance prediction and system modelling. There is considerable uncertainty associated with these models; for example, the effect that corrosion has on bond strength and how this might affect flexural and shear capacity. Hence, it is anticipated that serviceability reliabilities and associated life-cycle costs can be predicted with more certainty, and used with a greater degree of confidence, than structural strength (safety) reliabilities and associated life-cycle costs.

8. Conclusions

Risk–based approaches provide a meaningful measure of structural performance and a life-cycle cost or other decision analysis may be used to quantify the expected cost of a decision. The present paper focused on durability design specifications and their influence on serviceability (cracking and spalling) reliability. In conjunction with life-cycle cost analysis this provides a risk-based criterion for optimising repair strategies and various design parameters. It was found that, at least for this particular case, that life-cycle costs will be influenced considerably by serviceability considerations.

9. References

[1] Loss, J., "AEPIC Project: Update", *Journal of Performance of Constructed Facilities, ASCE*, Vol. 1, No. 1, 1987, pp. 11-29.

[2] Liu, Y. and Weyers, R.E., "Time to Cracking for Chloride-Induced Corrosion in Reinforced Concrete", *Corrosion of Reinforcement in Concrete Construction*, C.L. Page, P.B. Bamforth, J.W. Figg (Ed.), Royal Society of Chemistry, Cambridge, 1996, pp. 88-104.

[3] Vu, K. and Stewart, M.G., 'Structural Reliability of Concrete Bridges Including Improved Chloride-Induced Corrosion Models", *Structural Safety, Vol.* 22, No. 4, 2000, pp. 313-333.

[4] Stewart, M.G., *Risk-Based Optimisation of Repair Strategies for Concrete Bridge Decks Considering Cracking and Spalling Limit States and Life-Cycle Cost Analysis*, Research Report No. 190.03.2000**,** Dept. of Civil, Surv. and Env. Engg., The Uni. of Newcastle, Australia, 2000.

[5] Stewart, M.G. and Rosowsky, D.V., "Structural Safety and Serviceability of Concrete Bridges Subject to Corrosion", *Journal of Infrastructure Systems*, Vol. 4, No. 4, 1998, pp 146-155.

[6] Cady, P.D., "Bridge Deck Rehabilitation Decision Making", *Transportation Research Record*, No. 1035, 1985, pp. 13-20.

[7] Estes, A.C. and Frangopol, D.M., "Repair Optimization of Highway Bridges Using System Reliability Approach", *Journal of Structural Engg.*, ASCE, Vol. 125, No. 7, 1999, pp. 766-775.

[8] Koskisto, O. and Ellingwood, B.R., "Reliability-Based Optimization of Plant Precast Concrete Structures", *Journal of Structural Engineering*, Vol. 123, No. 3, 1997, pp. 298-304.

[9] Troive, S. and Sundquist, H., "Optimisation of Design Parameters for Lowest Life-Cycle Cost of Concrete Bridges", *Fifth International Conference on Short and Medium Span Bridges*, Canadian Society for Civil Engineering, Vol. 2, 1998, pp. 1101-1113.

[10] Ehlen, M.A., "Life-Cycle Costs for Fibre-Reinforced-Polymer Bridge Decks", *Journal of Materials in Civil Engineering*, ASCE, Vol. 11, No. 3, 1999, pp. 224-230.

[11] CIRIA, *Rationalisation of Safety and Serviceability Factors in Structural Codes*, Report No. 63, Construction Industry Research and Information Association, London, 1977.

Probabilistic Models of Cost for the Management of Existing Structures

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Keywords: Existing structures, probabilistic cost-benefit analysis, safety, serviceability.

Summary

This paper presents probabilistic models of costs and an approach to cost-benefit analyses to be used for planning interventions on existing structures. Costs due to inadequate performance with respect to both safety and serviceability are modelled and combined in the approach. Costs are modelled considering time dependent effects, such as deterioration, and the total cost is determined by summation over a given reference period, which is normally taken to be the expected remaining life of a structure. An approach is presented for determining the optimal intervention strategy for a structure with respect to time, place and level of intervention. The models and approach are illustrated with a simple example.

1. Introduction

Bridge maintenance costs are already high and are increasing. However, maintenance budgets will remain tight, and it is therefore very important to use resources efficiently. This can be achieved by implementing the concept of cost-benefit analysis, considering all expected costs over the remaining service life of a structure. This analysis is best carried out using probabilistic models in order to take account of uncertainty and to consider expected cost. A time-variant model can be developed for safety problems, and indeed much research has been conducted in this area [Frangopol 1997, Thoft-Christensen 1998, Val 1998]. However, existing approaches are based on the expected cost at a given point in time, without considering the risk of delaying an intervention. Such approaches suggest that it is advantageous to wait for as long as possible before repairing since the potential for instantaneous benefit is then greater, but this ignores the increased risk during the waiting period [Radojicic 1999]. Furthermore, with respect to serviceability there is currently no widely accepted approach to determining optimum interventions. Structural maintenance is normally determined in practice by serviceability problems and decisions on repairs are usually based on experience, without applying an analytical approach to finding an optimum strategy. The objectives of the research presented in this paper are therefore, firstly, to improve the existing cost-benefit model by incorporating the risk of delaying an intervention, secondly to address serviceability costs using probabilistic methods, and thirdly to develop an approach for using the new cost model in order to optimize intervention strategies.

2. Model Framework

The basic concept of the cost model proposed in this paper is that strategies for routine maintenance and repair should be based on the sum of the costs and benefits associated with interventions considered at all limit states and over a given period of time. The expected cost of intervention C_m consists of all tangible investments as well as user costs during interventions on the structure over a given period of time. The benefits due to intervention are the reduction of expected costs related to inadequate performance at each limit state, such as the expected cost of inadequate serviceability C_{ser} and the expected cost of structural failure C_{fail}. All costs can be expressed as a function of the set of parameters $\mathbf{X}$ that describe the structure (e.g. material properties, dimensions, traffic, loads, importance within the road network, etc.) and their variation over time, t. Costs and benefits are calculated using a discounting scheme in order to obtain their net present value. This analytical framework permits the comparison of alternatives with the objective of minimizing total expected costs and maximizing benefits over a given period of time.

2.1 Expected cost of intervention

The cost of intervention (routine maintenance, repair, rehabilitation, etc.), C_m, covers all user and owner costs and can be expressed as [Chang & Shinozuka 1996]:

$$C_m(\mathbf{X},t) = \sum_{i=1}^{N} (m_i(\mathbf{X},t)\cdot(1+r)^{-t} + C^i_{m,u}(\mathbf{X},t)) \tag{1}$$

where $m_i(\boldsymbol{X},t)$ is the material and labour cost of an intervention, r is the discount rate, $C^i_{m,u}(\boldsymbol{X},t)$ is the user costs for each intervention and N is the number of interventions.

2.2 Expected cost of structural failure

The incorporation of time-variant resistance and load models in classical approaches to evaluating structural reliability would provide only the estimated probability of failure at a certain point in time without taking into account the probability of failure sometime before that point. This fact becomes important when assessing existing bridges using a risk-based approach because it does not consider the cost of delaying an intervention and the corresponding risk. The cumulative-time probability of failure and failure-time probability should be used.

The cumulative-time probability of failure over a period $[t_1, t_2]$ is the probability that the structure will fail anytime within the period $[t_1, t_2]$[Mori & Ellingwood 1993]. If the conditional probability of failure at point t_i given that structure has already survived the period $[0, t_{i-1}]$, is denoted as point-in-time probability of failure, then applying the definition of conditional probability, the probability that failure will occur within the period $[t_{i-1}, t_i]$ or failure-time probability can be expressed as:

$$p^{LT}(t_i) = \left(1 - P_f^{cum}(t_{i-1})\right)\cdot p_f(t_i) \tag{2}$$

where $p^{LT}(t_i)$ is the failure-time probability or probability that failure occurs in the period $[t_{i-1}, t_i]$; $P_f^{cum}(t_{i-1})$ is the cumulative-time probability of failure over the period $[0, t_{i-1}]$ and $p_f(t_i)$ is the conditional probability of failure at time t_i given that a structure has already survived the period $[0, t_{i-1}]$. All three probabilities are illustrated in Figure 1.

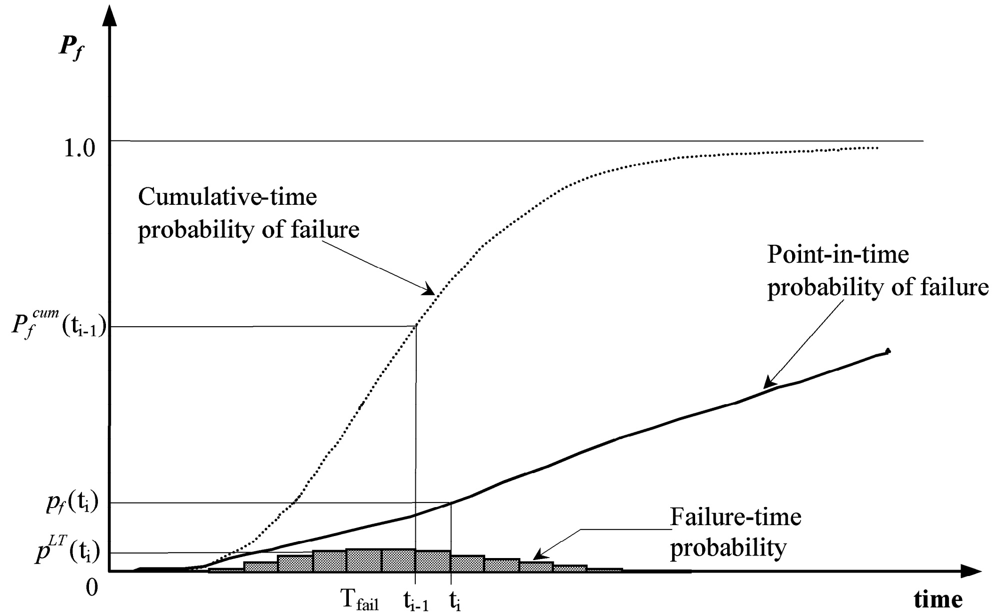


Figure 1. *Failure-time probability, cumulative-time and point-in-time probabilities of failure*

$p^{LT}(t_i)$ depends on the length of the interval $[t_{i-1}, t_i]$. If the interval increases, the probability that the structure would fail within that interval also increases. Note that initially the failure-time probability $p^{LT}(t)$ increases with time, reaches a maximum at T_{fail}, which is the most probable failure time, and then decreases. $p^{LT}(t)$ is low once the structure is highly deteriorated due to the fact that the structure would have probably already failed.

If the interval $\Delta t = [t_{i-1}, t_i]$ is finite:

$$pdf^{LT}(t_i) = \frac{p^{LT}(t_i)}{\Delta t} \tag{3}$$

where $pdf^{LT}(t)$ is the failure-time probability density function. The failure-time probability density function is related to the cumulative-time probability by the following integral:

$$P_f^{cum}(t_i) = \int_0^{t_i} pdf^{LT}(t)\ dt \tag{4}$$

Expected costs due to structural failure over a given period of time, C_{fail}, can be defined as the integral over time of the product of the failure-time probability density function and the costs associated with the failure C^f, discounted with the factor $(1+r)^{-t}$:

$$C_{fail}(\mathbf{X}, t_i) = \int_0^{t_i} pdf^{LT}(\mathbf{X}, t) \cdot C^f(t) \cdot (1+r)^{-t}\ dt \tag{5}$$

The failure-time probability density function $pdf^{LT}(\mathbf{X}, t)$ is a function of parameters describing the structure and their variation over time. The costs associated with failure depend only on the point in time when possible failure occurs, as shown in [Radojicic 1999].

2.3 Expected cost of inadequate serviceability

The expected cost of inadequate serviceability includes the loss of benefits to owners due to a deteriorated structure not being fully utilized as well as the cost to users due to inadequate serviceability. The level of inadequate serviceability of a structure can be represented by a factor r_{is}, which can vary between 0 and 1. It is equal to zero if serviceability is completely adequate and it is equal to 1 if the structure cannot be used. The level depends on a number of serviceability parameters (e.g. deflection). Thresholds on such parameters can be set that correspond to the upper and lower bounds on the factor r_{is} .

Deflection, or any other serviceability parameter, is a function of the set of structural parameters, X, and time. Therefore, $r_{is} = r_{is}(X, t)$. The cost of inadequate serviceability to users c_u expressed as a cost per unit of time is a function of the factor r_{is} , and can reach a maximum unit user cost, u. The relationship between c_u and r_{is} is not yet well defined. However, the intuition behind an upwardly sloping curve is clear, but the precise nature of the curve is not yet known and is the object of further research. For now it is assumed that the curve is parabolic as shown in Figure 2.

At a given point in time, t_i, the probability density function (PDF) of the factor $r_{is}(X, t_i)$ can be estimated considering the relevant state of the deteriorated structure. This PDF is shown in Figure 2 together with the user cost function $c_u(r_{is}, t_i)$. The total expected cost of inadequate serviceability over the period $[0, t_i]$ C_{ser} is then given by [Radojicic 1999]:

$$C_{ser}(\mathbf{X}, t_i) = \int_0^{t_i} (1+r)^{-t} \int_0^1 c_u(r_{is}, t) \cdot pdf^{r_{is}}(\mathbf{X}, t)\ dr\ dt \tag{6}$$

where r is the discount rate; r_{is} is the level of inadequate serviceability ($0 < r_{is} < 1$); c_u is the cost to users as a function of r_{is} ($0 < c_u < u$); u is the value of the structure to users; $pdf^{r_{is}}(\mathbf{X}, t)$ is the probability density function of r_{is} for the set of structural parameters $\mathbf{X}$ at time point t.

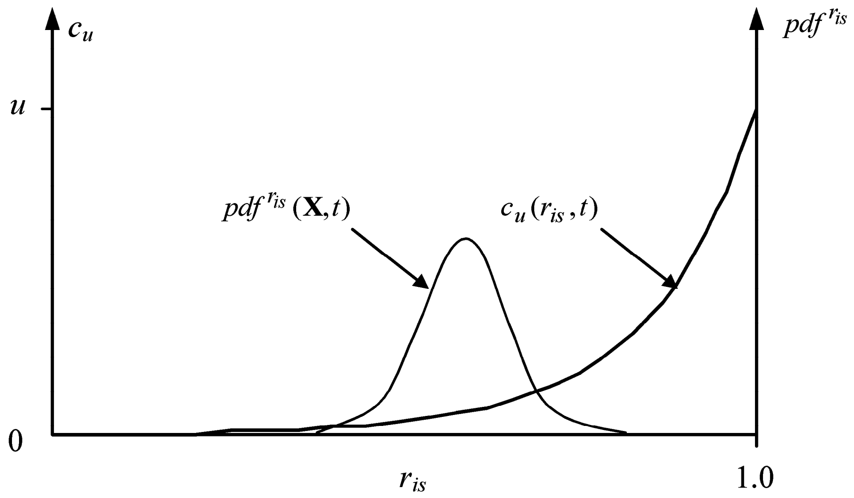


Figure 2. *User cost and the probability density function of inadequate serviceability*

2.4 Benefit of intervention

An intervention on a structure reduces the probability of failure and the corresponding risk. Assuming that the costs associated with the failure C^f are time-invariant and the discount rate is equal to zero, the total loss due to the risk of structural failure over a given period of time [0, t] can then be determined applying (4 & 5). An intervention at some point in time would decrease the future probability of structural failure and thus decrease the slope of the curve representing the associated expected cost as illustrated in Figure 3(a). The difference between the two curves "without intervention" and "with intervention" represents the benefit of the intervention due to the decreased risk of structural failure. The slope of the curve "with intervention" depends on the extent of intervention. Therefore, the benefit of intervention is a function of not only time, but also the level of intervention and can be expressed as follows:

$$B_{fail}(T_{int}) = C'_{fail}(\mathbf{X},T_{ref}) - C''_{fail}(\mathbf{X},T_{ref}) \qquad (7)$$

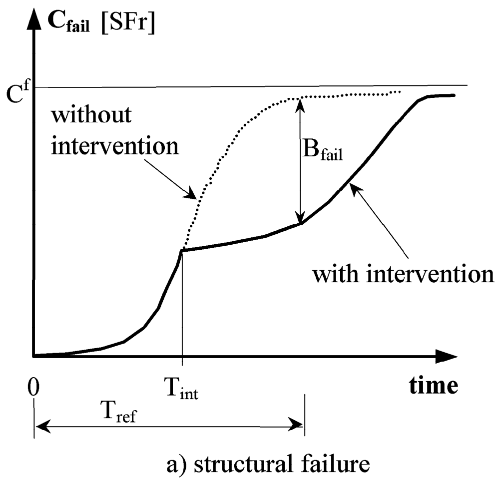
where $B_{fail}(T_{int})$ = benefit of intervention at time T_{int} with respect to the reduced expected cost of structural failure; $C''_{fail}(\mathbf{X},T_{ref})$ and $C'_{fail}(\mathbf{X},T_{ref})$ are the expected costs of structural failure at time T_{ref} with and without intervention, respectively; T_{int} and T_{ref} are the time of intervention and the reference period, respectively.

The benefit depends on the length of the reference period. However, the probabilities of failure for real-life structures rarely reach 1, meaning that the cumulative-time probability of failure, and thus curves of expected cost of structural failure for "without intervention" and "with intervention" rarely converge as is presented in Figure 3(a).

An intervention at some point in time would increase the future serviceability of a structure and thus decrease both the expected cost of inadequate serviceability (in cost unit per time) $c_{ser}(t)$ and the slope of the curve representing $c_{ser}(t)$ as illustrated in Figure 3(b). The slope of the new curve depends on the extent of intervention. The shaded area in Figure 3(b) represents the nominal benefit of intervention through increased serviceability over the interval [T_{int}, T_{ref}]:

$$B_{ser}(T_{int}) = \int_{T_{int}}^{T_{ref}} (c'_{ser}(t) - c''_{ser}(t))\ dt \qquad (8)$$

where $B_{ser}(T_{int})$ = benefit of intervention at time T_{int} with respect to reduced expected cost of inadequate serviceability; $c''_{ser}(t)$ and $c'_{ser}(t)$ are the expected cost of inadequate serviceability [cost per unit time] with intervention and without intervention, respectively.



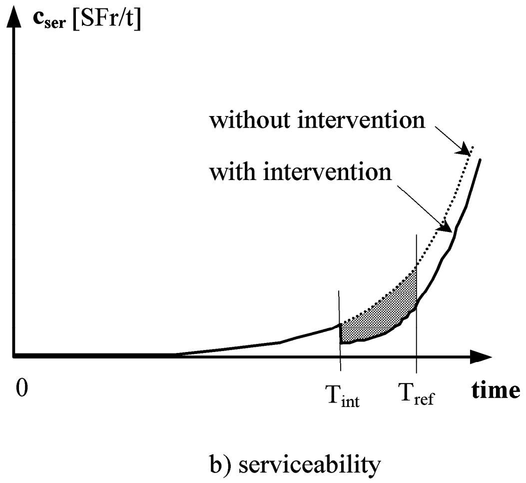


Figure 3. *Benefit of intervention*

3. Numerical Example

A simple structure has been treated to illustrate the model for optimizing the intervention strategy over a given reference period. The First Order Reliability Method was used to determine the time-variant reliability of individual elements that were then combined in order to consider the system. The effects of deterioration on structural parameters and post-failure load redistribution were considered in order to illustrate time dependent effects on the serviceability and safety of the structure as a system. Although simple, the example includes the aspects that would be treated when considering a more complex structure.

3.1 Description

The example treats a three-bar truss, as shown in Figure 4.

The deterioration of each element was modelled with the resistance degradation function:

$$R(t) = R_0 \cdot g(t) \tag{9}$$

where *R(t)* is the resistance as a function of time; R_0 is the initial resistance and *g(t)* is the resistance degradation function.

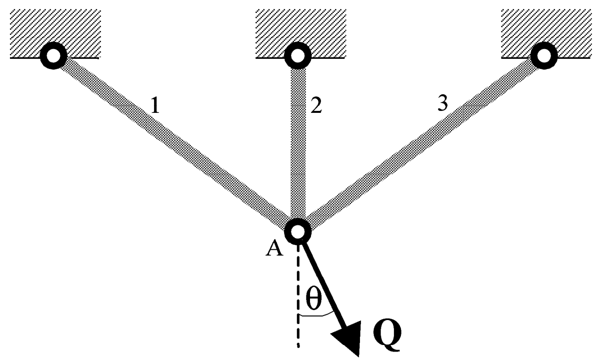


Figure 4. *Three-bar structure*

Resistance degradation functions for each element are shown in Figure 5. In this and all subsequent figures, time is expressed non-dimensionally as a proportion of the expected remaining service life. All functions were assumed to be parabolic and defined with initiation time for deterioration t_I and deterioration rate coefficients k_a and k_b:

$$g_i(t) = 1 - k_a^i \cdot (t - t_I^i) + k_b^i \cdot (t - t_I^i)^2, i = 1,2,3 \tag{10}$$

Mean values of the resistance degradation parameters are presented in Table 1. Initiation time for deterioration t_I is expressed as a proportion of the reference period and coefficients k_a and k_b are non-dimensional.

Table 1. Resistance degradation parameters

bar	t_I	k_a	k_b
1	0.25	0.045	0.00075
2	0.15	0.060	0.00060
3	0.15	0.055	0.00105

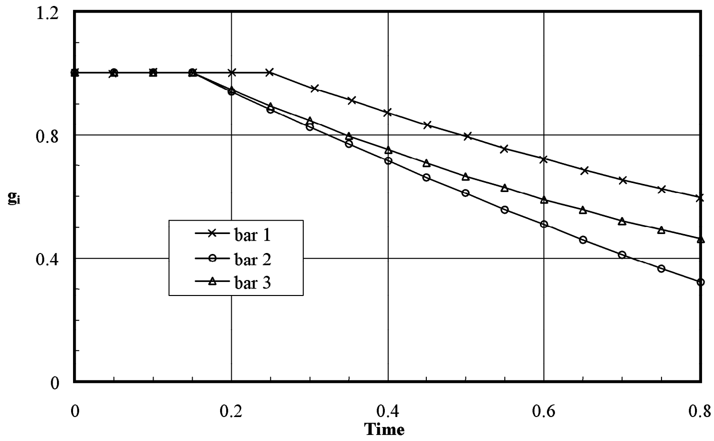


***Figure 5.** Resistance degradation functions*

3.2 Reliability analysis

The First Order Second Moment reliability method was used in order to determine reliability indices, β, and point-in-time failure probabilities for each limit state function. The reliability of the whole structure was then considered using a system approach, based on a system of parallel elements, whose combined failure can lead to collapse. A fault tree and event tree model was used for identifying all the potential failure modes and their respective consequences. The point-in-time (conditional) failure probabilities should be updated based on survival of previous time intervals, but this was not done in this example. This simplification leads to an over-estimation of failure probabilities. Cumulative-time probabilities of failure and failure-time probability density functions for all elements and the system were determined using the approach described in Section 2.2.

***Table 2.** Probabilistic models of basic variables*

Variable	Distribution	Nominal value	Bias	Cov
Steel, strength	Lognormal	460 N/mm^2	1.25	0.08
Steel, area	Normal	-	1.00	0.04
Deterioration factor	Beta	$g_i(t)$	1.00	0.10
Load	Gumbel	700 kN	1.00	0.10
Dimension error	Normal	10 mm	1.00	0.25
Force slope	Normal	15°	1.00	0.05

The probabilistic models of basic variables are presented in Table 2. Figure 6 illustrates how stresses in the bars develop over time without intervention. Note that failure is expected at point $T = 0.47$. The most critical element is bar 2, for two reasons: firstly, considering the geometry of the truss and the position and slope of the force, bar 2 carries the most load, and secondly, bar 2 deteriorates the fastest (see Figure 5).

Figure 7 shows how reliability indices β for all elements and the system change over time due to the simultaneous deterioration of all elements. Reliability indices are relatively high due to low stresses in the bars (the Swiss design codes imply a β of about 4.5 for a steel bar in tension at the ultimate limit state). Different deterioration scenarios were investigated, as the influence of assumptions about correlation between deterioration rates of different elements. Two cases were considered for determining the first-order bounds on reliability; either perfectly correlated, or uncorrelated, deterioration processes. It is interesting to note that the bounds on reliability index are narrow and the assumption about correlation in this case is not significant.

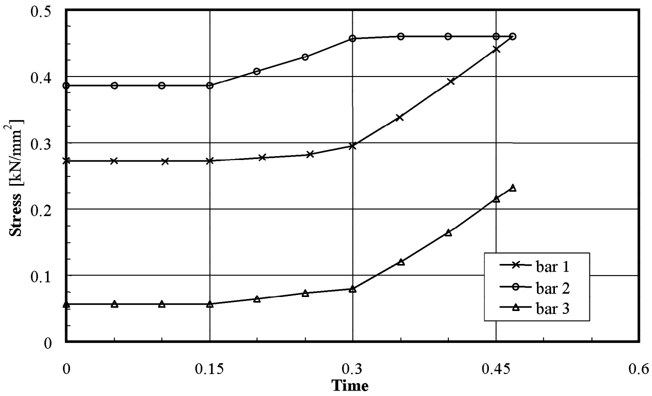


Figure 6. Stresses in the elements

Different repair scenarios were considered. Despite the fact that the structure consists of only three elements, the number of all possible intervention strategies is of the order of a million. Figure 8 shows how reliability indices β, vary as a function of the bar 2 repair level at time $T = 0.40$. The change in cross sectional area of bar 2, ΔA_2, is expressed as a percentage of the original cross sectional area at time $T = 0$.

Figure 8 illustrates that the safety of the system can only be increased up to some point by repairing just one element. Further repair of the same element does not change the dominant failure mechanism, and for that reason it would not be possible to increase the reliability of the system further with the same intervention (except a small part of it due to load redistribution).

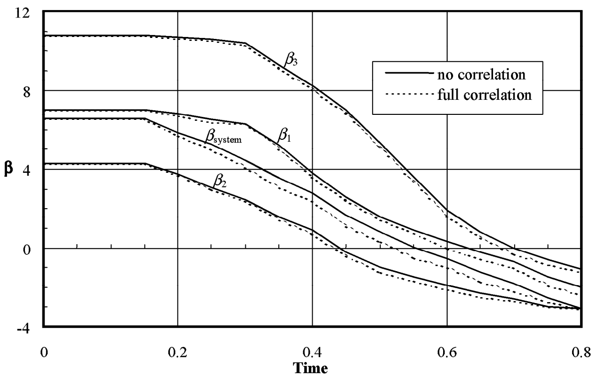


***Figure 7.** Reliability of individual elements and the system as a function of time*

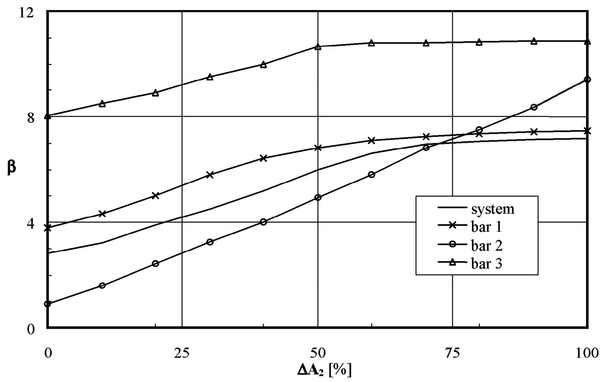


***Figure 8.** Reliability indices as a function of the bar 2 repair level*

3.3 Cost-benefit analysis

Three cost components are considered: intervention, safety and serviceability, of which the last two are used to evaluate the benefits of an intervention. Cost parameters are shown in Table 3. The costs considered in the example are arbitrary and were varied in order to investigate the sensitivity of results to assumptions about costs. All costs and benefits are expressed as a function of the costs associated with structural failure C^f.

***Table 3.** Cost parameters*

Variable	Value
Cost associated with failure, C^f	1.00
Value to users, u	1/year
Cost of bar 1 repair, m_1	0.03/unit*
Cost of bar 2 repair, m_2	0.05/unit
Cost of bar 3 repair, m_3	0.015/unit
User cost during repair, $C_{m,u}$	0.15
Discount factor, r	0.01

* unit = 15% of the initial section area

Expected cost of intervention

An intervention is considered as a combination of different repair levels at different points in time on each element. In order to simplify the example, routine maintenance costs were not considered. The expected cost of intervention was then determined by applying (1). It was assumed that the elements have different repair costs, where bar 2 is the most expensive to repair. Furthermore, the user cost during repair $C_{u,r}$, does not vary as a function of the repair level (for example, the intervention strategy that consists of 20 % repair of bar 1 at time t_a, 30 % repair of bar 2 at time t_a, and 10 % repair of bar 3 at time t_b, would have user cost during repair (without discounting) equal 2 x 0.15).

Expected cost of structural failure

The expected cost of structural failure (yield of at least two elements) is determined using (5). Figures 9 and 10 show how time-cumulative probability of failure and failure-time probability vary over time due to simultaneous deterioration of all elements for the three different intervention strategies presented in Table 4.

Table 4. *Repair levels and times of intervention for strategies A, B and C*

Strategy	Repair level [%]			Time of intervention		
	bar 1	bar 2	bar 3	bar 1	bar 2	bar 3
A	WP*	15	nothing	0.30	0.30	-
B	15	60	15	0.40	0.40	0.40
C	45	15	30	0.60	0.50	0.40

* protect element from further deterioration for next 20 years, without increasing the cross section

The slope of the curve that represents the time-cumulative probability of failure changes at times of intervention. The change in slope is a function of the repair level and importance of the repaired element. It can be seen that a high repair level on bar 2 (60 % in intervention strategy B) is the most efficient in order to delay system failure, which confirms the observation from the stress analysis (Figure 6) that bar 2 is the most critical.

Figure 10 shows that the most probable time of system failure is a function of the repair time, level and place. Note that the most probable failure point for the strategy "no intervention" is at $T = 0.55$ which differs from the result obtained with the deterministic analysis of stress development in the bar over time ($T = 0.47$, Figure 6). Figure 10 confirms that the high repair level on bar 2 (strategy B) is the most favourable among the presented intervention strategies from the structural safety point of view because it has the latest failure point.

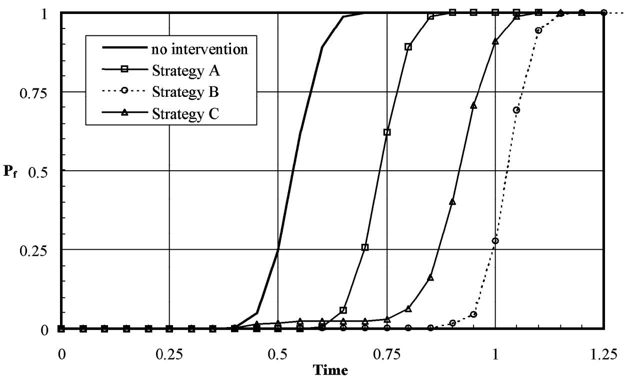


Figure 9. *Time-cumulative probability of failure for the system*

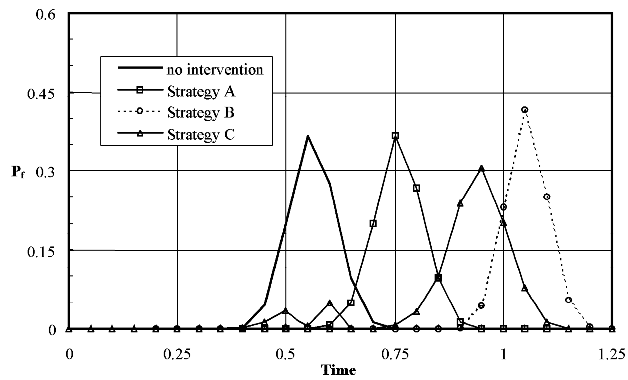


Figure 10. *Failure-time probability for the system*

Figure 11 shows how point-in-time reliability index varies over time as a function of the intervention strategy. An intervention is illustrated with the "jump" in the curves. Note that the repair of bar 3 has no significant influence on the system reliability (curve "strategy C" does not significantly differ from the curve "do nothing" after bar 3 repair at $T = 0.40$).

The system failure-time probability density functions are used to determine the expected cost of structural failure as a function of time applying (5). The cost associated with the failure C^f is considered to be time invariant in this example. Figure 12 illustrates how cumulative cost C_{fail} varies over time due to the simultaneous deterioration of all elements for three intervention strategies. Similar to the curve that represents the time-cumulative probability of failure (Figure 9), the cumulative cost C_{fail} curve changes slope when intervention takes place. The change in slope is a function of the repair level and importance of the repaired element. It can be seen that a high repair level on bar 2 (60 % in intervention strategy B) is the most efficient from the expected cost of failure point of view.

The curve C_{fail} is bounded due to the fact that the probability of failure cannot be greater than 1, thus the expected cost of structural failure C_{fail} cannot be greater than the cost associated with failure C^f discounted over time. The difference between the curves "no intervention" and "Strategy A" at $T = \infty$ (or at time where both curves become horizontal) represents the benefit of the Strategy A interventions with respect to the reduced expected cost of failure.

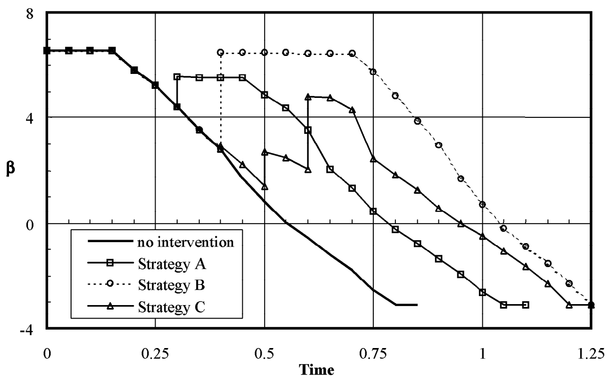


Figure 11. *Reliability index for the system*

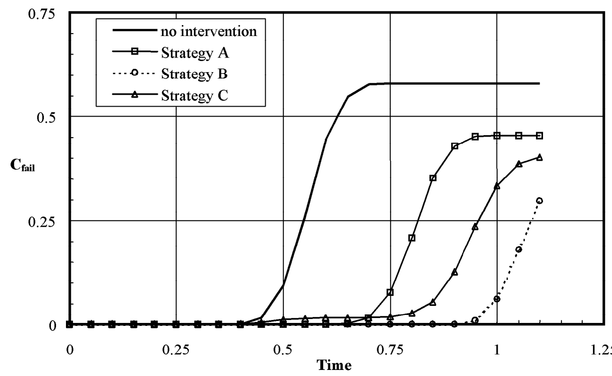


Figure 12. *Expected cost of structural failure*

Expected cost of inadequate serviceability

In this example the deflection at point A was taken as the only serviceability parameter and it is used to calculate the cost of inadequate serviceability (excessive deflection) considering a lower bound, an upper bound and cost function applying (6). The cost parameters are listed in Table 3. Figures 13 and 14 illustrate how deformation and the cost of inadequate serviceability develop over time due to simultaneous deterioration for different intervention strategies.

It can be seen that Strategy A involves an intervention immediately after the deflection exceeds the lower bound on acceptable serviceability, but the repair level is not high enough to keep the serviceability parameter under the lower bound for a very long time. Continuous deterioration causes a further increase of the deflection. On the other hand Strategy B involves an intervention later than Strategy A, but the extent of the intervention limits the deflection for longer. Figure 14 shows that Strategy B is more efficient than Strategy A with respect to the cost of inadequate serviceability because C_{ser} remains at a lower level.

The curves that represent the cost of inadequate serviceability C_{ser} for different repair strategies become parallel at some point in time when the upper bound on acceptable serviceability is reached by all curves (due to discounting, the curves become horizontal at $T = \infty$). The difference between the curves "no intervention" and "Strategy A" at the time when both curves become parallel represents the maximum benefit of Strategy A with the respect of the reduced cost of inadequate serviceability.

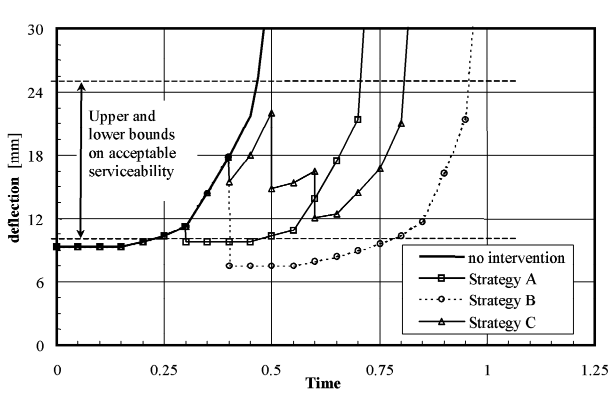


Figure 13. Vertical deflection at point A

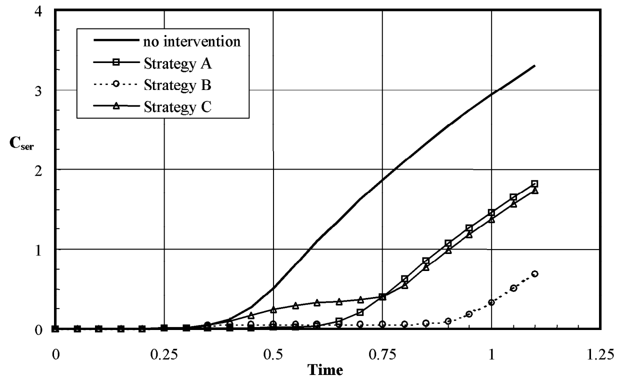


Figure 14. Expected cost of inadequate serviceability as a function of time and strategy

Benefit of an intervention

The benefits due to intervention are the reduction of expected costs related to inadequate performance at each limit state. An intervention on a structure reduces the risk of a **structural failure** and the corresponding costs and at some point in time increases the future **serviceability** of a structure and thus decreases the expected cost of inadequate serviceability. The total benefit of an intervention is the sum of the benefits with respect to the reduced costs of structural failure and inadequate serviceability. All benefits are calculated considering the cost parameters given in Table 3, applying (7) and (8). Due to the fact that one strategy can involve more than one time of intervention (different times of intervention on each element), the total structural life-span (from t = 0 until t = 1) is considered as T_{ref} in (7) and (8).

Table 5. Cost and benefits of intervention

Strategy	B_{fail}	B_{ser}	B_{tot}	C_m
A	0.1377	1.5857	1.7234	0.1873
B	0.2840	2.3286	2.6126	0.3158
C	0.1808	1.6714	1.8522	0.5512

Table 5 presents the expected cost of intervention C_m and benefits of the intervention with respect to the reduced expected cost of structural failure B_{fail} and inadequate serviceability B_{ser} for strategies A, B and C.

It is important to note that the benefits of intervention with respect to serviceability, B_{ser}, are about tens times greater than those for structural failure, B_{fail}, for all three strategies. The high ratio is due to the fact that the expected cost of inadequate serviceability is not bounded (unlike the expected cost of structural failure) and therefore can be high when accumulated over a long time interval T_{ref}. Naturally, this ratio depends on the assumptions made with respect to the value of the structure. In this example, the value to users over one year of service of the structure was assumed to be equal to the cost of structural failure (essentially the reconstruction cost in this case).

A comparison of the costs and benefits illustrates that strategy B is more suitable than strategy C because it has smaller cost C_m and greater benefit B_{tot}. On the other hand, both strategies A and B show some advantages when compared. Strategy A has smaller cost, but the strategy B has greater benefit. The relative merits of each strategy are discussed in the next section.

3.4 Optimum intervention strategy

A cost-benefit analysis is generally used to decide whether a number of investment projects (like an intervention on a single bridge, or a number of interventions on a bridge network) should be undertaken and, if funds are limited or projects are mutually exclusive (different types of intervention on the same bridge), which among these projects should be selected. In this example, any possible combination of intervention options on each element is regarded as a project with its associated cost and benefit.

Considering eight different levels and fifteen different times of intervention produces more than 1,700,000 different intervention options. Figure 15 shows a random sample of points that represent the cost and total benefit of intervention strategies. The envelope of the points represents the **efficient line**, which defines the most efficient strategies. The search for an optimum strategy thus consists of determining the efficient line, which is done in this example by random search (alternative search algorithms could be more effective).

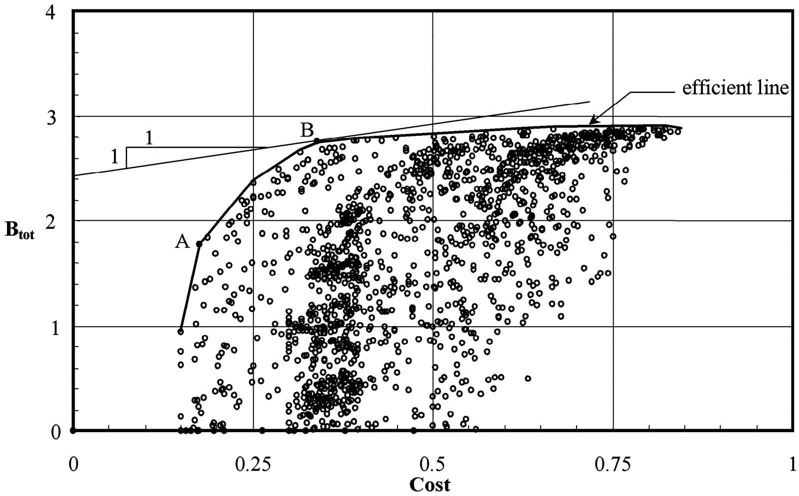


Figure 15. *Set of intervention strategies with respect to their cost and benefit*

Since the objective of the cost-benefit analysis is to increase expected benefits and reduce the costs, only the investment strategies that lie along the efficient line are of interest. It is up to the investor to choose which of the **efficient strategies** is best, depending on the available budget. For illustration purposes, two strategies are highlighted: strategy A with the highest benefit/cost ratio and strategy B (point at which the efficient line has a slope of 1:1) as having the highest benefit that has a positive incremental benefit/cost ratio.

The interventions in strategies A and B are included in Table 6, which presents efficient strategies with their expected cost of intervention C_m and benefits B_{tot}. Note that all efficient strategies include a high repair level of bar 2, implying that bar 2 is the most critical from the cost-benefit point of view, which agrees with the conclusion drawn from the reliability analysis. It can also be seen that an intervention on bar 3 is not significantly beneficial to the system, which also agrees with the reliability analysis results. The optimum intervention times for most efficient strategies are between points $T = 0.30$ and $T = 0.40$, when the system reliability index β reaches about 3 (see Figure 11). Furthermore, this is the period of time when the curve representing serviceability crosses the lower bound (see Figure 13) and deflection begins to have increase. As expected, the optimum time of intervention from the cost point of view is before the serious deterioration occurs. However this analysis suggests that it is beneficial to wait as long as possible before intervening.

Table 6. *Repair levels and times of intervention for efficient strategies*

Strategy	Repair level [%]			Time of intervention			Cost C_m	Benefit B_{tot}
	bar 1	bar 2	bar 3	bar 1	bar 2	bar 3		
1	15	15	nothing	0.50	0.50	-	0.1521	0.9854
2□A	WP*	15	nothing	0.30	0.30	-	0.1873	1.7234
3	WP	30	WP	0.40	0.40	0.40	0.2515	2.3612
4	nothing	60	nothing	-	0.30	-	0.2985	2.4102
5□B	15	60	15	0.40	0.40	0.40	0.3158	2.6126
6	60	90	nothing	0.35	0.35	-	0.4484	2.7621
7	30	90	45	0.30	0.30	0.55	0.6212	2.8481

* protect element from further deterioration for next 20 years, without increasing the cross section

4. Discussion and Conclusions

The model presented above extends current approaches to cost-benefit analysis [Thompson 1998] by taking into account the cost due to the inadequate serviceability of a structure. Furthermore, both expected cost due to structural failure and inadequate serviceability are time-cumulative, incorporating the risk and corresponding cost of delaying an intervention. The framework presented in this paper enables an evaluation of the costs and benefits over a reference period for any possible combination of intervention options.

It is very difficult to assess benefits for public sector activities [Sage 1995], such as transportation. Therefore, most studies on bridge intervention optimisation are focused on choosing the best strategy rather than justifying that benefits exceed cost. This concept of "cost-effectiveness" is used when benefits cannot be expressed in purely economic terms, and has the goal of determining the degree to which objectives are achieved rather than questioning the economic value. Unlike the concept of "cost-effectiveness", the concept of "cost-benefit" is based on the returns of an intervention being expressed in economic terms. This is the advantage of the approach presented in this paper, which thus enables a quantitative economic optimisation of intervention strategies.

Many parameters considered in this type of cost-benefit analysis, for example deterioration processes, the probability of collateral damage and associated costs, are subject to a high level of uncertainty. Perhaps the greatest challenge is to express the value of a structure to users as a unit cost per unit of time. These matters need to be studied further. Nevertheless, the application of the model presented with estimated costs and benefits would represent a significant advance compared to decision making based on arbitrary criteria and experience alone.

5. References

[Chang & Shinozuka 1996] Chang, S. E. and Shinozuka, M., “Life-cycle cost analysis with natural risk hazard”, *Journal of Infrastructure Systems*, Vol. 2, No. 3, 1996, pp. 118-126

[Enright & Frangopol 1998] Enright, M. P. and Frangopol, D. M., “Service-life prediction of deterioration concrete bridges”, *Journal of Structural Engineering*, Vol. 124, No. 3, 1998, pp. 309-317

[Frangopol 1997] Frangopol, D. M., Lin, K.-Y. and Estes, A. C., “Reliability of reinforced concrete girders under corrosion attack”, *Journal of Structural Engineering*, Vol. 123, No. 3, 1997, pp. 286-297

[Mori & Ellingwood 1993] Mori, Y. and Ellingwood, B. R., “Reliability-based service-life assessment of aging concrete structures”, *Journal of Structural Engineering*, Vol. 119, No. 5, 1993, pp. 1600-1621

[Radojicic 1999] Radojicic, A., Bailey, S. F. and Brühwiler, E., “Consideration of the serviceability limit state in a time dependent probabilistic cost”, *Application of statistics and probability*, Vol. 2, Balkema, Rotterdam, 1999, pp. 605-612

[Sage 1995] Sage, A. P., *System management*, John Wiley & Sons, New York, 1995

[Thoft-Christensen 1998] Thoft-Christensen, P., “Assessment of the reliability profiles for concrete bridges”, *Engineering Structures*, Vol. 20, No. 11, 1998, pp. 1004-1009

[Thompson 1998] Thompson, P. D., Small, E. P., Johnson, M. and Marshall, A. R., “The Pontis Bridge Management System”, *Structural Engineering International*, Volume 8, 1998, pp. 303-308

[Val 1998] Val, D. V., Stewart, M. G. and Melchers, R. E., “Effect of reinforcement corrosion on reliability of highway bridges”, *Engineering Structures*, Vol. 20, No. 11, 1998, pp. 1010-1019

Indicators for Assessment and Inspection Planning

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Keywords: Condition indicators, concrete, reliability, corrosion, Bayes, degradation, half-cell potential measurements, maintenance.

Summary

Based on an idea introduced by Benjamin and Cornell [1] and previous works by the authors it is demonstrated how condition indicators may be formulated for the general purpose of quality control and for assessment and inspection planning in particular. The formulation facilitates quality control based on sampling of indirect information about the condition of the considered components. This allows for a Bayesian formulation of the indicators whereby the experience and expertise of the inspection personnel may fully utilized and consistently updated as frequentistic information is collected. The approach is illustrated on an example considering a concrete structure subject to corrosion. It is shown how half-cell potential measurements may be utilized to update the probability of excessive repair after 50 years.

1. Introduction

Condition control of existing structures by means of inspection and testing forms a corner stone in the maintenance management of structures. It is generally accepted that the information collected by inspection and testing is the basis for the estimation of the condition of the structure and moreover for the estimation of the future development of the condition of the structure, i.e. the future structural degradation.

Ideally the principle for the planning of inspection and testing activities is to minimize the service life economical risks, i.e. the sum of the costs of inspection and testing, the costs due to future maintenance and strengthening works and the costs due to future failures.

To be able to plan inspection and testing efforts optimally in the above-mentioned sense it is necessary that the effect of inspections and tests can be related to the those particular states of the structure, which have impact on the service life economical risks for the structure. I.e. the results of the inspections and tests must be related to the service life economical risks.

Traditionally in applications such as bridge management the results of inspections and tests are expressed in both linguistic terms such as “corrosion has initiated in xx % percent of the structure”, “the membrane is defect”, etc.. This information is then processed and results in an overall grading by assigning a certain number (e.g. 1, 2, 3,..) to the overall condition of the structure. The further treatment of such information for the purpose of estimating the service life economical risk is less than obvious, and even though some systematic may be developed on this basis there is no theoretical basis ensuring that the information is consistently utilized for the purpose of maintenance management.

Based on an idea introduced by Benjamin and Cornell [1] and previous works by the authors (Faber and Sorensen [8]) it is demonstrated how condition indicators may be formulated for the general purpose of quality control and for assessment and inspection planning in particular. The formulation facilitates quality control based on sampling of indirect information about the condition of the considered components. This allows for a Bayesian formulation of the indicators whereby the experience and expertise of the inspection personnel may fully utilized and consistently updated as frequentistic information is collected. The approach is illustrated on an example considering a concrete structure subject to corrosion where it is shown how half-cell potential measurements may be utilized to update the probability of excessive repair after 50 years.

2. Information sampling

As a basis for the further derivations it is assumed that the defect rate can be used as a measure of the general condition of a series of components. Defect rate is here understood in a broad sense as the rate of occurrence of any state of the considered components, which is of special interest. Components are considered for which the unknown defect rate is θ, i.e. the probability that a considered component is defect is θ and the probability that it is not defect is 1-θ. On the basis of sample (inspection) results and subjective information the estimate of the defect rate can be updated and decision-making regarding the inspection and maintenance of the components may be undertaken on this basis.

2.1 Random Sampling

The usual sampling technique from classical quality control is to test (inspect) samples, which are taken randomly from the considered population. Here this approach is shortly described within a Bayesian statistical framework utilising to the highest degree possible all available information.
It is assumed that a well defined population with constant defect rate θ is considered. n components are selected randomly from the population and inspected with the result that n_F ($\leq n$) components are defect.
Letting the random variable Y model the number of defect components n_F in a random sample of size n, Y is binomial distributed with probability density function given by

$$f_Y(y|\theta) = \binom{n}{y} \theta^y (1-\theta)^{n-y} \tag{1}$$

However, the defect rate θ is not known with certainty and must hence be considered as an uncertain variable Θ. Initial information regarding the defect rate θ is modeled by the prior probability density function $f'_\Theta(\theta)$, see e.g. Lindley [5]. If no initial information is available or if the initial information is very uncertain a diffuse prior may be used, i.e. $f'_\Theta(\theta) \propto 1$. Through the prior probability density function it is possible to incorporate subjective information regarding the general defect rate. Having inspected a sample of n components with the result that n_F components are defect, this information can be utilized to update the available information regarding θ.
The updated probability density function (the posterior probability density function) for θ can be derived on the basis of Bayes formula. The posterior probability density is denoted $f''_\Theta(\theta|Y = n_F)$ and expresses the updated knowledge about the general defect rate. The posterior probability density function is given by

$$f''_\Theta(\theta|Y = n_F) = \frac{1}{c} f_Y(n_F|\theta) f'_\Theta(\theta) \tag{2}$$

where c is a normalizing constant such that $\int_0^1 f''_\Theta(\theta|Y = n_F)d\theta = 1$

The posterior probability density function for the defect rate may readily be used in the decision analysis, see e.g. Raiffa and Schlaifer [7] either on its own or by using it together with Equation (1) whereby the predictive probability density function for the number of defects may be achieved as

$$f_Y(y|Y=n_F)=\int_0^1\binom{n}{y}\theta^y(1-\theta)^{n-y}f''_\Theta(\theta|Y=n_F)d\theta \tag{3}$$

2.2 Sampling Based on Indicators

Inspection and maintenance planning of engineering systems such as structural systems, pipelines and process systems is complicated by the fact that the systems often are very large, in terms of components. Furthermore, the components of the systems often belong to a larger number of different populations. Application of classical quality control procedures for the quantification of the effect of inspections on such systems is thus hardly practical and hence not used in practice.

In practice, inspections are performed on a rather limited number of components. The components inspected are typically selected on the basis of criteria such as criticality, expected condition, inspectability and experience. Following such schemes little possibility is left to evaluate the effect of the sample size and even worse to quantify and justify the rationale behind the inspection scheme.

In the following it is hence attempted to outline a statistical framework for an alternative approach based on Bayesian statistics and so-called indicators. Initially the case is considered where knowledge is included in the estimate of the failure rate. Letting D_i denote that indicator no. i is inspected and indicates a defect. It is assumed that prior information may be represented by the probabilities

$P(F|D_i)=p_i$: the probability that the inspected component is defect (F) given the observation that indicator no. i indicates a defect.

$P(F|\overline{D_i})=q_i$: the probability that the inspected component is defect (F) given the observation that indicator no. i does not indicate a defect.

The number of components for which the indicator has been inspected and found to indicate a defect is denoted N_D and the number of components inspected where the inspected indicator does not indicate a defect is denoted $N_{\overline{D}}$. On the basis of these probabilities and information regarding the total number of inspected components the probability density function for the defect rate may be updated. The updated probability density function is denoted $f_\Theta'''(\theta)$. In $f_\Theta''(\theta)$ both prior information regarding θ and results of random inspections may be included. $f_\Theta''''(\theta)$ is the updated probability density function for the general failure rate when all information is included (prior information, results of random samples and results of targeted samples).

The probability density function may be updated approximately by

$$f_\Theta'''(\theta|D_i)\cong c\left(f_\Theta''(\theta|F)p_i+f_\Theta''(\theta|\overline{F})(1-p_i)\right) \tag{4}$$

for the case where the inspection indicates a defect and

$$f_\Theta'''(\theta|\overline{D}_i)\cong c\left(f_\Theta''(\theta|F)q_i+f_\Theta''(\theta|\overline{F})(1-q_i)\right) \tag{5}$$

for the case where the inspection indicates that there is no defect. c is a normalising constant.

As input for the updating information regarding the probabilities p_i and q_i is required. If these are not known with certainty Bayesian statistics may also be applied to model this. p_i and q_i are then modelled as uncertain variables and their uncertainty, modelled by the prior probability density functions, may be updated when information is available.

The approximation in Eqs. (4) and (5) may be expressed by the assumptions that

$$P(D_i|F\cap\Theta=\theta)\cong P(D_i|F) \tag{6}$$

$$P(D_i|\overline{F} \cap \Theta = \theta) \cong P(D_i|\overline{F}) \tag{7}$$

which means that it is assumed that the probability of finding an indication of a defect, given that the component is defect (or not defect) is independent of the defect rate. This assumption appears to be non-restrictive for practical applications.

If inspection results are available the probabilities $f_\Theta^{''}(\theta|F)$ and $f_\Theta^{''}(\theta|\overline{F})$ may be determined by using $f_\Theta^{''}(\theta|Y = n_F)$ as prior probability density function:

$$f_\Theta^{''}(\theta|F) = \frac{1}{c'}\theta f_\Theta^{''}(\theta|Y = n_F) \tag{8}$$

$$f_\Theta^{''}(\theta|\overline{F}) = \frac{1}{1-c'}(1-\theta) f_\Theta^{''}(\theta|Y = n_F) \tag{9}$$

where $c' = \int_0^1 \theta f_\Theta^{''}(\theta|Y = n_F)d\theta$ is a normalising constant. If no inspection results are available $f_\Theta^{'}(\theta)$ may be used as prior probability density function instead of $f_\Theta^{''}(\theta|Y = n_F)$.

First the situation where the observation of N_D indicate that that the component is defect is considered. The N_D indicators are denoted D_i $, i = 1, N_D$. The updated probability density function is determined recursively by

$$f_\Theta^{''''(i)}(\theta|D_1,...,D_i) = c_i\left(\frac{p_i\theta}{c_i^{'}} + \frac{(1-p_i)(1-\theta)}{1-c_i^{'}}\right) \times f_\Theta^{''''(i-1)}(\theta|D_1,...,D_{i-1}) \quad ,i = 1,..,N_D \tag{10}$$

where c_i is determined such that $\int_0^1 f_\Theta^{''''(i)}(\theta|D_1,..,D_i)d\theta = 1$ and $c_i^{'}$ is a normalising constant:

$$c_i^{'} = \int_0^1 \theta f_\Theta^{''''(i-1)}(\theta|D_1,...,D_i)d\theta \tag{11}$$

Furthermore there is $f_\Theta^{''''(0)}(\theta) = f_\Theta^{''}(\theta|Y = n_F)$ or $f_\Theta^{''''(0)}(\theta) = f_\Theta^{'}(\theta)$.

Secondly the situation where observations of $N_{\overline{D}}$ components do not indicate that the component is defect. The $N_{\overline{D}}$ indicators are denoted $\overline{D}_j$ $, j = 1, N_{\overline{D}}$. The updated probability density function is determined recursively by

$$f_\Theta^{''''(j)}(\theta|\overline{D}_1,...,\overline{D}_j) = c_j\left(\frac{q_j\theta}{c_j^{'}} + \frac{(1-q_j)(1-\theta)}{1-c_j^{'}}\right) f_\Theta^{''''(j-1)}(\theta|\overline{D}_1,...,\overline{D}_{j-1}) \quad ,j = 1,..,N_{\overline{D}} \tag{12}$$

In the above derivations it has been assumed as a prerequisite that the considered components belong to the same population.

2.3 Updating Knowledge about Indicators

If inspections of the condition of interest is conducted fully for the components for which one or more indicators have been found to indicate a defect the result of the inspections can be used to update the probabilities $p_i = P(F|D_i)$ and $q_i = P(F|\overline{D}_i)$ as described in the foregoing and by letting p_i and q_i be modelled by the random variables P_i and Q_i.

If more populations are considered p_i and q_i may be assumed to be the same for the same indicators in the different populations. In this case it is possible to update the statistical knowledge regarding p_i and q_i on the basis of one population and to use this updated knowledge on the other populations. If on the other hand p_i and q_i are different for a given indicator for different populations the improved knowledge regarding p_i and q_i can only be used for the indicator in the inspected population.

3. Inspection of concrete structures

In the present example a concrete structure, subject to chloride induced corrosion of the reinforcement is considered. It is assumed that the effect of the corrosion is mainly related to the serviceability condition of the structure. The repair strategy for the structure specifies that the structure shall undergo a major repair if by visual inspection more than 50% of the surface of the structure exhibits initiated corrosion, see e.g. Estes [3]. Furthermore it is assumed that the considered structure in terms of its corrosion characteristics may be modelled as an assembly of 100 components. I.e., when any 50 of these components exhibit initiated corrosion a major repair will be performed. The time until corrosion becomes visual may be modelled as the time till initiation of corrosion T_I plus a certain corrosion propagation time T_P, i.e. $T_V = T_I + T_P$. In the following the time till initiation T_I is assessed assuming that the ingress of chlorides through the concrete cover d may be described by a diffusion process with diffusion coefficient D and that corrosion initiation will take place when the concentration of chlorides exceed a certain critical concentration C_{CR}. The time till initiation may thus be written as, see e.g. Engelund et al. [2]

$$t_I = d^2\left(4\cdot D\cdot\left[erf^{-1}(\frac{C_{CR}}{C_S}-1)\right]\right)^{-1} \tag{13}$$

where C_S is the concentration of chlorides on the surface of the concrete. The propagation time till corrosion becomes visual is for simplicity modelled as a Log-normal distributed random variable with mean value and standard deviation as shown in Table 1.

Based on the safety margin

$$M = g(\mathbf{X},t) = X_I\cdot T_I + T_P - t \tag{14}$$

where X_I is a model uncertainty associated with the corrosion initiation time and the probabilistic model given in Table 1 the probability of corrosion initiation as well as visual corrosion may be readily calculated using e.g. FORM/SORM methods, see Madsen et al. [9].

Table 1. Modelling of parameters in the corrosion limit state.

Description	Cover thickness d	Diffusion Coefficient D	Surface conc. C_S	Critical conc. C_{CR}	Model unc. X_I	Propagation time T_P	Stat. unc. μ_D	Stat. unc. μ_{C_S}
Distribution	Log-normal	Log-normal	Log-normal	Log-normal	Log-normal	Log-normal	Normal	Normal
Mean value	55.0	μ_D	μ_{C_S}	0.15	1.0	7.5	40.0	0.4
Std. Dev.	11.0	10.0	0.08	0.05	0.05	1.88	4.0	0.04

Due to statistical uncertainty the mean value of the diffusion coefficient D and the surface concentration C_S are assumed to be uncertain themselves.

The ensemble of the 100 components may thus be considered as 100 components with a common failure probability (evaluated on the basis of the physical uncertainties only) which is uncertain in itself due to the remaining statistical and model uncertainties. Assuming for the sake of illustration that the individual components may be assumed statistically independent the probability of having more than 50% of the components exhibiting visual corrosion may be assessed through

$$P(n_F(t) \geq 50) = 1 - \int_0^1 B(50-1,100,\theta(t)) f_{\theta(t)}(\theta(t)) d\theta(t) \tag{15}$$

where $B(50-1,100,\theta(t))$ is the cumulative Binomial distribution (see Faber and Rostam [4]) and $\theta(t)$ is the uncertain probability of visual corrosion of the individual components at time t determined by simulation and nested FORM/SORM analysis on the limit state equation given in Equation (14). $f_{\theta(t)}(\theta(t))$ is the prior or posterior PDF for the probability of visual corrosion depending on the state of knowledge. The prior PDF for the probability of visual corrosion for one component after 50 years is shown in Figure 2.

As a part of the planned condition monitoring of the structure half-cell potential measurements are performed on a regular basis. It is the aim to utilize the measurements in order to update the probability of repair and thus the expected future maintenance costs.
In order to use the Eqs. (11) - (13) to establish a posterior PDF for $\theta(t)$ given the results of inspections the probabilistic characteristics of the indicator corresponding to the half-cell potential measurements needs to be established.

According to tests performed by Marschall [6] the probability distribution function for the half-cell measurement may be described by a normal distribution function. Given that corrosion has initiated the mean value and standard deviation are –0.354 Volts and 0.08 Volts, respectively. Given that no corrosion has initiated the mean value and the standard deviation are -0.207 Volts and 0.0804 Volts, respectively.

A central question concerns what measurement result to assign with an observation of "corrosion" or "no corrosion", i.e. the choice of the indicator. Choosing somewhat arbitrarily as an indicator for corrosion initiation a potential reading corresponding to the lower 10 % fractile value for the potential measurements given corrosion this value corresponds to -0.2515 Volts. We then have $P(I_{\overline{D}}|CI) = 0.10, P(I_{\overline{D}}|\overline{CI}) = 0.71$ where CI denotes the event of corrosion initiation at the time where the measurement is performed and $I_{\overline{D}}$ the event of no indication. The probability of getting an indication of no-corrosion initiation given that corrosion exists is 0.1 and the probability of getting an indication of no corrosion initiation given that no corrosion exists is 0.71. Correspondingly we have $P(I_D|CI) = 0.90, P(I_D|\overline{CI}) = 0.29$ where I_D denotes the event of an indication of corrosion. Later on the significance of the choice of the indicator will be assessed further.

Denoting by CV the event of visual corrosion after 50 years, (see also Figure 1a) the strength q of the indicator "no corrosion" for the half-cell potential measurement may now be expressed as

$$q = P(CV|I_{\overline{D}}) = \frac{P(CI \cap CV \cap I_{\overline{D}}) + P(\overline{CI} \cap CV \cap I_{\overline{D}})}{K} \quad \text{with} \tag{17}$$
$$K = P(CI \cap CV \cap I_{\overline{D}}) + P(\overline{CI} \cap CV \cap I_{\overline{D}}) + P(CI \cap \overline{CV} \cap I_{\overline{D}}) + P(\overline{CI} \cap \overline{CV} \cap I_{\overline{D}})$$

Noting that in the present case $P(CV|CI \cap I_{\overline{D}}) = P(CV|CI \cap I_D) = \dfrac{P(CI \cap CV)}{P(CI)}$ we have

$$q = P(CV|I_{\overline{D}}) = \frac{P(CV \cap CI) \cdot P(I_{\overline{D}}|CI)}{K_1} + \frac{P(CV \cap \overline{CI}) \cdot P(I_{\overline{D}}|\overline{CI})}{K_1} \quad \text{with}$$
$$K_1 = P(I_{\overline{D}}|CI) \cdot \left[P(CI \cap CV) + P(CI \cap \overline{CV})\right] + P(I_{\overline{D}}|\overline{CI}) \cdot \left[P(\overline{CI} \cap CV) + P(\overline{CI} \cap \overline{CV})\right] \tag{18}$$

Correspondingly the strength p of the indicator "corrosion initiation" for the half-cell potential measurement is

$$p = P(CV|I_D) = \frac{P(CV \cap CI) \cdot P(I_D|CI)}{K_2} + \frac{P(CV \cap \overline{CI}) \cdot P(I_D|\overline{CI})}{K_2} \quad \text{with}$$
$$K_2 = P(I_D|CI) \cdot \left[P(CI \cap CV) + P(CI \cap \overline{CV})\right] + P(I_D|\overline{CI}) \cdot \left[P(\overline{CI} \cap CV) + P(\overline{CI} \cap \overline{CV})\right] \tag{19}$$

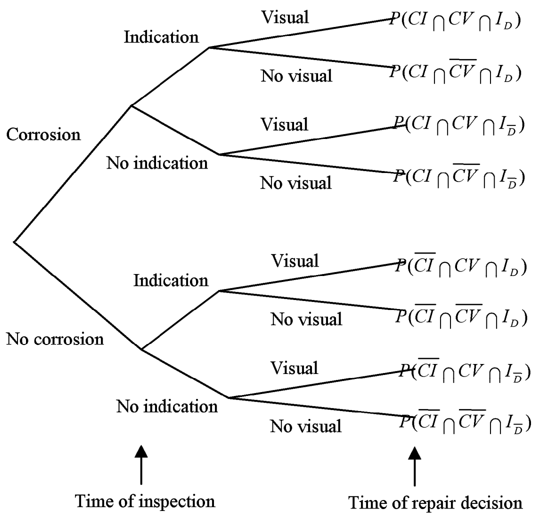


Figure 1a. Event tree for observations and concrete conditions.

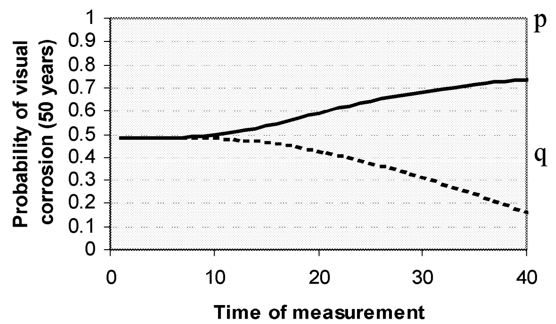


Figure 1b. Strength of the indicators p and q as function of the inspection time.

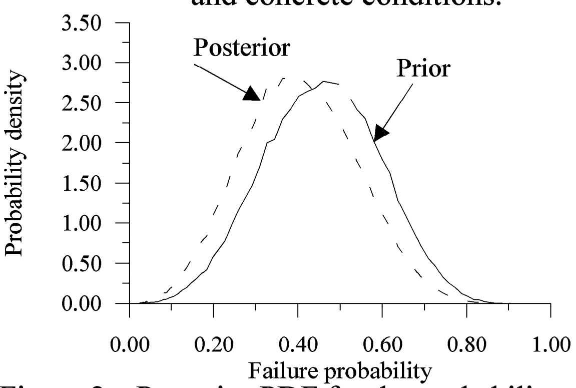


Figure 2a. Posterior PDF for the probability of visual corrosion of one component given no indication.

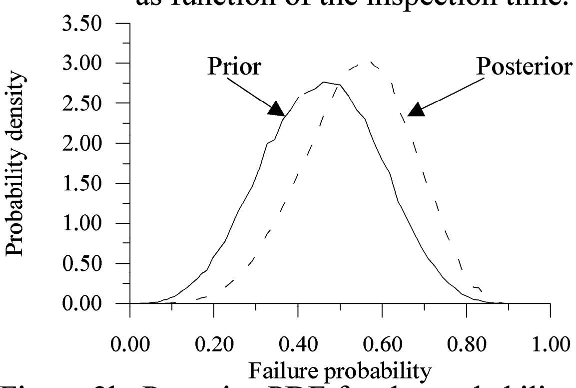
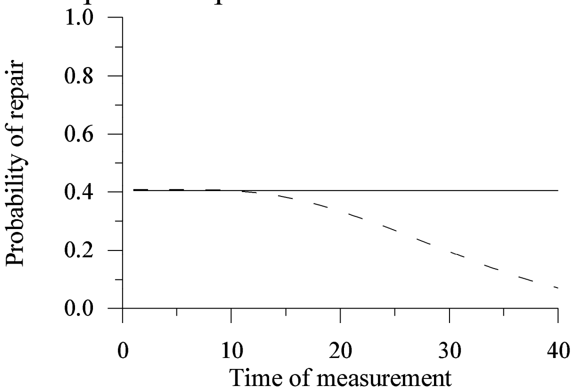


Figure 2b. Posterior PDF for the probability of visual corrosion of one component given indication.

The strength of the indicator p "no corrosion initiation" and q for "corrosion initiation" for the half-cell potential measurement are shown as a function of time in Figure 1b.

Half-cell potential measurements are performed for 10 components after 25 years with the result that no corrosion has initiated. Based on the indicator for "no corrosion initiation" using half cell potential measurements the posterior probability density function for the uncertain probability of visual corrosion is evaluated using Eqs. (11)-(13), see Figure 2a and 2b.

The probability of extensive repair, i.e. more than 50 % of the structure exhibiting visual corrosion is evaluated using Equation (15) and illustrated in Figure 3 as function of the time where the inspection is performed.



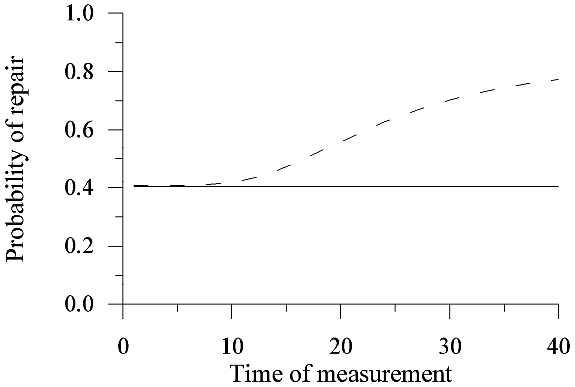
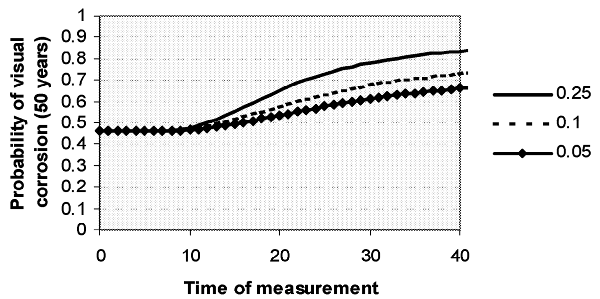


Figure 3a. No indication of corrosion. Figure 3b. Indication of corrosion.

From Figures 3a and 3b it is seen that the inspection using half-cell measurements will not provide any knowledge in regard to the development of excessive visual corrosion until after about 10 years. This is because the probability of initiation of corrosion is negligible until this point in time. Thereafter the indication of no-corrosion and corrosion respectively is seen to have a significant impact on the probability of more than 50 % of the concrete surface exhibiting visual corrosion. The updated probabilities may readily be applied in the planning of inspections and maintenance following the same principles as outlined in e.g. Faber and Rostam [4].

Returning now to the question regarding the choice of indicator the strength of the indicators p and q is depicted in Figure 4 for different choices (5%, 10% and 25%) of the fractile values in the probability distribution function for the hall-call potential reading given corrosion has initiated.



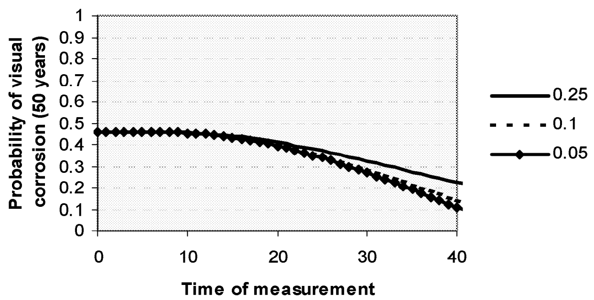


Figure 4a The strength of the indication "no-corrosion" for different choices of the indicator.

Figure 4b The strength of the indication "corrosion" for different choices of the indicator.

From Figure 4 it is seen that the strength of the indicator depends significantly on the choice of the indicator. This dependency is also carried over to the updated probability of excessive repair as shown in Figure 5.

Figure 5 (kurver som I figur 3 men for forskellige/flere p og q kurver)

The optimal choice of the fractile value in the distribution function for the half-cell potential reading given corrosion, which is associated with an indication of corrosion, is, however, not a trivial matter. To solve this requires the simultaneous consideration of the effect on the updated probability of excessive repair as shown in Figure 5, but also consideration of the maintenance activities, which may be implemented in order to reduce the risk of excessive repair after 50 years.

A suitable framework for addressing this problem complex is the pre-posteriori analysis from the Bayesian decision theory, see e.g. Raiffa and Schlaifer [7], where the assignment of the indicators is treated as a decision variable in the same way as decisions on when to inspect and how to maintain the structure.

4. Discussion and Conclusions

A Bayesian approach to assessment and inspection planning has been formulated on the basis of a general methodology for indicator based quality control. The approach allows for the consideration of problems where the components have uncertain failure rates and where tests are performed on condition indicators rather than the condition of interest directly. The approach may readily be utilized for the optimal planning of tests, inspections and maintenance activities of components within the framework of Bayesian decision analysis. The Bayesian formulation of condition indicators is especially well suited to consistently incorporate subjective knowledge in the test and inspection planning and the treatment of failure rates as being uncertain facilitates the development of quality control schemes for populations of components for which little if any frequentistic information is available.

5. References

[1] Benjamin, J.R. & Cornell, C.A. (1970) Probability, statistics and decision for civil engineers, McGraw-Hill, NY.

[2] Engelund, S. Sørensen, J.D. & Sørensen, B. (1998) Evaluation of repair and maintenance strategies for concrete coastal bridges on a probabilistic basis, ACI Materials Journal.

[3] Estes, A.C. & Frangopol, D. (2000) An Optimized Lifetime Reliability-Based Inspection Program for Deteriorating Structures. Proc. 8th ASCE Spec. Conf. on Prob. Mech. and Struct. Rel., Notre Dame, USA.

[4] Faber, M.H. & Rosdam, S. (2001) Durability and Service life of Concrete Structures - the Owners Perspective. To be presented at the IABSE Conf. on Safety an Risk in Engineering, Malta 2001.

[5] Lindley, D.V. (1976) Introduction to probability and statistics from a Bayesian viewpoint, Cambridge, Cambridge University Press.

[6] Marschall, S.J. (1996) Evaluation of Instrument-Based, Non-Destructive Inspection Methods for Bridges, M.Sc. thesis, University of Colorado, Boulder, Colorado.

[7] Raiffa, H. & Schlaifer, R. (1961) Applied Statistical Decision Theory. Harward University Press, Cambridge University Press, Cambridge, Mass.

[8] Faber, M.H. & Sorensen, J.D. (2001), Bayesian Sampling using Condition Indicators, Submitted to ICOSSAR, July 2001, Newport Beach, California.

[9] Madsen, H.O., Krenk, S. & Lind, N.C. (1986), Methods of Structural Safety, Prentice-Hall, Englewood Cliffs, NJ 07632.

Probabilistic Modelling of Corrosion Processes and Inspections using SBRA Concept

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Keywords: Monte Carlo method, Corrosion, Structural Reliability, Risk based inspection planning

Summary

Using the Simulation Based Reliability Method (SBRA), the paper demonstrates the durability analysis of an axially loaded steel bar exposed to degradation due to corrosion. A combination of three variable time-dependent load effects (dead-, short- and long-lasting load effects represented by load effect duration curves) is considered. The scatter of geometrical and material properties is expressed by bounded histograms. The time dependent reliability function $RF(t) = R(t) - S(t)$ is analyzed (applying a selected corrosion model) using the direct Monte Carlo technique and the time dependent probability of failure function $P_f(t)$ is obtained. Using this a-priori estimation of probability of failure, an inspection was planned taking into account the assigned target probability of failure P_d. Incorporating the data resulting from the inspection into the analysis enabled to adjust and improve the corrosion model and to obtain a corrected prediction of the function $P_f(t)$.

1. Introduction

Drastic development in computer technology in last decade enables to develop and to use probabilistic reliability assessment concepts based on simulation methods leading to qualitatively new approach to the assessment of safety, serviceability and durability.

The simulation technique, namely the direct Monte Carlo method, and the Limit States philosophy are the basis of so-called Simulation Based Reliability Assessment Method (SBRA) documented in textbook [1]. Considering various reliability assessment methods (see Figure 1), the SBRA approach represents a fully probabilistic approach, which uses bounded histograms for representation of individual stochastic variables. The computer programs (M-Star, AntHill, LoadCom, Rescom, DamAc) can be linked to databases containing such histograms.

Applicability of SBRA approach has been demonstrated in many different situations (see [1], [2]). This paper concerns a reliability assessment of a steel bar affected by the corrosion. The SBRA procedure allows for calculating the time-dependent probability of failure function $P_f(t)$. Such approach indicates how to use SBRA method, for example, in case of bridge inspection planning. Moreover, the inspection data can be subsequently incorporated into the analysis and an adjustment of the previous estimates of the service life of the structure can be obtained.

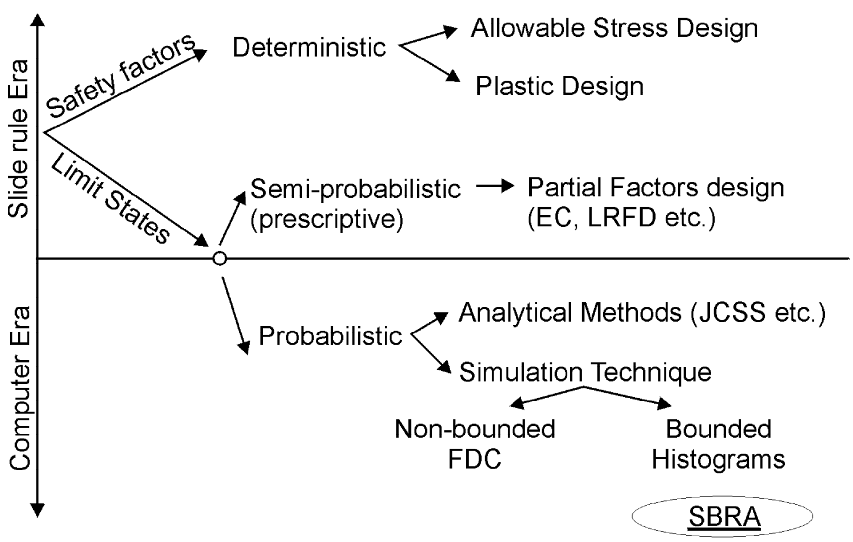


Figure 1. SBRA vs. other methods in reliability analysis

2. Description of the assigned problem

A simple structural member subjected to tension was chosen for the analysis. Figure 2 shows schematically input data used in this example. All parameters are described in detail next.

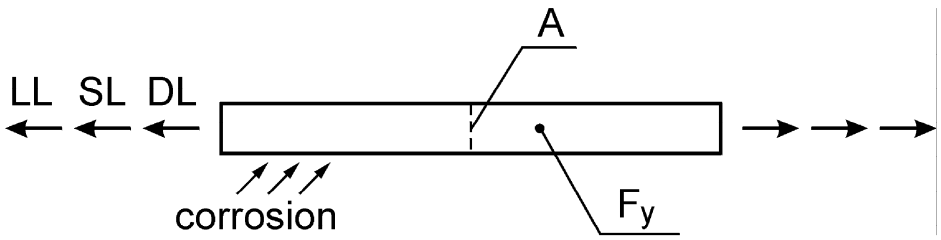


Figure 2. A bar exposed to tension: A - cross-sectional area; F_y - yield stress; DL - dead load; LL - long lasting load; SL - short lasting load

2.1 Dimensions

The geometrical characteristics of the bar are expressed by the cross-sectional area. The initial nominal value was A_{nom} = 450 mm^2 and the variation of this area (considering a range of ±10%) was assumed. The variable cross-sectional area A was expressed by equation $A = A_{nom} * A_{var}$, where A_{var} described the scatter of area and was defined by a non-parametric bounded histogram. This histogram, marked in the database as AREA-S, is shown in Figure 3.

Figure 3. Histogram AREA-S representing variability of the cross-sectional area

2.2 Loads

Load consisted of three components – dead load (*DL*), long lasting load (*LL*) and short lasting load (*SL*). Each load component was described by a product of the extreme value and a variable coefficient represented by a histogram, as indicated in Table 1. The applied histograms are shown in Figure 4.

Load	Maximum value		Variable	
	Description	Value [kN]	Description	Histogram
Dead	DL_{max}	50	DL_{var}	DEAD2
Long lasting	LL_{max}	30	LL_{var}	LONG1
Short lasting	SL_{max}	40	SL_{var}	SHORT1

Table 1. Characteristics of laods using non parametric histograms.

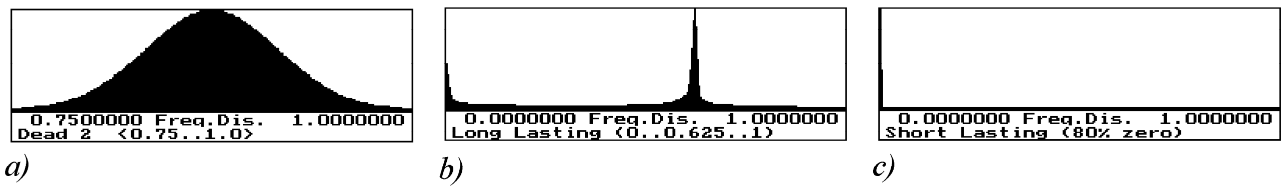


Figure 4. Histograms used for loads: a) DEAD2 – Dead load, b) LONG1 – Long lasting load,

c) SHORT1 – Short lasting load

The extreme values of the dead and long lasting load were assumed as time dependent quantities. In case of dead load, the extreme value increased abruptly after 10 years by 10%, i. e. the extreme value was 55 kN for $t > 10$ years. This fact can be expressed by equation:

$$DL_{max}(t) = DL_{max} + 0.1 * DL_{max} * \mathrm{NEG}(10 - t) \text{ [kN]} \tag{1}$$

Where NEG(10 – t) is a special function available in programs AntHill and M-Star and has following meaning:

$$\text{a) } \mathrm{NEG}(10 - t) = 1 \quad \text{for} \quad 10 - t < 0, \text{ i. e. } 10 < t \tag{2}$$

$$\text{b) } \mathrm{NEG}(10 - t) = 0 \quad \text{for} \quad 10 - t \geq 0, \text{ i. e. } 10 \geq t$$

Finally, the dead load was given as a product of time dependent extreme value $DL_{max}(t)$ and variable DL_{var}:

$$DL = DL_{max}(t) * DL_{var} \text{ [kN]} \tag{3}$$

For the long lasting load, a continues increase of the extreme value was assumed according to the equation:

$$LL_{max}(t) = LL_{max} * (1 + 0.25*(t/t_{tot})^{1.5}) \text{ [kN]} \tag{4}$$

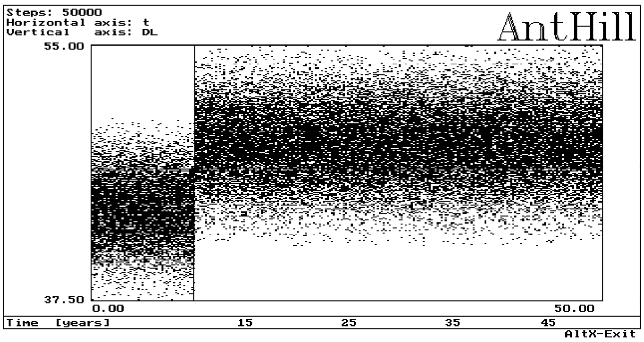
Where $t_{tot} = 50$ years and for $t = t_{tot}$ was the increase of extreme value 25% of the initial state. Similarly as for dead load, the long lasting load was expressed by:

$$LL = LL_{max}(t) * LL_{var} \text{ [kN]} \tag{5}$$

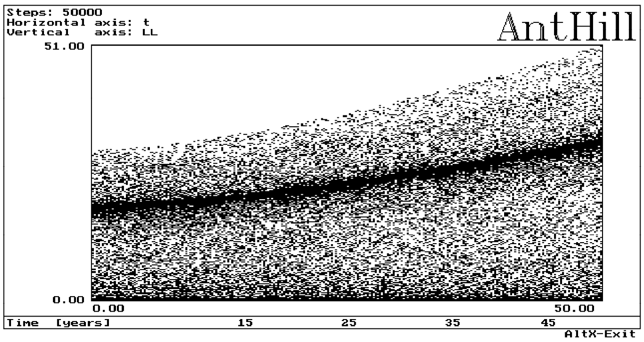
Resulting load combination $S(t)$ of all three components was expressed:

$$S(t) = DL_{max}(t) * DL_{var} + LL_{max}(t) * LL_{var} + SL_{max} * SL_{var} \text{ [kN]} \tag{6}$$

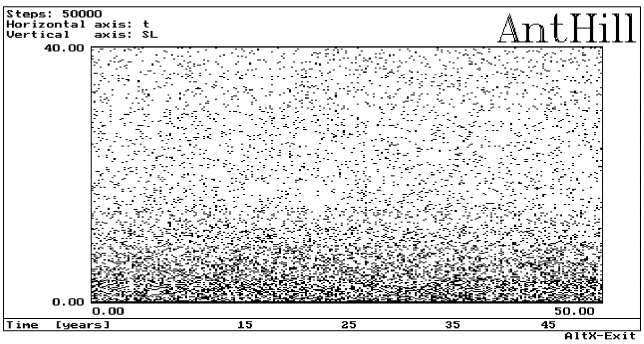
Figure 5 shows time dependence and scatter of the assumed load components as well as their combination.



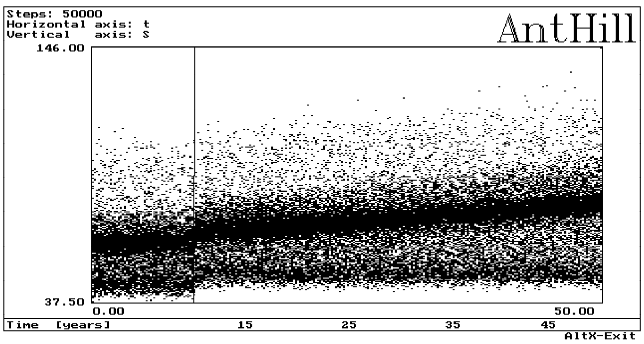
a) Dead load



b) Long lasting load



c) Short lasting load



d) Resulting load combination

Figure 5. Time- dependent loads and load combination

2.3 Resistance

The product of the yield stress $F_y = F_{ynom} * F_{yvar}$ and area $A = A_{nom} * A_{var}$ expresses the time independent resistance R of the bar. In case of a bar exposed to a time dependent corrosion effect, the resistance $R(t)$ can be expressed by equation:

$$R(t) = R * C(t)/1000 \quad \text{[kN]} \tag{7}$$

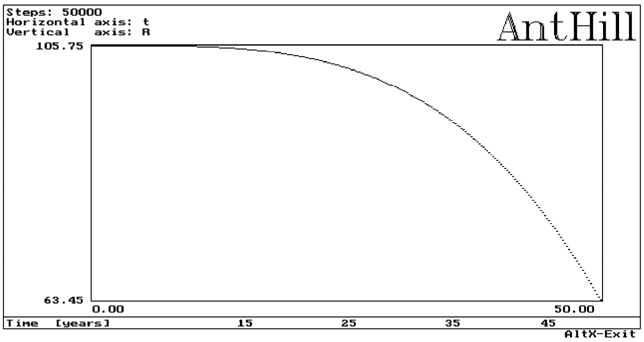
where $C(t)$ is the time dependent corrosion model.

Figure 6. Histogram FY235A

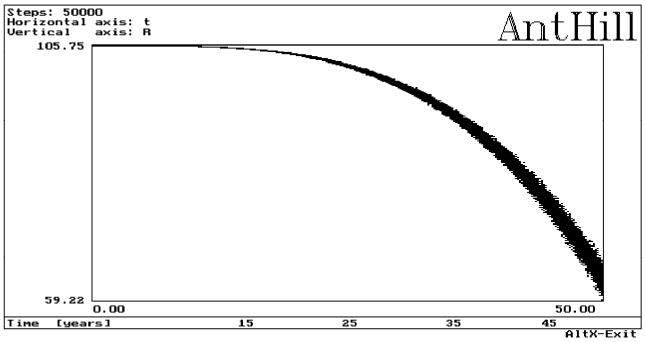
Nominal value of the yield stress F_{ynom} was 235 MPa. The scatter of the yield stress, represented by the variable F_{yvar}, was expressed by histogram FY235A (See Figure 6). The function $C(t)$ expresses the degradation due to corrosion. A simple model was selected. The effect of corrosion was expressed by the loss of cross-section area. The function $C(t)$ can be written in a form:

$$C(t) = 1 - K * (t/t_{tot})^n \tag{8}$$

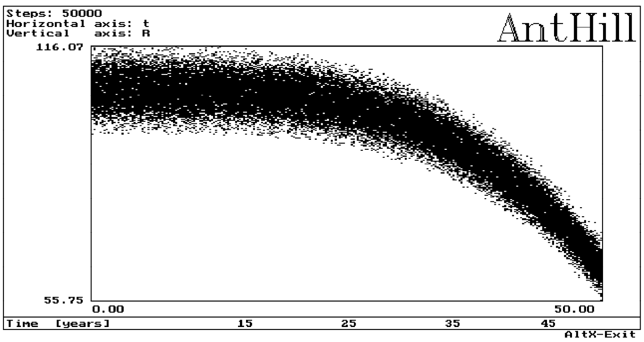
where the coefficient K was assumed $K_{nom} * K_{var}$. The value of K_{nom} was chosen 0.4 and the variable K_{var} , which describes approximately the scatter of K, was given by the same histogram as in case of A_{var} (see Figure 3). Exponent n was assumed as deterministic value 3.5. The influence of variability of coefficient K, area A and yield stress F_y on the resistance $R(t)$ is shown in Figure 7.



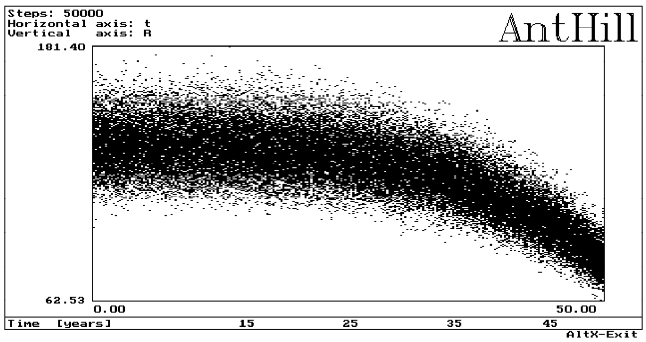
a) K, A, F_y and n are deterministic constants



b) Variable K (A, F_y and n are constants)



c) Variables K and A (F_y and n are constants)



d) Variables K, A and F_y (n is a constant)

Figure 7. Input parameters affecting the resistance, see eq. (7)

Combining the resistance $R(t)$ of the bar (Figure 7d) and the resulting load effect combination $S(t)$ (Figure 5d), using the AntHill program, two "streams" of dots can be obtained, see Figure 7.

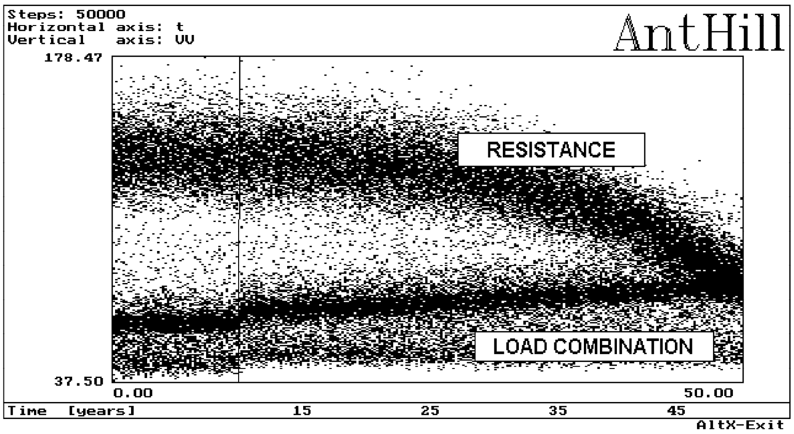


Figure 8. Resistance R(t) and Load combination S(t) as function of time

3. A priori estimate of probability of failure

The a priori estimate of probability of failure was determined using a safety function $SF(t)$. This function was defined as:

$$SF(t) = R(t) - S(t) \tag{9}$$

where functions $R(t)$ and $S(t)$ are given by equations (7) and (6), respectively. The probability of failure results from the analysis of $SF(t)$ using Monte Carlo simulation. For each point in time t_i,

$P_f(t) = P[R(t) < S(t)]$. This result corresponds to the analysis of an analogous situation and condition:

$$P_f(t) = P[SF(t) < 0] \tag{10}$$

The equation (10) was analysed using program M-Star. A distribution of the safety function $SF(t)$ was calculated for selected time points. As an example, the result corresponding to time $t = 25$ years is shown in Figure 9. The probability of failure for this case is $P_f = 0.000\ 283$.

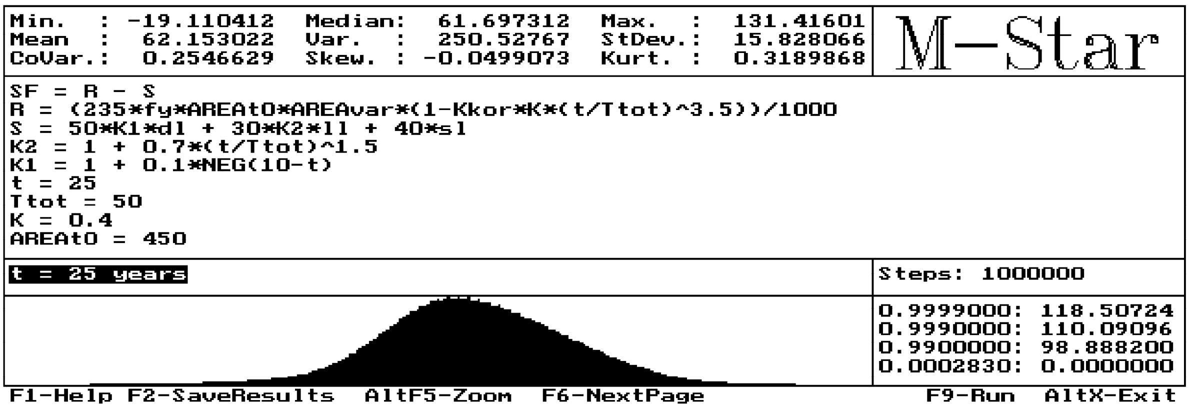


Figure 9. Safety function SF(t) for time t = 25 years

Using results of several calculations, a time dependent probability of failure $P_f(t)$ can be plotted (see Figure 10). This is the a priori estimation. The word "a priori" is used because the original estimated data in the corrosion model were used. Correction of the corrosion model could be made on the basis of inspections. The inspections provide real information about the degradation process and enable to suppress uncertainties of original corrosion estimate.

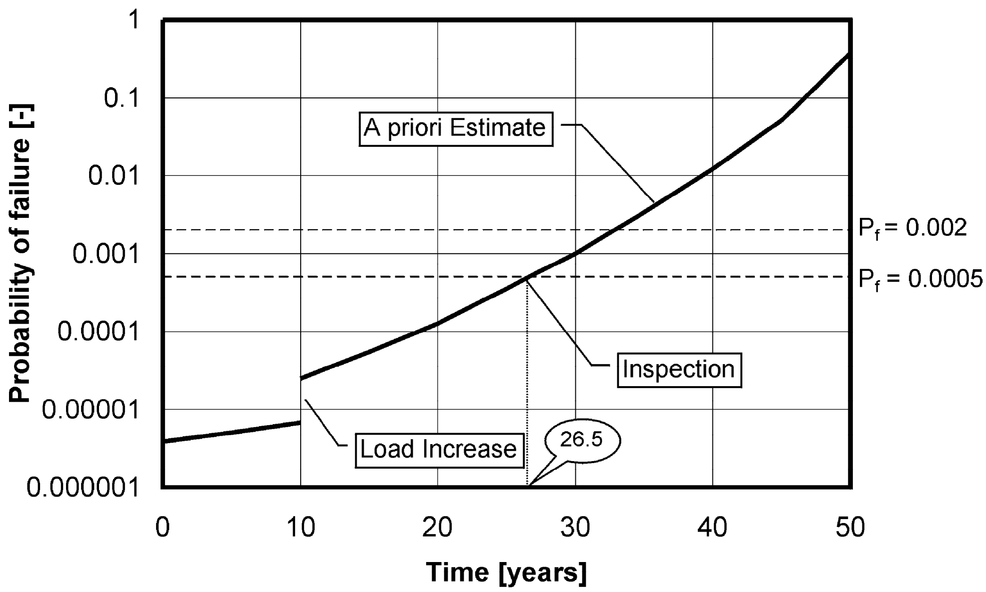


Figure 10. A priori estimate of probability of failure

4. Inspection plan using the a priori estimate of probability of failure

The inspection time could be determined using the a priori estimate of probability of failure assuming that $P_d = 0.002$ is the target probability of failure. This value would be given for example by the operator or by code. When the calculated probability of failure gets near the target value, an inspection should be performed. In this example, the criterion was chosen that the inspection should be performed in time when the calculated probability of failure would reach, for example, a quarter of the target value. It means, the inspection was recommended in the point of time when probability of failure reached value 0.0005. Using Figure 10, it could be determined that the inspection would be performed in point of time $t_i = 26.5$ years.

5. Correction of the corrosion model

Let us demonstrate how to use the data found out by inspection. Let be the loss of the cross-sectional area due to corrosion, as observed by the inspectors, denoted ΔA.

The measurement after 26.5 years determined the maximum lost of the cross-section ΔA_{max} = 8 mm^2. Because this information is influenced by any recording inaccuracies (errors), it is well-founded to assume this parameter to be stochastic. With the assumption that the recorded (measured) quantity ΔA has exponential distribution, the loss of cross-section could be expressed in form:

$$\Delta A = \Delta A_{max} * \Delta Avar = \Delta A_{max} *(1 + (Expon1)/20) \quad (11)$$

The histogram Expon1 is shown in Figure 11.

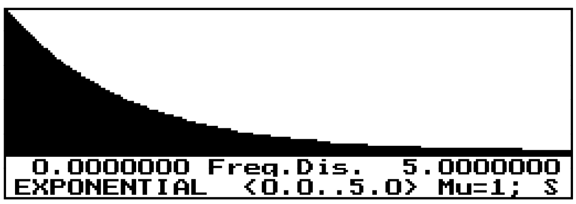


Figure 11. Bounded histogram Expon1; $\mu = 1$, $\sigma = 1$

The equation (11) was incorporated into the probability of failure calculation and a new estimate for time t > 26.5 years was determined. The exponent n in equation (8), as a deterministic parameter, was evaluated using relation (12). The corrected value was denoted n_c.

$$n_c = \frac{\log \frac{\Delta A_{max}}{K_{nom} * A_{nom}}}{\log \frac{t_i}{t_{tot}}} = \frac{\log \frac{8}{0.4*450}}{\log \frac{26.5}{50}} = 4.95 \tag{12}$$

Application of similar way, the distribution of K_{var} was modified. Using equation (13), the value of K_{varc} was evaluated in each simulation (for A_{nom} and A_{var} see chapter 2.1).

$$K_{varc} = \frac{\Delta A_{max} * \Delta A_{var}}{A_{nom} * A_{var} * \left(\frac{t_i}{t_{tot}}\right)^{n_c}} \tag{13}$$

These corrected parameters provide new values of the safety function $SF(t)$ for $t > 26.5$ years. The results are shown in Figure 12. Because of the conservative a-priori estimate of the probability of failure, the corrected values are lower.

Similarly as in the previous case, next inspection could be planned. Using the same procedure, the next inspection should be performed in time $t_i = 30.5$ years. Using the inspection data, the correction of the corrosion model is possible again and next calculation of probability of failure could be performed. Generally, it is possible to repeat the procedure as many times as it is necessary. But in this case, the described procedure leads to very short inspection intervals for time $t > 30.5$ years. Because the main reason was to demonstrate the possible applicability of the simulation technique, only simple inspection plan with two inspections was indicated.

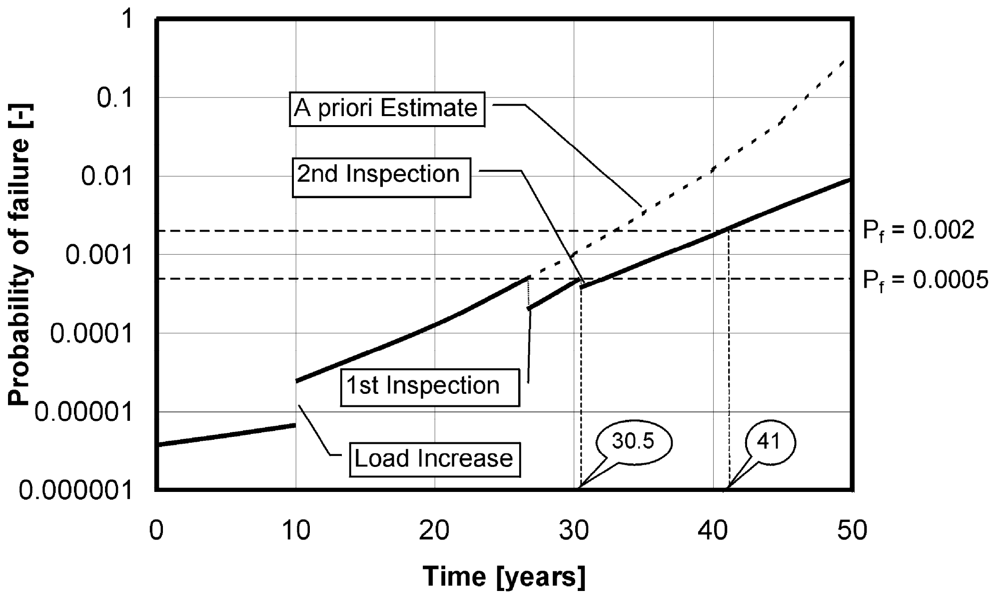


Figure 12. Correction of the probability of failure estimate

The correction of the corrosion model represents a modification of the resistance $R(t)$. The dependence of resistance $R(t)$ on time is changed, as is evident from comparing Figures 13 and 8.

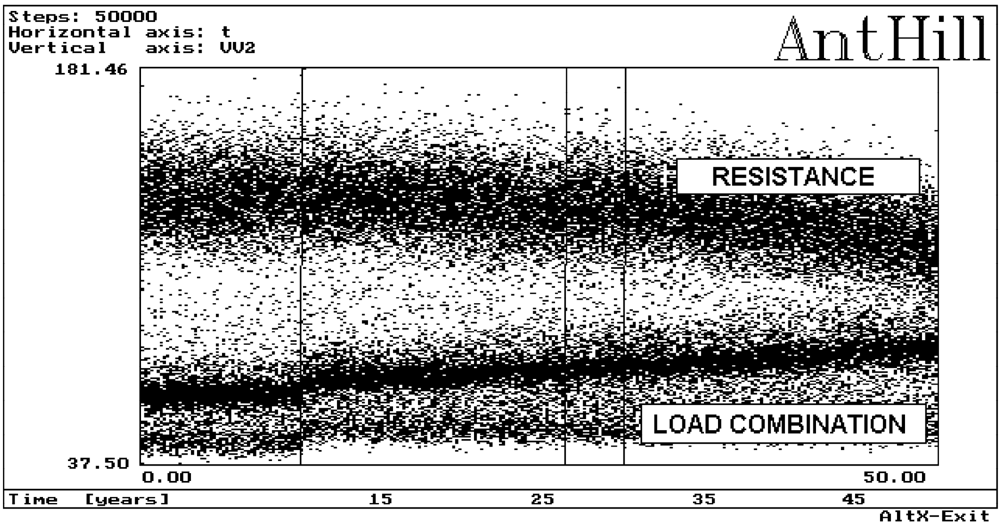


Figure 12. Time dependence of corrected resistance R(t) and load combination S(t)

6. Conclusions

A simple example of a steel bar affected by corrosion demonstrates the application of the SBRA simulation technique in case of the reliability assessment and inspection planning. The intervals of inspections were proposed on the basis of probability of failure. Because the inspection provided important data about degradation, the correction of corrosion was possible and more precise probability of failure analysis could be performed.

In the discussed example, only two inspections were planned. However, this procedure indicates significant increase of the lifetime of the steel bar. According to the Figure 11, the expected lifetime corresponding to the target probability $P_d = 0.002$ would be 41 years (compare to 33 years corresponding to the a priori probability of failure in case of “no inspection”).

Proposed procedure is strongly dependent on corrosion model. A sensitivity analysis should be performed and different corrosion models should be compared. Also the inspection data could influence results considerably. In the example, the conservative a priori estimate was assumed. But inspection can find out more intensive corrosion than it was expected. In such case, replacement of the structural member would be necessary or, if possible, a decrease of loading should be ensured.

7. References

[1] Marek P., Guštar M., Anagnos T., *Simulation-Based Reliability Assessment for Structural Engineers*, CRC Press, Boca Raton, Florida, 1995, 365 pp.

[2] Marek P., Guštar M., Bathon L., *Tragwerkbemessung von deterministischen zu probabilistischen Verfaren*, Academia, Prague, 1998, 263 pp.

Reliability-based Condition Testing: An Owner's Point of View

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Keywords: Existing Structures, Concrete, Deterioration, Management Systems, Inspection Models, Long-term budgeting, Inaccuracy, probability.

Summary

Optimisation of repair and maintenance of bridges has been a hot topic throughout the last decade.

The purpose of carrying out optimisation is:
To initiate repair works at the right time to ensure the lowest cost to society without affecting the integrity and safety of the bridge itself and the entire bridge stock in general.

Most bridge management systems are based on an inspection routine consisting of visual inspections carried out at intervals of 3 to 6 years. Supplementary special inspections are carried out when the visual inspection reveals a need for repair works. The special inspection is carried out to determine the extent of repair, the budget and the optimum time of execution.

The combination of visual and special inspections is used for the long-term budgeting of future repair costs for the entire bridge stock.

Inaccuracies related to the inspections can have a crucial effect on the long-term budget estimates and ultimately on the bridge stock condition.

Reliability-based condition testing and evaluation is a means of dealing with the inaccuracies and enhancing the quality of the outcome of the inspection - long-term service life and budget prognoses.

In the following sections this approach is discussed from the owner's point of view.

1. Introduction

Optimisation and reliability-based condition testing and evaluation became hot topics for two reasons:

1. **Insufficient funding requires the postponement of repair works.**
2. **Lack of fundamental knowledge regarding deterioration.**

To correctly evaluate the consequences of a postponement of a repair work, experience in the fields of in-place testing and service life modelling is required.

This knowledge is lacking. To deal with this problem the use of reliability-based assessment becomes relevant. The use of the technique, however, does not eliminate the fundamental problems and should therefore help to focus on the fields where knowledge is inadequate and in need of improvement.

In the following these fields are highlighted.

2. Existing management systems

Management systems are meant to optimise the investment of available funding based on technical evaluations.

However, the evaluations are affected by a combination of political, legal and technical aspects.

2.1 The political aspect

In a political system, problems cannot be approached purely technically. Road-user safety and road-user inconvenience during the execution period and the aesthetic appearance of the structure during service are factors which can have a decisive impact to the final decision on how and when to initiate repair works. They often overrule the purely technical approach.

2.2 The legal aspect

The general technical attitude and approach to a bridge exhibiting damage is: Repair works have to be initiated as soon as possible. To accept deteriorated structures and allow for a compromise on safety without measures being initiated to arrest the further progress of deterioration creates a conflict with codes and standards.

The party to be responsible for such a decision has to be clearly defined.

2.3 The technical aspect

The technical aspect involves substantial inaccuracies. The technical aspect is subdivided into the visual inspection and the special inspection.

2.3.1 The visual inspection

Most management systems are based on a visual inspection, in which each structural member is ranked according to a scale, e.g. from 0 to 5. A 0 ranking indicates an undamaged structural member with no need of repair for many years whereas 5 indicates a member in urgent need of repair.

The ranking is a means of helping the assessment of the short-term need for repair works to be initiated.

However, as a long-term budget tool the visual inspection has limitations. The ranking tool was developed on the assumption that there is a linear relationship between degree of deterioration and visual appearance.

Experience with deterioration mechanisms such as corrosion, frost attack and alkali-silica reactions reveals that deterioration is initiated several years before any visible signs of the ongoing deterioration become detectable.

Furthermore, the visible signs of deterioration arise overnight. Signs of deterioration (cracks, spalling etc.) develop instantaneously when the tensile strength of the concrete is exceeded due to the formation of of corrosion products, silica gel or ice.

An expected consequence is that a number of bridges with a 0 ranking might change to a 2 or 3 ranking at the next inspection.

At this damage level extensive repair measures are required and the benefit that could have been obtained by initiating preventive measures is lost.

The visual inspection therefore might need an enhancement to be a reliable basis for the preparation of long-term budget prognoses.

2.3.2 Special inspection

Decisions concerning postponements of repair works generally have to be based on a series of non-destructive tests which are either complicated to carry our or do not have widespread acceptance and documentation.

Furthermore, concrete is a non-homogeneous material and may be cast with hidden and randomly located execution errors. Experience reveals that deterioration is often related to execution errors.

To obtain representative test results is therefore difficult.

An example is illustrated in Figure 1. The Figure highlights the results of an estimate of the future number of cases of damage to bridge columns. The estimate is based on two special inspections carried out on the same bridge columns in 1990 and 1995 respectively.

TIME INTERVAL	1990-ESTIMATE FOR THE NUMBER OF BRIDGES WITH CORROSION	1995-ESTIMATE FOR THE NUMBER OF COLUMNS WITH CORROSION
1990-1995	97	181
1996-2000	0	40
2001-2005	48	20
2006-2010	0	0
2011-2020	16	0
2021-2050	32	20
2051-	128	60

Figure 1.

The Figure illustrates the uncertainty related to long-term budgeting based on available test methods and service life models.

A major problem is related to the assessment of post-tensioned structures. For various reasons, the available test methods are not adequate. Together with a very limited experience on the deterioration of post-tensioned structures this represents a major future problem.

3. Approach to future development

The approach to future assessment has to be divided into two approaches which are further subdivided as follows:

1. the local approach:
   - the durability approach
   - the structural approach
2. the global approach:
   - the durability approach
   - the structural approach

The local approach addresses the problems of correctly assessing the condition and service life of the individual structure.

The global approach addresses the problems of using the results of the assessment of a limited number of structures to predict the long-term repair budget for an entire stock of bridges.

To enhance the process it is important to distinguish between the durability and structural approaches and the definition of service life.

It has to be stressed that the structural aspect is crucial to the optimisation and the preparation of long-term budgets. The optimisation can be reliable only if the structural consequences of the deterioration are known. This requires that the ultimate limit state be calculated in each situation.

The structural and durability approaches are described below.

3.1 The service life

The definition of the concept service life often covers a combination of durability and structural service life.

When service life is in question the discussion generally refers to durability only.

The definition has to be strengthened.

Definition:
Durability = The time -T1- until a deterioration process has been initiated.
Structural service life = The time - T2 - until the load-carrying capacity has been reduced by the deterioration to an unacceptable level.

3.1.1 Durability

The time T1 can be estimated from e.g. diffusion models. The estimate is uncertain due to the uncertainties related to the models in use, the test methods and the sampling of test data.

3.1.2 Structural service life

The assessment of T2 is considerably more complicated as the time is dependent on the actual load-carrying capacity of the structure and the development of the damage mechanism.

Discussions on service life have often been based on experience with the deterioration of structural components relatively easy to deal with e.g. columns or edge beams exposed to corrosion due to chloride ingress. These components are usually uncomplicated to handle and do not involve huge investments compared to e.g. bridge deck repair works.

Often repair works have to be initiated on these components at the time T1 for aesthetic reasons or road user safety reasons (e.g. to prevent pieces of concrete from falling off) several years before the repairs become necessary from a load-carrying capacity point of view.

On the other hand, the deterioration of a bridge deck with post-tensioned cables involves a huge repair cost and a postponement of the repair from the optimum time results in severe impact on the load-carrying capacity, safety and the budget.

Furthermore, as the cables are hidden inside the concrete cross-section and are not accessible to visual inspection, available test methods are insufficient and little or no data are available regarding deterioration.

The assessment of the structural service life of such structural components involves major uncertainties.

4. Problems to highlight

Based on Sections 1-4, a number of topics have to be addressed if further progress is to be made:

1. To evaluate the long-term budget effects of relying on visual inspections and eventually to propose modifications to enhance the quality of the visual inspection.
2. To develop deterioration models in general and especially for post-tensioned structures.
3. To develop reliable methods for testing of post-tensioned structures.
4. To evaluate the uncertainty related to the combination of:
   - effects of concrete non-homogeneity
   - service life modelling
   - testing of condition (planning, test methods, evaluation, execution etc.)
   - effect of deterioration on the load-carrying capacity
   - load-carrying capacity calculation models
5. To develop a technical-economic assessment model to include and capitalise the safety and the road-user inconvenience during the execution of the repair works.
6. To evaluate the long-term effects related to different available economic models.

Focus has to be put on structural members with a load-carrying system consisting of post-tensioned cables, components covered by e.g. a waterproofing membrane and inaccessible (buried or submerged) components.

Furthermore, more attention must be given to the calculation of the ultimate limit state. The consequences of the postponement of repair works can be evaluated correctly only if compared with the consequences of reaching the ultimate limit state.

The reliability-based approach can help in this process.

Planning of Inspections of Concrete Structures

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Keywords: Concrete structures, inspection planning, deterioration, reliability, statistical analysis.

Summary

Planning of inspections of concrete structures may be performed within the framework of Bayesian decision analysis. However, in many cases it is not possible to perform a full decision analysis. This may e.g. be due to lack of funds, time or the necessary information concerning the deterioration of the structure. Even in such cases it is possible to plan the inspection such that the amount of information gained by the inspection is optimised. The most important parameters may be identified by sensitivity analysis and the measurements of these important parameters should be carried out in a way, which ensures that the observations are independent. Further, the measurements should be carried out in a way, which ensures that only the random variation of these parameters in the most critical region is measured. Here, a few simple rules for the planning of inspections of concrete structures are given. Also, the statistical analysis of the inspection results and the use of the inspection results as a basis for reliability analysis are discussed.

1. Introduction

Planning of inspections of concrete structures may be performed within the framework of Bayesian decision analysis. This method assures that the inspection plan leading to the lowest expected cost is identified. This method requires that the probability of some failure event and the probability of repair of the considered structure can be obtained. These probabilities may be determined on the basis of structural reliability analysis using a prior probabilistic model and limit state functions describing the failure and repair events. Bayesian decision analysis has e.g. been used to determine optimal inspection plans for offshore structures subject to fatigue, see e.g. Madsen [5] and Faber et al. [4]. Bayesian decision analysis has in recent years also been used for planning of inspections and repairs of concrete structures, see e.g. Engelund, Sørensen and Sørensen [2] and Enevoldsen and Jensen [1].

It is not always possible to determine an optimal inspection plan on the basis of Bayesian decision analysis. Often the cost of performing the Bayesian decision analysis will exceed the savings obtained by identifying the optimal plan. Further, the purpose of an inspection may be to identify the critical deterioration mechanism. Obviously, in these cases the inspection cannot be planned on the basis of decision analysis. If the deterioration mechanism is unknown, i.e. it is not possible to use decision analysis because a limit state function describing the failure event cannot be formulated.

The purpose of this paper is to give a few simple rules for the planning of such inspections. These rules are meant to assure that the results of the inspections may be used as a basis for a probabilistic analysis and to assure that the largest amount of information is obtained from the inspection. Further, use of the information obtained from such an inspection as a basis for reliability analysis and reliability-based planning of rehabilitation actions is discussed.

2. Types of inspections

In general two different types of inspections are carried out

- General inspections
- Detailed inspections

General inspections are carried out with a fixed time interval, e.g. every five years. This inspection consists of a thorough visual inspection of the entire load-carrying structure and all other elements such as rails and road signs. On the basis of this inspection it is decided if it is necessary to perform a detailed inspection of selected parts of the structure such as e.g. a bridge deck, piers or other parts of the structure.

By the detailed inspection of a given part of a structure, a number of different measurements and/or experiments are usually carried out. These measurements may e.g. consist of measurements of the carbonation depth at different locations, measurements of the chloride concentration, the concrete compressive strength and/or corrosion activity. On the basis of the results of the detailed inspection, decisions concerning the future maintenance, repair and possible future inspections are made.

The detailed inspection should, naturally, always be planned such that a maximum amount of information is obtained at the lowest cost. The results of the detailed inspection should also be in a form suitable for statistical analysis in order to allow for a reliability-based planning of the future maintenance, repair and inspection actions.

3. Planning of detailed inspections

As mentioned, inspections of concrete structures are often carried out in order to determine the type of deterioration affecting the structure and the criticality of the deterioration. The planning of the inspection must be performed without information about the deterioration mechanism.

Even though little information concerning the deterioration mechanism is known prior to the detailed inspection it is possible to plan the inspection on the basis of a few simple rules. The following points should be considered

- Sensitivity analysis
- Performing measurements in the critical region
- Ensuring that the test results are independent
- Plan the tests so that the uncertainty related to the estimated parameters is minimised

These points are treated in more detail below.

3.1 Sensitivity analysis

Prior to the inspection a sensitivity analysis with respect to the basic variables may be carried out. In this way the most important parameters may be identified. A sensitivity analysis will e.g. reveal that the load carrying capacity of a concrete beam only to a relatively little extent depends on the compressive strength of the concrete and that the load carrying capacity to a large extent depends on the bond between the concrete and the reinforcement. Hence, the inspection should focus on the detection of delamination and loss of bond and not on the compressive strength of the concrete. Taking into account that delamination and loss of bound may be detected by hammer testing and that substantial effort is involved in testing the compressive strength of the concrete this becomes even more evident.

3.2 Critical region

It is often possible to identify a critical region on a given bridge structure where the probability of observing a given deterioration mechanism is high. Chloride ingress will usually occur on surfaces subject to water spray from the road surface, i.e. the lower part of the piers. Corrosion due to carbonation will occur in areas where the humidity is low enough for carbonation to occur and high enough for the corrosion process to progress, i.e. on the underside of the bridge deck. Such prior information shall naturally be applied by the planning of measurements. However, the identification of the critical regions cannot be based solely on information about the environment. On bridge piers in a marine environment the cover thickness is usually largest in the splash zone where the chloride surface concentration is largest. Hence, it is necessary to consider both the environment and structural details in order to locate the critical region. In some cases it may be necessary to perform measurements at more than one critical region. In such cases it will usually be necessary to treat the measurements from the different regions separately. This will be treated in more detail below.

3.3 Independent observations

The statistical analysis of the test results is facilitated if the test results are independent. According to test results reported by Hergenröder [6] outcomes of the material properties of concrete may be assumed to independent if the distance between the observations is larger than 0.5-1.0 m. Hence, the distance between measurements of the concrete properties should always be larger than about 1 m. Further, it should be noted that the material properties also may depend on the environment. For example, measurements of the concrete chloride diffusion coefficient will in general exhibit a substantial variation between different environments such as the splash zone and the atmospheric zone (3-4 m above sea level). If the deterministic variation of the parameters is not modelled and taken into account the results of the measurements are not suitable for statistical analysis.

3.4 Minimising statistical uncertainty

Results of detailed inspections are often used as a basis for predicting the service life of the considered structure on the basis of some degradation model (see the following section). In many cases the degradation with time may only be predicted if measurements are made at a number of different points in time. The statistical uncertainty related to the parameters of the degradation model is minimised if the distance in time between these measurements is large.

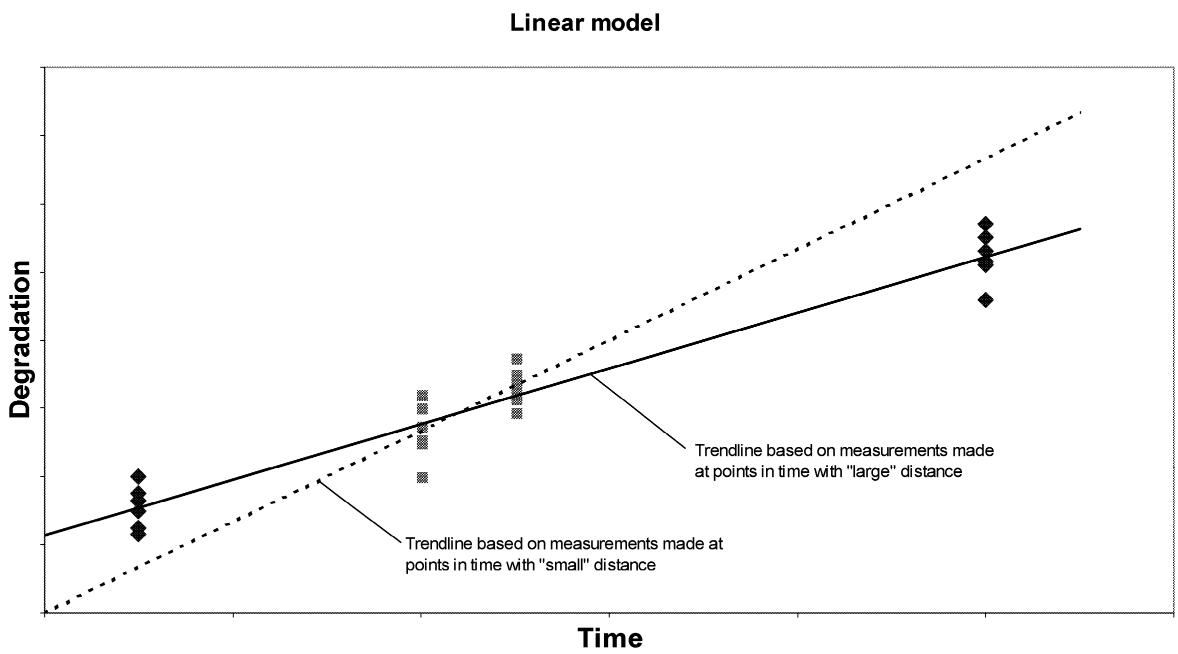

Figure 1: Linear regression.

In figure 1 the results of a linear regression analysis based on two different sets of measurements are shown. Evidently, the trendline determined solely on the basis of the measurements made at points with a large distance in time provides a good fit to all measurements including the ones made with a small distance in time. In contrast, the trendline determined solely on the basis of the measurements made at points with a small distance in time provides a very poor fit to the other measurements. This clearly demonstrates the benefit of performing measurements at a very early stage of deterioration of a structure. Such measurements made at an early stage of deterioration, provides an excellent basis for service life predictions because these measurements assures that the statistical uncertainty related to the estimated parameters is minimised.

4. Degradation models

A decision concerning the future inspection and repair of a concrete structure subject to deterioration must be based on a model predicting the deterioration of the structure. In recent years a large number of different models for predicting the deterioration of concrete structures has been suggested by various researchers. It is beyond the scope of this work to go into a detailed discussion of the merits and disadvantages of these models.

For a probabilistic analysis it is important to select the model with the lowest number of unknown parameters which provides the best fit to the available data. Ideally, the model should reflect the underlying physical process. However, this is not a necessary condition.

Consider the following two models describing the chloride concentration in a concrete structure as a function of the distance, x, from the exposed surface and time, t

$$c(x,t) = c_s\left(1-\mathrm{erf}\left[\frac{x}{2\sqrt{Dt}}\right]\right) \qquad \text{"Traditional" model}$$

$$c(x,t) = c_s\left(1-\mathrm{erf}\left[\frac{x}{2\sqrt{D_0 t\left(\frac{t_0}{t}\right)^n}}\right]\right) \qquad \text{Duracrete model}$$

where c_s is the chloride surface concentration and D denotes the diffusion coefficient. In the so-called DuraCrete model the effective diffusion coefficient is assumed to be a function of time, i.e. $D = D_0\left(\frac{t_0}{t}\right)^n$ where D_0 is the diffusion coefficient measured at the time t_0 and where n is a time factor. For a concrete made from ordinary Portland cement, the age factor will normally be about 0.3.

If a set of measurements of the chloride concentration has been made at $t = 50$ years and these measurements are used as a basis for the prediction of the development of chloride ingress the following 10 years the results of the two models will be virtually identical. The diffusion coefficient used in the Duracrete model will be $D_0\left(\frac{50}{60}\right)^{0.3} = 0.947 D_0$. Whereas, the diffusion coefficient used in the "traditional" model will be D_0. Evidently, the "traditional" model will in this simple example provide the best fit with the lowest number of parameters.

5. Limit state function

Inspections are carried out to ensure that the considered structure is in an acceptable state. This implies that the structure must have an acceptable reliability both with respect to collapse and serviceability failure. However, the owner of the structure may have other requirements as well. Often the visual appearance of the structure is important. If for example a bridge shows severe signs of deterioration it may lead to loss of prestige for the bridge owner. Finally, the structure must have a sufficient level of reliability with respect to events, which may lead to other risks to the users. For example spalling of the concrete of a road bridge may be a risk to cars passing below the bridge.

For all these events relevant limit state functions must be formulated and the acceptance criteria must be identified. Acceptance criteria for events having an impact on the visual appearance of the structure may be chosen by the owner, provided that the structure has a sufficient reliability with respect to events which may lead to loss of life and limb.

6. Reliability analysis

The evaluation of the probability of collapse or serviceability failure is in general a time-variant problem because both the loads and the resistance exhibit a random variation in time, see Figure 2. This failure probability may be determined by the following expression

$$P_f(t) = 1 - P(g(\mathbf{x}, t) > 0 \quad \forall t \in [0;T])$$

where $g(\mathbf{x}, t)$ is a limit state function which is less than zero if and only if the failure event occurs, $\mathbf{x}$ is a vector of time-variant stochastic variables and t denotes time.

Time-variant reliability problems are in general difficult to solve. However, an approximation to the solution to the problem may be obtained if the resistance is assumed to be constant within a period of one year. In that case the load may be represented by the yearly maximum. The error made using this approximation will always be less than one year. If the resistance is assumed to be equal to the value at the end of the considered period of one year the method will always be conservative (given that the resistance is reduced with time).

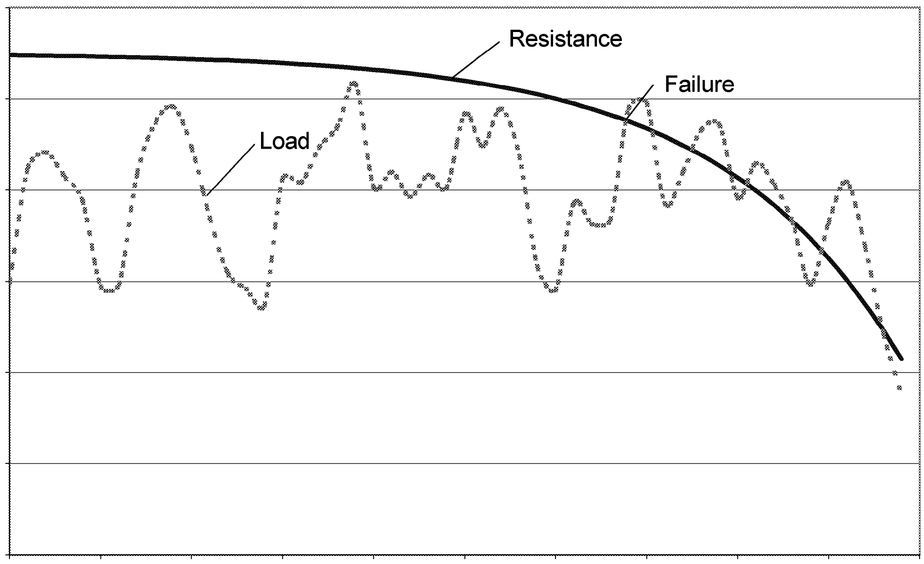

Figure 2: Time-variant reliability problem.

The evaluation of the acceptability of the appearance of a structure may e.g. be based on the probability that a given percentage of the structure exhibits signs of corrosion or some other deterioration mechanism. By the evaluation of the probability that a given percentage of the structure shows signs of deterioration the random spatial variation of the material properties and the environment must be taken into account as well as the deterministic variation of both material properties and environment.

The spatial variation may e.g. be taken into account by partitioning the considered structure into elements of about 1 x 1 m. Within an element of this size the material properties and environment may be assumed to be constant. On the basis of a reliability analysis the probability of signs of deterioration in each of these elements may be determined. Using information concerning the correlation between the event of discolouring in each of the elements the probability that a given number of failures have occurred may be evaluated.

The cover thickness cannot be assumed to be constant within an element of 1 x 1 m. However, corrosion will always be initiated at the point with the lowest cover thickness (because the environment and material properties are assumed to be constant within the considered area). The cover thickness within a given element may therefore be represented by the distribution of the minimum value of the cover thickness.

For a more detailed description of the evaluation of the probability that discolouring of the concrete surface occurs see e.g. Engelund and Sørensen [1].

7. Reliability-based management plan

Once sufficient information concerning the deterioration of the structure is available, i.e. the deterioration mechanism(s) has (have) been identified and the relevant parameters have been measured the future rehabilitation actions may be planned. The future rehabilitation actions are often determined on the basis of a deterministic analysis. However, it is possible on the basis of the information gained from the inspection to determine an optimal rehabilitation plan using reliability-based decision theory. Experience has shown that the savings obtained by reliability-based planning of rehabilitation actions may be significant, see e.g. Jensen, Knudsen and Enevoldsen [ ]. This is naturally only the case if a deterministic analysis indicates that the cost of rehabilitation is large.

The development of a reliability-based management plan may be performed in the following steps, which have been discussed in the previous sections.

1. Fact-finding (previous inspections, analyses, etc.)
2. Sensitivity analyses (deterministic model)
   - Identification of critical failure modes
   - Identification of critical combinations of failure modes and deterioration
3. Perform inspection
4. Probabilistic model
   - Material parameters (strength, resistance to deterioration)
   - Environment (loads, concentration of aggressive substances)
   - Model uncertainty
5. Identify acceptance criteria
6. Reliability analysis
7. Analysis of various repair and rehabilitation options
8. Evaluation of the cost-optimal management plan using decision analysis

For a more detailed presentation of reliability-based management plans see e.g. Enevoldsen and Jensen [1].

8. Conclusions

Reliability-based inspection planning based on Bayesian decison theory is an attractive approach for planning of inspections of concrete structures subject to deterioration. However, in many cases it is not possible to perform a full decision analysis in conjunction with the inspection planning.

A number of practical recommendations for inspection planning have been presented. On the basis of these simple rules it is possible to plan inspections such that the most important measurements are made and such that the measurements are made at the most critical locations.

Also the use of the measurement results as a basis for reliability analysis and planning of rehabilitation actions have been discussed.

References

[1] Enevoldsen, I. & F. M. Jensen , (2001): “Safety-based Bridge Maintenance Management ” Proceedings, IABSE conference on safety risk and reliability - Trends in engineering, Malta, March 21-23, 2001.

[2] Engelund, S., Sørensen. J. D., Sørensen, B., Evaluation of Repair and Maintenance Strategies for Concrete Coastal Bridges on a Probabilistic Basis, ACI Materials Journal, 96, 2, pp. 160-166.

[3] Engelund, S., Sørensen, J. D., A probabilistic Model for Chloride-Ingress and Initiation of Corrosion in Reinforced Concrete Structures, Structural Safety, 20, pp. 69-89.

[4] Faber, M. H., OMAE 2000

[5] Madsen, H. O., Model Updating in First Order Reliability Theory with Application to Fatigue Crack Growth, in: Proceedings to Second International workshop on stochastic methods, University of Pavis, Pavia, Italy, 1985.

[6] Hergenröder, M. Zur Statistischen Instandhaltungsplannung für bestehende Bauwerke bei Karbinatisierung des Betons und möglicher Korrosion der Bewehrung, Berichte aus dem Konstruktiven Ingenieurbau, TU München 4/92, 1992 (in German).

New trends in composite bridge design, assessment and maintenance

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Keywords: Composite bridges, reliability, inspection, maintenance, fatigue, fracture.

Summary

This paper presents a probabilistic maintenance optimisation procedure for welded joints damaged by fatigue. The linear elastic fracture mechanics stands as a basis for fatigue crack growth while the crack depth at failure is determined according to brittle or ductile fracture. The first order reliability method being applied, the assessment is then based on reliability indices. At an inspection instant, different events can occur: no crack detection, crack detection, and repair. All these events are uncertain and are expressed in terms of probabilities. As they are cost-dependent, the total expected cost of maintenance can be obtained. The inspection instant is therefore searched by minimising that total expected cost. Such an approach appears as a good compromise between reliability and costs. The paper provides some sensitivity studies.

1. Introduction

A French national research project, the MIKTI project [1], has been undertaken in 2000 for studying new types and new designs of composite bridges. This project, involving numerous partners (public owners, public and private laboratories, universities, private companies), is divided into five main Working Group:

- short span composite bridges,
- slabs,
- two-girder composite bridges,
- tubular welds,
- assessment and inspection.

One of the most interesting feature of this project is that new construction forms will be analysed through their relative reliability. Furthermore, the project is intended to assess and eventually to reduce failure risk versus fatigue and fracture not only by using high performance material and construction quality, but also through inspection and maintenance. This especially concerns two girder composite bridges which allow very low degrees of redundancy. The question is therefor to assess the reliability of a component as far as a crack is detected. Inspections have to be chosen in such a way that failure risk is controlled. But, maintenance actions are also cost-dependent and a rational maintenance must therefore rely on a balanced compromise between cost and risk.
For all these reasons, WG 5 is dedicated to the development of probability-based models for fatigue and fracture assessment (including parameters determination with extensive tests), crack detection and whole life costing.
The purpose of this paper is to present preliminary results and research subjects which will be carried out within WG5 of the MIKTI project.

2. Reliability criterion

As mentioned previously, there is today a strong demand to evaluate two-girder composite bridge safety versus fatigue risk. Indeed, their low degrees of redundancy - and some international expert comments [2] - have led to undertake some researches on that problem. For this purpose, a model based fracture mechanics has been chosen for assessing fatigue crack growth and brittle/ductile rupture.

2.1 Fatigue and fracture modelling

As mentioned above, one of the most important problem is the crack growth in a flange of a « transverse stiffener-bottom flange » welded joint. The model is the kinetic Paris law corresponding to the opening of a semi-elliptical crack:

$$\frac{da}{dN} = C\left(Y(a)M_k(a)\Delta S\sqrt{\pi a}\right)^m \tag{1}$$

where a is the crack depth, c the crack half-length, b the flange thickness, N the number of cycles, ΔK the stress intensity factor range, ΔS the stress range at the hot spot for the non cracked section, C and m being two material parameters. The stress intensity correction factor $Y(a)$ is the solution from Newman and Raju, as used for the Eurocodes. The stress concentration factor $M_k\,(a)$ represents the magnification factor to take account of stress concentrations due to a specific structural detail. Its parameters are calculated according to the Hobbacher formula. This model is compared to test results and gives a fair prediction of the crack growth [2].

2.2 Safety margin

The safety margin expresses the frontier between damage and non damage. A convenient safety margin for computations - based on an integral form of Eq.(1) - has been used:

$$\hat{M}(t) = \int_{a_0}^{a_c} \frac{dx}{\left(Y(x)M_k(x)\sqrt{\pi x}\right)^m} - C\,N(t)\,\mathrm{E}(\Delta S)^m \tag{2}$$

where a_0 corresponds to the beginning of the propagation period. The failure probability is given by $P_f(t) = \mathrm{P}\left(\hat{M}(t) \le 0\right)$. Up to now different possibilities have been proposed for a_c among which the two most frequently used have been to choose the critical crack depth equal to the half-thickness or the full-thickness of the bottom flange. Such choices can provide optimistic values compared to results with a critical crack depth determined using fracture mechanics criteria. The approach adopted was to perform an analysis based on the R6 rule (introducing ductile and brittle fracture) in order to obtain the critical crack depth [2]. Two criteria (fracture and plastic collapse) limit the load capacity of a cracked structure. The elastic and plastic components of the analysis are separated in a way that aids calculation, facilitates a sensitivity analysis and provides an insight into the way in which a structure will perform. Firstly, the stress intensity factor should not exceed the fracture toughness; secondly, the applied load should not exceed the plastic yield load. Interactions nevertheless exist between those two modes of failure and the R6 rule proposes the simplified expression:

$$g(L_r, K_r) = \begin{cases} \dfrac{1}{\sqrt{1+0.5L_r^2}} - K_r & L_r \le 1 \\ 0 & L_r > 1 \end{cases} ;\, K_r = \frac{K}{K_c} \le 1;\; L_r = \frac{P}{P_L} \le L_{\max} \tag{3}$$

with K the stress intensity factor, K_c the fracture toughness, P and P_L the applied and plastic loads. The measure of proximity to plastic yield L_r is a measure of how close to plastic yield is the structure containing the flaw. This effect must be included in evaluating the plastic yield load:

$$L_r = \frac{S_G + S_Q}{f_y} \frac{bd}{bd - \pi ac/2} \tag{4}$$

where S_G, S_Q, f_y are the dead load stress, the peak traffic load stress and the yield strength. The measure of proximity to linear elastic fracture mechanics failure K_r can be assessed by the linear elastic stress intensity factor K which is evaluated from the elastically calculated stress field in the uncracked body at the location of the crack:

$$K = Y(a)\left[M_k(a)\left(S_G + S_Q\right) + S_S\right]\sqrt{\pi a} \qquad [5]$$

where S_S represents the residual stress. The fracture toughness is expressed as a function of the Charpy-V transition temperature:

$$K_c\left[\mathrm{MPa}\sqrt{\mathrm{m}}\right] = 20 + \mathsf{E}(K_c)\left[11 + 77\,e^{0.019\left[T_e - T_{K28} + 18\right]}\right]\left(\frac{25}{2c-2}\right)^{1/4} \qquad [6]$$

$\mathsf{E}(K_c)$ is the fracture toughness uncertainty factor, T_e the material temperature, and T_{K28} the test temperature for 28J minimum average Charpy V-notch impact energy. The value a_c such as $g(K_r, L_r) = 0$ is the critical crack depth. The evolution of the reliability index as well as the evolution of the failure probability with time are given in Fig.1. A conventional crack depth at failure (*b* or *b*/2) provides overestimated reliability indexes in comparison with the approach based on fracture mechanics criteria.

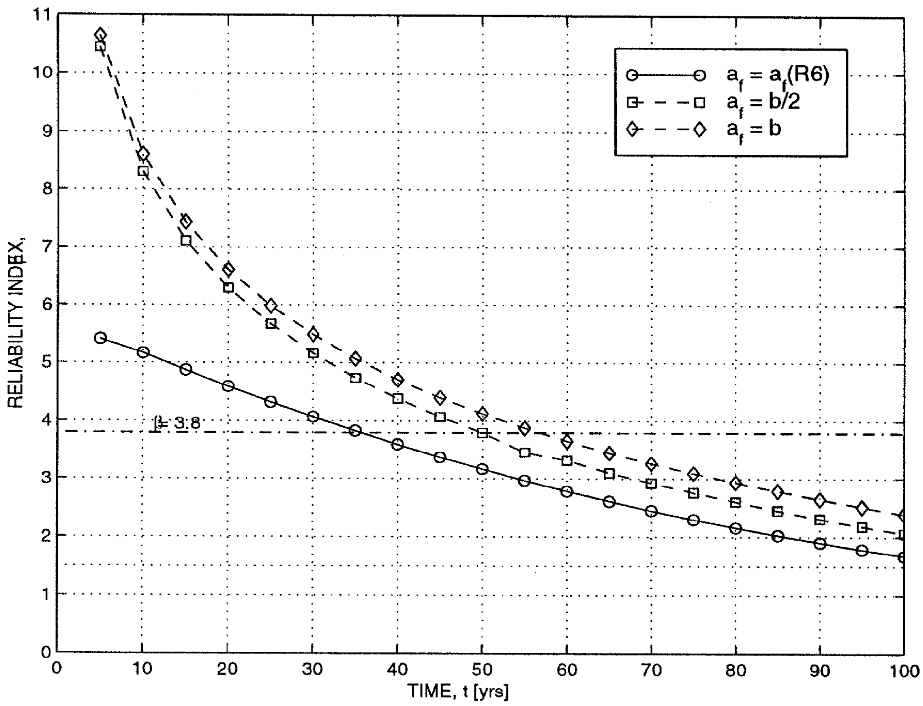


Fig.1 Influence of the critical crack size

The previous developments have been applied to a set of hot points in composite bridges. Basic variables to be taken into account are obtained in three different ways: from the literature, from the drawings and from the measures and subsequent computations. The full description of the variables is given in [3]. Nine representative bridges have been chosen (Fig.2a). Fig.2b gives the evolution of their reliability indexes versus time.

2.3 Acceptable safety levels

From Fig.2b, it can be noticed that, except one bridge (Auxerre), the reliability indexes belong to a narrow zone. Such curves are interesting to fit target reliabilities.

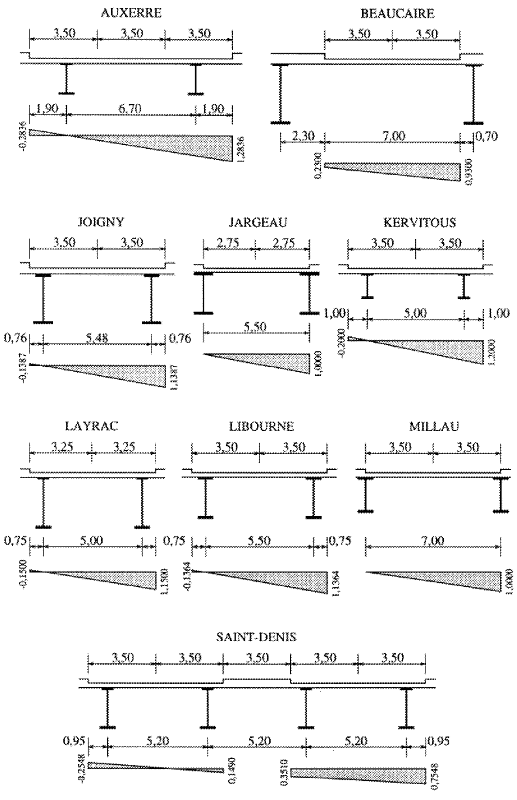


Fig. 2a Set of composite bridges

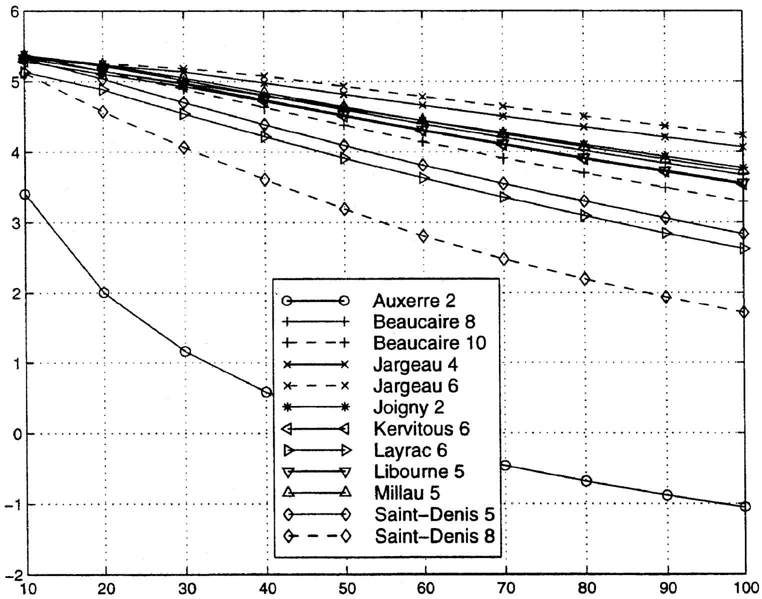


Fig. 2b Reliability index evolution with years

When performing a reliability analysis, it is necessary to compare calculated values to acceptable reliability levels. Let us note that this target reliability level is meaningless when failure probabilities are just calculated for relative comparisons between different designs. Nevertheless, target reliabilities are essential for determining safety levels for a specific bridge. This problem is difficult to handle. If we assume that reliabilities from computations in Fig.2b are relative to a bridge stock in good state (except for the Auxerre one), the calculated values can be used to calibrate a target value. One of the possible approach is to calculate:

$$P_f^* = \frac{1}{\sigma}\int_{-\infty}^{+\infty}\Phi(-\beta_{100})\,\varphi\left(\frac{\beta_{100}-\mu}{\sigma}\right)d\beta \qquad [7]$$

But other approaches can also be used for calibrating this target reliability such as utility theory. One of the objectives of the project will be to target these acceptable probabilities of failure.

3. Reliability of inspection results

3.1 Inspection Efficiency

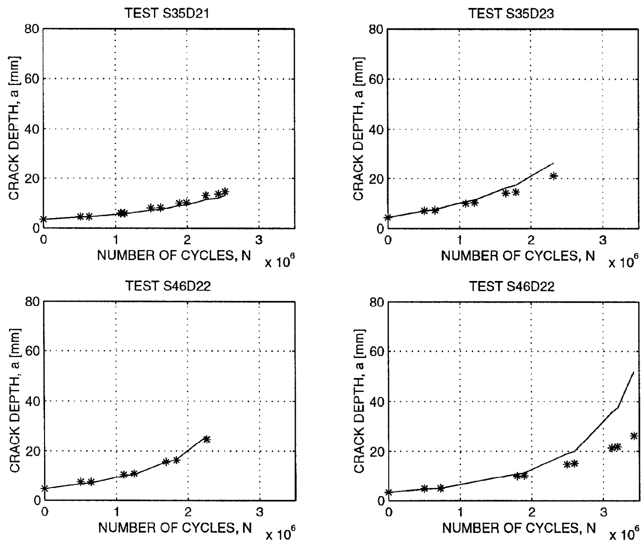
The efficiency of an inspection depends on a certain number of factors, inspection method and inspection team for instance, and is attached to the inspection quality. In the case of fatigue, this quality is attached to the probability to detect a crack of a given size. Therefore, a variable $q \in [0,\infty[$ may be defined as follows: $q = 0$ corresponds to no inspection while $q \to \infty$ corresponds to a perfect inspection, where infinitely small defects can be detected. The reliability of any non-destructive inspection technique represents the measure of its efficiency in detecting defects of specified type, shape and size. After having proceeded to an inspection there is a probability that the component is "clear" from defects. A crack detecting device cannot detect a too small crack. In fact, there is a threshold - the smallest detectable crack depth, a_d - bellow which the detection is no more reliable. Crack detection depends both on crack depth and on inspection efficiency. They are never perfectly known: a_d is therefore a random variable. The probability of crack detection is therefore equal to the probability that the crack depth is greater then or equal to a_d. Let $a(t)$ be the crack depth in time *t* and let $F_{a_d}(.)$ the cumulative distribution function of a_d. The probability of crack detection is then:

$$P_d(x) = P(x \geq a_d) = F_{a_d}(x) \qquad [8]$$

This function can be modelled according to different laws: e.g. exponential, Weibull, lognormal. Cremona [3] shows that the nature of the probabilistic model has little influence on final results. The latter taken into account, this study is limited to a lognormal law supposed to cover the most conveniently the randomness of this kind. On the other hand, it is important to point out that any inspection technique has its own detection threshold and its own precision as well. It is therefore sufficient to know the probability of detection function, P_d, in order to determine the distribution of a_d necessary to determine the event margins.

3.2 Experiments

Within the MIKTI project, different experiments will be scheduled in order to evaluate the statistical characteristics of the variables introduced in the crack growth model. Preliminary tests are today performed in order to test the experimental procedure. Fig. 3 highlights some results obtained on RWTH specimen [2].



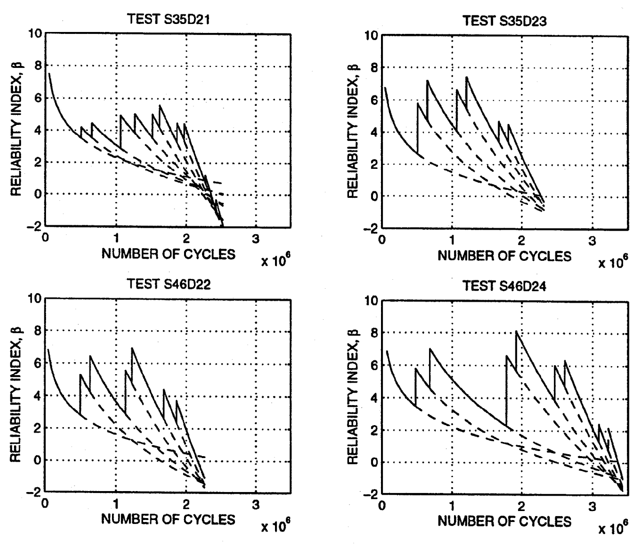


Fig.3a Comparison between predictions and tests *Fig.3b Reliability updating for specimen*

The experimental programme has for objective to detect, by different means (strain gauge, ultrasonic, magnetic,...), crack depths on painted or unpainted T-joints. Until now, 12 initial specimen are under tests (Fig.4).

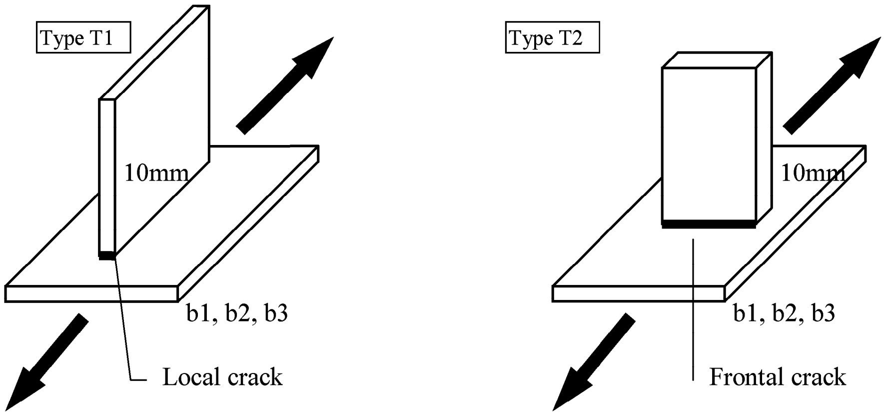


Fig.4 TestedT-joints

Each specimen is first cracked under high fatigue amplitude. Detection will be performed by strain gauges near the welding. When cracked, the length and the depth will be measured at regular intervals. The crack mark will be marked by overloads, the crack growth parameters being assessed Between each interval, a fatigue sequence (equivalent to a 1-year service) is applied to the joint. The fatigue loading is a sequence of elementary loads which are characterised by the amplitude S_k and the number of cycles N_k. Each sequence is ended by an overload. The sequence is repeated M times. The elementary loads are randomly performed with respect of amplitude.

4. Whole life costing

The maintenance model which has been used, is the conditional maintenance with regular inspection planning. A conditional maintenance requires to define criteria or conditions according to which maintenance actions will be engaged. This has to be made by quantifying crack sizes by a detection method and by ranging crack sizes in a finite number of severity classes. Let us assume that this finite number of class intervals is limited to (n_R+1) where the first class corresponds to undetected cracks. Let us call $I_0=[0,a_d[$ this class where a_d corresponds to the smallest crack size which can be detected. If there are n_R types of possible repairs, then it comes that any crack with size $a\in I_i=[a_{i-1},a_i[$ is repaired by the repair technique N. *i*. Let us note that the decision interval $I_1=[a_d,a_1[$ can correspond to a detection followed by no repair actions. T_s is a reference period which can be the next inspection time or the conventional lifetime T_f. The inspection events are therefore defined as it follows:

- $\{H^0\le 0\}=\{H_{nd}(a_d)\le 0\}$, $\{H^1\le 0\}=\{H_d(a_d)\le 0\cap H_{nd}(a_1)\le 0\}$
- $\{H^i\le 0\}=\{H_d(a_{i-1})\le 0\cap H_{nd}(a_i)\le 0\}$ for $i=2,\cdots,n_R-1$
- $\{H^{n_R}\le 0\}=\{H_d(a_{n_R-1})\le 0\}$.

Nevertheless, by sake of simplicity, the first notation will be kept in the following developments. Each action at each inspection has an effect on the event and safety margins at the next inspection time. It is therefore necessary to introduce another notation for describing the event sequences. For instance, with an action *k* at time t_1 and an action *l* at time t_2, the safety margin at time $t_2\le t\le t_3$ will be noted $M^{k,l}$ as well as the event margin at time t_2 will be noted $H^{k,l}$. $M(t)$ will still define the safety margin before the first inspection time. The probability of failure in the time interval $]t_i,t]$ is given by:

$$P_f(t)=\hat{P}_f(t)=\mathrm{P}\ (M(t)\le 0)\ \text{for}\ 0\le t\le t_1 \qquad [9i]$$

$$P_f(t) = P_f(t_1) \\ + \mathrm{P}\left(M(t_1) > 0 \cap H^0 \leq 0 \cap M^0(t) \leq 0\right) + \cdots \text{ for } t_1 \leq t \leq t_2 \\ + \mathrm{P}\left(M(t_1) > 0 \cap H^{n_R} \leq 0 \cap M^{n_R}(t) \leq 0\right) \qquad [9ii]$$

and so on for each inspection time. The probability of repair N. r ($r \geq 2$) is determined by:

$$P_R(t_1) = \mathrm{P}\left(M(t_1) > 0 \cap H_r \leq 0\right) \qquad [10i]$$

$$P_R(t_2) = \mathrm{P}\left(M(t_1) > 0 \cap H^0 \leq 0 \cap M^0(t_2) \cap H^{0,r} \leq 0\right) + \ldots \\ + \mathrm{P}\left(M(t_1) > 0 \cap H^{n_R} \leq 0 \cap M^0(t_2) \cap H^{n_R,r} \leq 0\right) \qquad [10ii]$$

and so on, for each inspection time.

4.1 Optimisation of the next inspection time

The problem consists in the determination of an optimal inspection time t_1 which minimises the total expected cost. The inspection time t_1 must fulfil the condition $t_1 \leq T_s$, where T_s corresponds to the time with a reliability equal to the minimum reliability $\beta_{\min}$ which can be accepted. The inspection, repair and failure expected cost models are therefore the following:

$$C_I(t_1) = C_{ins}\left(1 - P_f(t_1)\right)\frac{1}{(1+\alpha)^{t_1}}\ ;\ C_R(t_1) = \sum_{r=2}^{n_R} C_{rep}(r) P_R^r(t_1)\frac{1}{(1+\alpha)^{t_1}} \qquad [11i]$$

$$C_F(t_1) = C_f\left(P_f(t_1) - P_f(t_0)\right)\frac{1}{(1+\alpha)^{t_1}}\ ;\ C_F(T_s) = C_f\left(P_f(T_s) - P_f(t_1)\right)\frac{1}{(1+\alpha)^{T_s}} \qquad [11ii]$$

where C_{ins}, $C_{rep}(r)$, C_f are respectively the expected inspection cost, the expected repair cost the expected failure cost, and α is the rate of interest. The inspection time t_1 is therefore determined as the optimal solution of the minimization problem:

$$\min_{t_1} C_T(t_1) = \min_{t_1}\left(C_I(t_1) + C_R(t_1) + C_F(t_1) + C_F(T_s)\right) \qquad [12]$$

The time t_0 can be any time after the putting in service of the welded joint. The time t in the models has nevertheless to be adjusted in order to take into account of this delay. The results are given on *Fig.5*. That shows the evolution of the expected inspection, repair, failure and total costs versus the inspection instant, for different detection threshold mean values, ranging from 0.2 mm to 5.0 mm, with coefficients of variation of 10 %, and 50 %. For the interpretation of the results it must not be forgotten that the relative inspection costs are supposed constant: they do not depend on the inspection quality. This is evidently wrong since more elaborated non destructive inspection techniques have generally higher costs than those which detection qualities remain low. *Fig.6* gives the results corresponding to the minima of the total expected cost values for different detection thresholds. *Fig.7a* shows the minimal total costs for different detection thresholds, i.e. its different mean values and different coefficients of variation. The non destructive inspection techniques providing the lowest costs for the adopted strategy and initial data seems to be those for which detection threshold mean values lie between 0.5 and 1.5 mm. *Fig.7b* presents the instants of that first inspection for which the total expected costs are the least. So, for the techniques with detection threshold mean values between 0.5 and 1.5 mm: the corresponding inspection instants lie between 20 and 30 years approximately. Interest rates can significantly change the minimal total expected cost. When the applied interest rate is moved from 4 % to 8 % the minimal total expected costs fall approximately three times. That can prove that it is difficult to justify the construction of more and more expensive bridges in order to obtain ”zero maintenance” except if the maintenance costs are very high or if the applied interest rate value is inferior to 8 %.

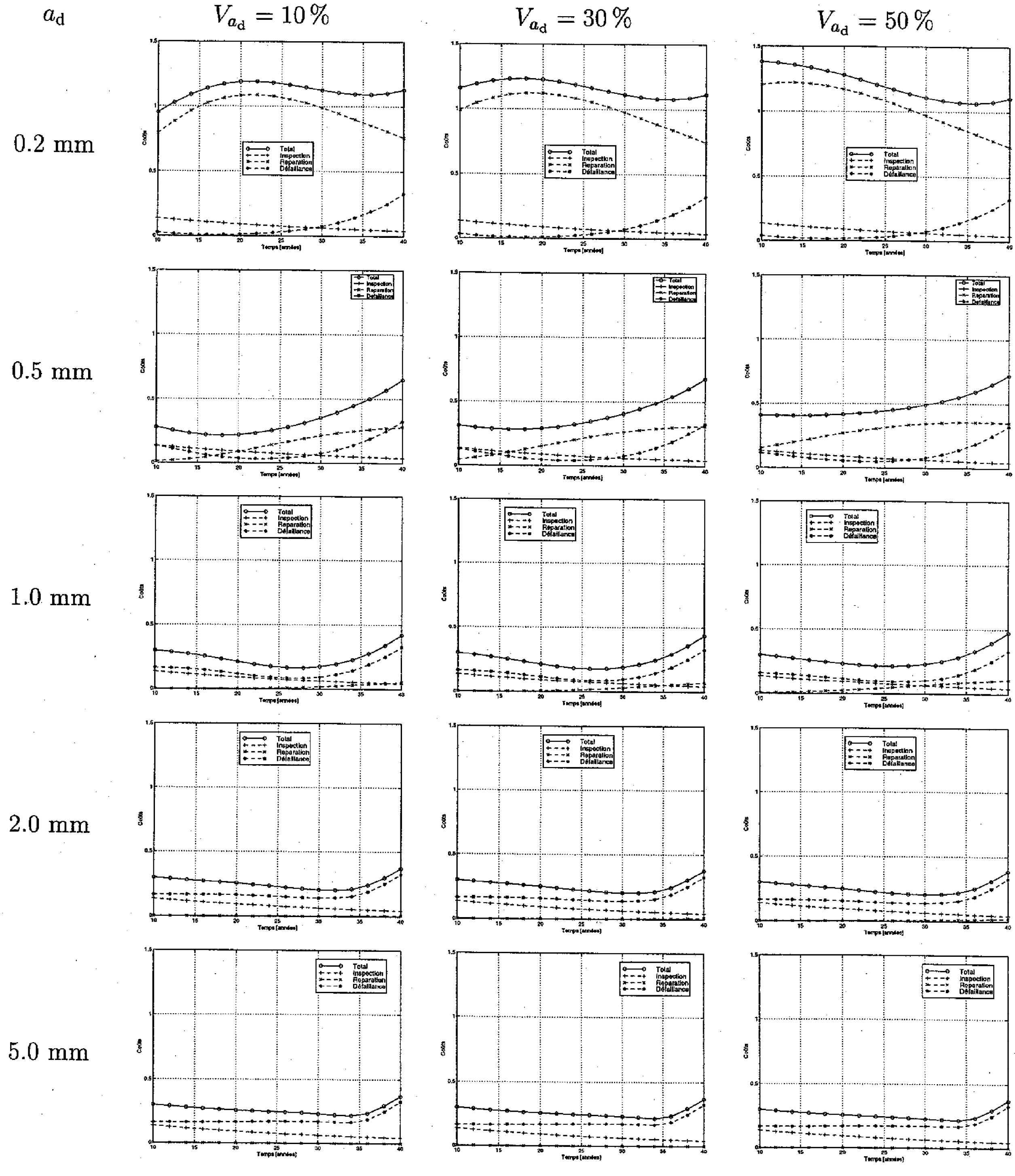


Fig.5 Cost evolution

The influence of the inspection cost is rather insignificant. The repair cost does not considerably affect the total expected costs, except for the small values of the detection threshold. Failure costs include numerous parameters: component, structure, lives ... costs. The total expected costs considerably change with that parameter, especially for high detection threshold mean values. For the lowest values of the failure cost, the minimal total expected costs are found among the more important detection thresholds. With the increase of this failure cost a minimum becomes apparent: it is around 0.5 mm. Inspection instants vary from 35 to 20 years [5].

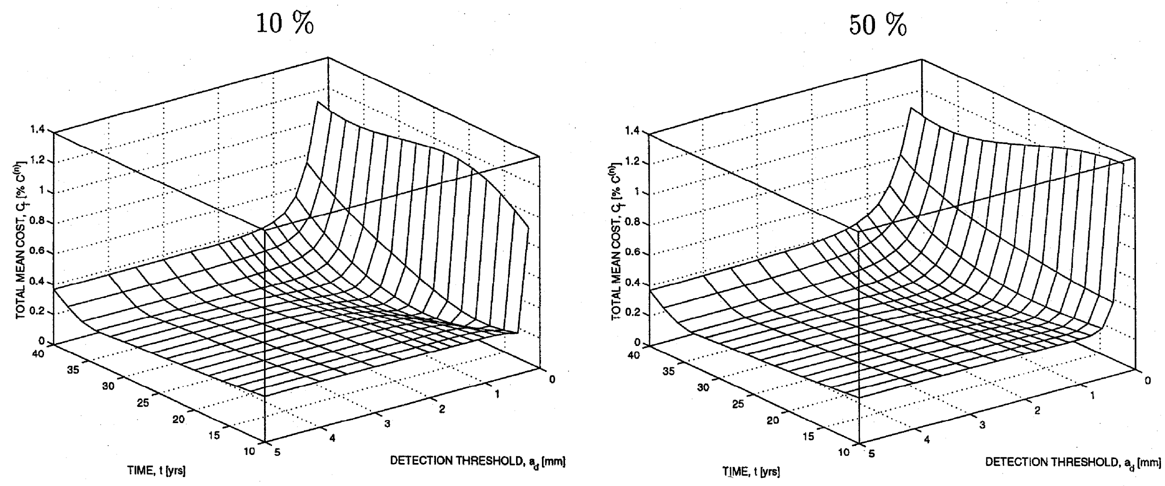
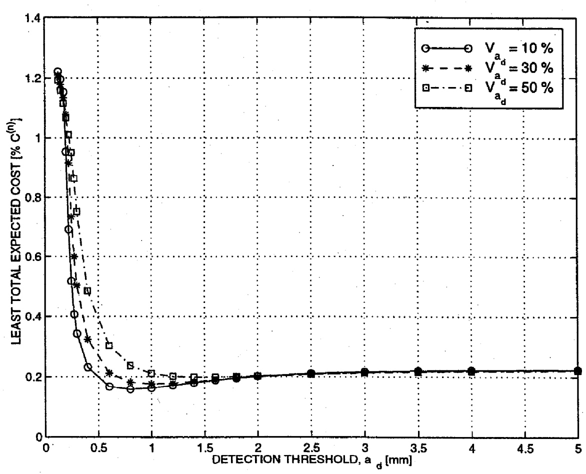


Fig.6 Optimisation results



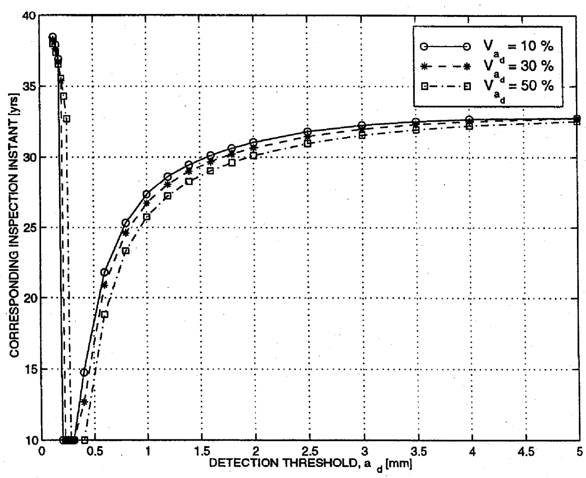


Fig.7a Optimal maintenance costs

Fig.7b Optimal inspection times

4.2 Inspection interval optimization

Here, the problem consists in the determination of an inspection interval Δt which induces a minimal maintenance expected cost during the conventional lifetime T of the component. For this purpose, the number of inspections n is first given, and then the total expected cost is evaluated. The procedure is performed for different number of inspections and the different expected costs are compared; the value n which provides the smallest cost gives to the optimal inspection period. Let us precise that some constraints have to be also fulfilled, as the optimisation problem beneath highlights:

$$\min_{\Delta t} C_T(\Delta t) = \min_{\Delta t}\left[\sum_{i=1}^{n}\left[C_I(\Delta t)+C_R(\Delta t)+C_F(\Delta t)\right]+C_F\left(T_f\right)\right] \qquad [13]$$

under constraints $\beta(T_f) \geq \beta_{\min}$. The problem treated here is not only to assess the optimal intervals between two inspections, but also to take into account different repair strategies. *Strategy 1* correspond to the repair of all cracks while *Strategy 2* leads to only repair cracks over a conventional fixed length. The conventional lifetime T is 60 years. The minimal reliability index in that case is 3.5. *Table 1* provides the optimization results and the optimal calendars for the two strategies. Let us note that strategy 2 (-which authorizes the case of no repair under some specific crack depths) leads to increase the number of inspections while strategy 1 provides less numerous inspections. The number of inspections is increased as soon as the decision of repair is less authoritative.

	Strategy 1					***Strategy 2***				
Number of inspections	**1**	**2**	**3**	**4**	**5**	**1**	**2**	**3**	**4**	**5**
Inspection cost	*0.13*	*0.28*	*0.43*	*0.60*	*0.75*	*0.13*	*0.28*	*0.43*	*0.60*	*0.75*
Repair cost	*1.22*	*2.26*	*2.7*	*2.75*	*2.88*	*0.72*	*0.95*	*1.00*	*1.11*	*1.12*
Failure cost	*2.77*	*0.02*	*0.0001*	*0.0022*	*0.022*	*3.8*	*0.66*	*0.11*	*0.02*	*0.27*
Total cost	*4.12*	*2.56*	*3.14*	*3.34*	*3.65*	*4.66*	*1.89*	*1.6*	*1.73*	*2.14*
Inspection N°1	*30*	*20*	*15*	*12*	*10*	*30*	*20*	*15*	*12*	*10*
Inspection N°2		*40*	*30*	*24*	*20*		*40*	*30*	*24*	*20*
Inspection N°3			*45*	*36*	*30*			*45*	*36*	*30*
Inspection N°4				*48*	*40*				*48*	*40*
Inspection N°5					*50*					*50*
final β	*2.24*	*3.76*	*4.9*	*4.1*	*3.52*	*1.98*	*2.73*	*3.37*	*3.64*	*2.78*

Table 1 Inspection calendars – Strategies 1 & 2

As it can be noticed, whole life costing is strongly dependent on a cost analysis: repair, inspection, failure costs as well as interest rates. For this reason, a cost study will be performed through the MIKTI project.

5. Conclusion

This paper has briefly presented the studies performed within the MIKTI project, WG.5. It has pointed out the research axis but also the required information. The purpose of WG.5 is to improve composite bridge reliability not only by mean of material and design performance, but also by appropriate maintenance actions. For this reason, a risk-based inspection procedure or whole life costing approach, offers an interesting and pertinent procedure for managing risk.

6. References

[1]. MIKTI, *Nouvelles formes de mixité*, National Research Programme, 2000.

[2]. Lukic M., Cremona C., 2000, "Probabilistic assessment of welded joints versus fatigue and fracture", *ASCE Journal of Structural Engineering*, February 2001.

[3]. Cremona C., *Applications de la théorie de la fiabilité à la sécurité d'élément structuraux d'ouvrages d'art*, N°OA17, LCPC Press, France, 1995

[4]. Lukic M., *Evaluation et maintenance probabilistes des assemblages soudés vis-à-vis de la fatigue et de la rupture : Application aux ponts mixtes*, PhD Thesis, ENPC, France,1999

[5]. Cremona C., Lukic M., "Risk-based inspection of welded joints in composite bridges", *IABSE Conf. on Safety, Risk and Reliability*, Malta, 2001.

Probabilistic Modelling of Carbonation Effects

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Keywords: Carbonation depth, concrete cover, measured data, cooling towers, probabilistic models, probability of protection failure, optimisation.

Summary

Extensive measurements are used to propose prior distributions for carbonation depth and concrete cover. It appears that the carbonation depth may be well described by the gamma distribution, the concrete cover by the beta distribution. Probabilistic methods are applied to verify the protection of reinforcement, defined as the carbonation depth being less than the concrete cover. The proposed method of stochastic optimisation indicates that the optimum thickness of the concrete cover is significantly dependent on the cost ratio C_f/C_1 of the malfunction cost C_f to the marginal cost C_1, and on the required design life of a structure. It appears that commonly used concrete covers may not be adequate.

1. Introduction

It is well recognised that carbonation of concrete may jeopardise durability of reinforced concrete structures by diminishing the protection of reinforcement provided by the concrete cover [1, 2, 3, 4] (CEB (1997), Funk and Reinhard (1997), Funk (1996), RILEM (1997)). However, carbonation is dependent on several unknown factors and, consequently, it can hardly be fully described by appropriate theoretical models. A number of new models has been developed recently [1, 4] and further improvement may be expected from computerised modelling [2, 3].

However, as a rule calibration of any theoretical model against actual observations seems to be necessary in order to guarantee a truthful prediction of the carbonation process. Experimental measurements on the carbonation depth under different conditions may be therefore extremely valuable. Moreover, available data on the carbonation depth have always a great scatter although they are obtained from measurements on one structure under the same conditions. Thus, a probabilistic approach is indispensable in order to analyse the carbonation process and its possible unfavourable effects on structural safety.

Extensive measurements of the carbonation depth have been carried out recently by the Klokner Institute of the Czech Technical University in Prague as a part of structural assessment of a number of cooling towers [5, 6, 7, 8]. The aim of this contribution, which is an extension of the previous study [9] written by Holický and Mihashi (1998), is to summarise data available up to now and to propose probabilistic models needed for subsequent probabilistic studies. A review of available data is supplemented by a short probabilistic analysis using recommended probabilistic models.

It appears that an unresolved, nevertheless the key question is the level of acceptable probability of ~~the~~ protection failure when the carbonation depth exceeds the concrete cover within a specified design life of a structure. Obviously an increased thickness of the concrete cover, which would extend the protection of reinforcement, would also lead to an increasing weight of a structure and to its greater overall cost. It is shown that methods of probabilistic optimisation may provide a rational tool for the specification of adequate requirements.

It should be mentioned that a loss of reinforcement protection due to neutralised (carbonated) concrete cover might not necessarily lead to subsequent corrosion of reinforcement and consequent loss of bearing capacity. The corrosion of reinforcement itself is a complex deteriorating process, which is not directly covered here. However, the proposed probabilistic models for the carbonation depth and the concrete cover are obviously needed to analyse the more complex deterioration processes.

2. Experimental data

2.1 Carbonation depth

When investigating cooling towers in the Czech Republic, 31 homogeneous samples having the total of 2197 measurements were gathered during the last ten years (see *Table 1*). Although the obtained results can hardly be generalised, they can be used as representative prior information on the carbonation depth and its distribution.

Table 1: Experimental data on carbonation depth.

Sample	Age [years]	Sample size n	Dep.$d(t)$ [mm]	St. dev. s [mm]	Coef. v_w	Skew. a	Coef. A [mm/y$^{0.2}$]	Coef. K [mm/y$^{0.5}$]
1	16.3	68	9.07	3.31	0.36	0.52	5.19	2.25
2	17.4	55	9.82	2.03	0.21	-0.24	5.55	2.35
3	16.4	38	7.34	2.45	0.33	0.34	4.19	1.81
4	19.7	33	5.70	1.70	0.30	0.14	3.14	1.28
5	18.7	38	4.84	2.09	0.43	0.84	2.69	1.12
6	18.7	36	5.75	2.76	0.48	0.84	3.20	1.33
7	17.7	38	5.08	2.25	0.44	1.00	2.86	1.21
8	14.4	87	11.97	2.84	0.24	0.31	7.02	3.15
9	13.4	79	14.15	3.10	0.22	0.44	8.42	3.87
10	12.4	80	12.16	3.36	0.28	0.91	7.35	3.45
11	11.3	94	16.18	4.14	0.26	1.04	9.96	4.81
12	10.3	100	14.98	3.79	0.25	1.04	9.40	4.67
13	9.3	89	15.40	3.47	0.23	0.77	9.86	5.05
14	21.8	78	5.76	4.25	0.74	1.40	3.11	1.23
15	21.0	79	7.72	3.46	0.45	0.73	4.20	1.68
16	22.1	67	7.55	4.43	0.59	1.52	4.07	1.61
17	16.1	65	6.88	2.13	0.31	0.40	3.95	1.71
18	15.0	51	6.37	2.21	0.35	0.24	3.71	1.64
19	15.8	79	6.14	2.41	0.39	0.52	3.54	1.54
20	15.8	104	9.85	4.50	0.46	0.93	5.67	2.48
21	6.9	84	7.74	2.25	0.29	0.36	5.26	2.95
22	5.9	62	8.23	2.04	0.25	0.44	5.77	3.39
23	4.9	63	9.13	2.17	0.24	0.31	6.64	4.12
24	6.9	62	9.08	3.13	0.34	-0.20	6.17	3.46
25	20.1	72	10.33	4.68	0.45	1.23	5.67	2.30
26	17.9	166	8.95	4.93	0.55	0.90	5.03	2.12
27	19.1	74	7.19	2.71	0.38	0.58	3.99	1.65
28	18.1	60	8.37	3.15	0.38	0.44	4.69	1.97
29	19.9	79	8.73	3.77	0.43	0.67	4.80	1.96
30	19.9	74	11.05	3.75	0.34	0.59	6.08	2.48
31	39.9	43	7.12	2.99	0.42	0.12	3.41	1.13
Sum		2197						
Mean	16.2		8.99	3.10	0.37	0.62	5.31	2.44

In a recent CEB document [1] the following general relationship is considered

$$d(t) = A \; t^{0.5 - n} \tag{1}$$

where $d(t)$ [mm] denotes carbonation depth, t[years] time in years, A [mm/year$^{0.5-n}$] can be understood as a carbonation coefficient and n is a parameter of climatic conditions. In accordance with [1] n is assumed to be 0 in indoor and 0.3 in unsheltered outdoor conditions. Thus, to analyse data in *Table 1* $n = 0.3$ should be applied. Alternatively the following "square root" rule is also considered

$$d(t) = K\ t^{0.5} \tag{2}$$

where K [mm/year$^{0.5}$] is a special value of carbonation coefficient A for $n = 0$. *Table 1* shows both types of the carbonation coefficient A and K.

2.2 Concrete cover

When investigating the cooling towers 27 homogeneous samples of concrete cover having the total of 3349 measurements were gathered during the last ten years (see *Table 2*). Although the obtained results can hardly be generalised, they can be considered as prior information on the distribution of concrete cover under similar conditions. Note that the mean is approximately equal to the nominal value given in design documentation.

2.3 Correlation of carbonation depth and-concrete cover

Table 2 shows also coefficients of correlation of the carbonation depth and the concrete cover. The obtained data, which are of a limited size and not very convincing, indicate that the coefficient of correlation may be expected within the interval from -0.5 up to 0.5. In particular a possible negative dependence is alarming. It would obviously have an unfavourable effect on durability and reliability of structures and, therefore, should be taken into account. Unfortunately, the available measurements do not include all information that would make it possible to analyse all the possible hypotheses of dependence of the carbonation depth on the concrete cover.

3. Probabilistic models

Both the carbonation depth and the concrete cover seem to have a positive skewness α. Several types of the probability distribution have been tested and the most suitable ones are indicated in *Figure 1*.

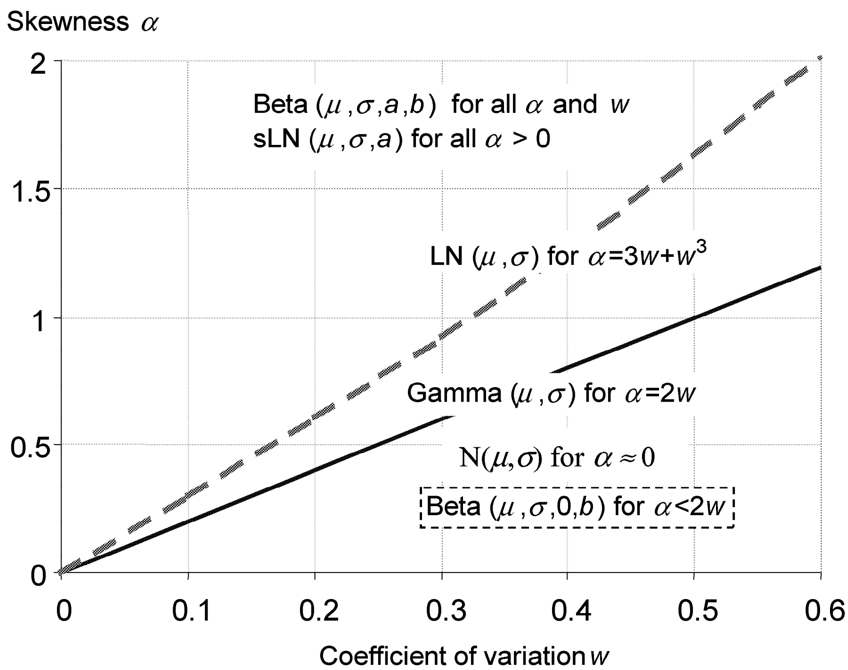

Figure 1. Probability distributions applicable for carbonation depth and concrete cover.

If the skewness is close to zero then the normal distribution, denoted N(μ, σ), or the beta distribution Beta(μ, σ, a, b) may be applied. An obvious disadvantage of the normal distribution is the occurrence of negative values. Then the beta distribution Beta(μ, σ, 0, 2 μ) with the lower bound a = 0 and upper bound b = 2 μ may be useful. The beta distribution with the lower bound a = 0 can be used when α < 2 w (for α = 2 w it becomes the gamma distribution Gamma(μ, σ)).

Table 2: Experimental data on concrete cover.

Sample	Tower	Sample size n	Mean m	Standard deviat. s	Coef. of var. ~~v~~w	Skewness a	Regres. sample n	Carbon. d. vs. cover
1	ECH	64	18.56	6.88	0.37	-0.01	11	0.615
2		117	21.74	4.87	0.22	0.62	14	-0.023
3		97	24.13	7.08	0.29	0.37	10	0.365
4	ED□	64	16.05	4.75	0.30	-0.53	16	0.667
5		92	20.13	10.63	0.53	1.13	19	0.108
6		80	19.29	5.82	0.30	0.55	20	-0.028
7		72	15.63	5.64	0.36	0.06	18	0.352
8	EDU	160	31.20	12.33	0.39	-0.10	59	0.201
9		151	17.94	5.48	0.31	0.59	53	0302
10		173	29.39	9.44	0.32	-0.25	60	-0.282
11		162	18.69	7.90	0.42	0.47	44	-0.091
12		154	24.86	8.02	0.32	0.04	50	0.268
13		169	26.92	10.23	0.38	0.97	56	0.051
14	EPO□	-	-	-	-	-	-	-
15		-	-	-	-	-	-	-
16		157	26.81	11.13	0.42	0.42	36	0.575
17		-	-	-	-	-	-	-
18	EPRU	-	-	-	-	-	-	-
19		206	26.61	6.20	0.23	0.53	65	-0.107
20		184	27.01	6.07	0.22	0.53	80	-0.178
21	ETE	144	39.72	11.62	0.29	0.06	48	0.098
22		150	32.60	11.89	0.36	0.30	50	0.234
23		151	37.74	11.70	0.31	0.21	51	-0.168
24		155	36.19	11.70	0.32	0.12	49	-0.372
25	ETU	106	25.99	8.27	0.32	0.35	27	0.537
26		-	-	-	-	-	-	-
27		108	22.68	7.23	0.32	0.35	25	-0.101
28		76	22.32	5.19	0.23	-0.20	19	0.027
29	EVO	117	24.66	11.20	0.45	0.33	33	-0.055
30	EVOSEP	109	23.70	7.84	0.33	0.14	22	0.168
31	CHEM	131	18.09	7.67	0.42	0.37	31	0.136
	Sum	3,349					966	
	Mean	129		8.34	0.34	0.29	37.15	0.13

The most general type applicable for any α > 0 is the three parameter lognormal distribution denoted sLN(μ, σ, a) with a general lower bound a. This type of distribution was considered in the previous study [9]. A disadvantage of this distribution is the occurrence of negative values. The lognormal distribution with the lower bound at the origin has however a rather high skewness $\alpha = 3w+w^3$ that leads to unrealistic exaggeration of positive deviations.

Available statistical data for the carbonation depth indicate that in average $\alpha = 2\,w$. Thus the gamma distribution Gamma(μ, σ) seems to be the most suitable candidate as a prior distribution for the carbonation coefficient A (*or* K). Note that the mean μ_A = 4 to 6 mm. Both the coefficient of variation w and the skewness α seem to be time dependent quantities that can be expressed as

$$w(t) = \alpha(t)/2 = 0{,}1\; t^{0{,}5} \tag{3}$$

Available statistical data for the concrete cover also have a positive skewness, which is equal to the coefficient of variation. $\alpha \approx w$. Two types of distribution are therefore applicable: the three parameter lognormal or the beta distribution. As the concrete cover is obviously a both side limited random variable, the Beta distribution Beta(μ, σ, 0, b) with

$$a = 0,\; b = \mu\,(1+w(2+2w))/(2w-\alpha) \approx \mu\,(3 + w^2) \approx 3\,\mu. \tag{4}$$

may be applied as the most suitable prior distribution.

4. Probability of protection failure

The probability of protection failure $p_f(t,c)$ is defined in a traditional way by the integral

$$p_f(t,c) = \int_{-\infty}^{\infty} \varphi_d(t,x)\Phi_c(x)\mathrm{d}x \tag{5}$$

where $\varphi_d(t,x)$ is the probability density function of the carbonation depth $d(t)$ and $\Phi_c(x)$ is the distribution function of the concrete cover c, x is an integration variable.

Figure 2 shows the probability of protection failure $p_f(t,c)$ given by equation (5) for selected input data indicated in equations (1) and (3). Alternatively also the lognormal distribution for both the basic variables is considered. Obviously the difference between the combination of beta and gamma distributions and the lognormal distributions is not significant. However, the effect of the coefficient of correlation r between the concrete cover and the carbonation depth (see *Figure 2*) seems to be more important than the applied type of distribution.

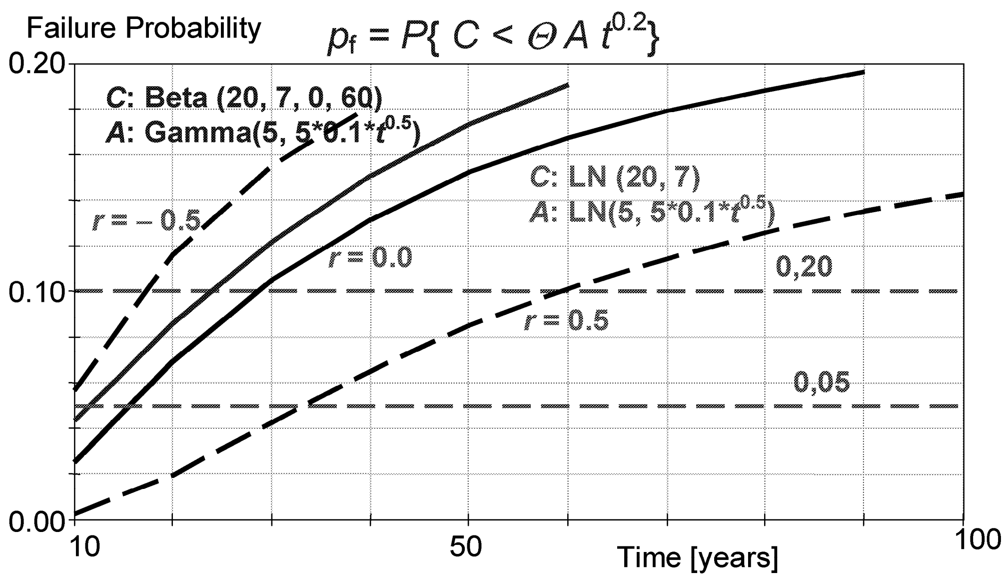

Figure 2. Probability of protection failure.

The probability $p_f(t,c)$ may be a useful reliability measure, however, an important question of the required value for this probability is still open. It appears, however, that the method of stochastic optimisation using the total cost as an objective function may provide some additional information.

5. Optimisation

The objective function is expressed in terms of costs considering the total value, the cost of the concrete cover and the cost of the protection failure including all expenses due to repair and necessary strengthening. The total cost C_{tot} is thus expressed as a sum of the initial (installation) cost C_0, marginal cost cC_1, where c denotes the concrete cover and C_1 the cost for its unit thickness, and of an expected cost of protection failure given as $p_f(t,c)C_f$, where $p_f(t,c)$ is the probability of protection failure corresponding to the condition (1) and C_f is the cost due to the protection failure. The expected total cost C_{tot} may be then expressed as

$$C_{tot} = C_0 + c\, C_1 + p_f(t,c)\, C_f \tag{6}$$

It is assumed that the initial cost C_0 and the failure cost C_f are independent of c. Further, the standard deviation s_c is assumed to be almost independent of c and, therefore, the derivative of the probability $\partial p_f(t,c)/\,\partial c$ can be approximated by a derivative with respect to the mean $\partial p(t,c)/\,\partial \mu_c$. Then the necessary condition for the minimum total cost is

$$C_1 / C_f = -\frac{\partial\, p_f(t,c)}{\partial\, \mu_c} \tag{7}$$

The partial (comparative) cost $c + p_f(t,c)\, C_f / C_1$ for a design life of 50 years and recommended probabilistic models for the carbonation depth and the concrete cover are shown in *Figure 3*. The results presented in *Figure 3* including the minimum given by partial derivative (7) were calculated using the MATHCAD programme (called CARBON) and the software COMREL (1999) [10]. Assuming $C_1 = 1$ it follows from *Figure 3* that for $C_f = 100$ (which is a rather low value) the optimum cover (indicated by a dot) is about 24 mm. For $C_f = 500$ the optimum cover is 42 mm, for $C_f = 1000$ the optimum cover is 50 mm.

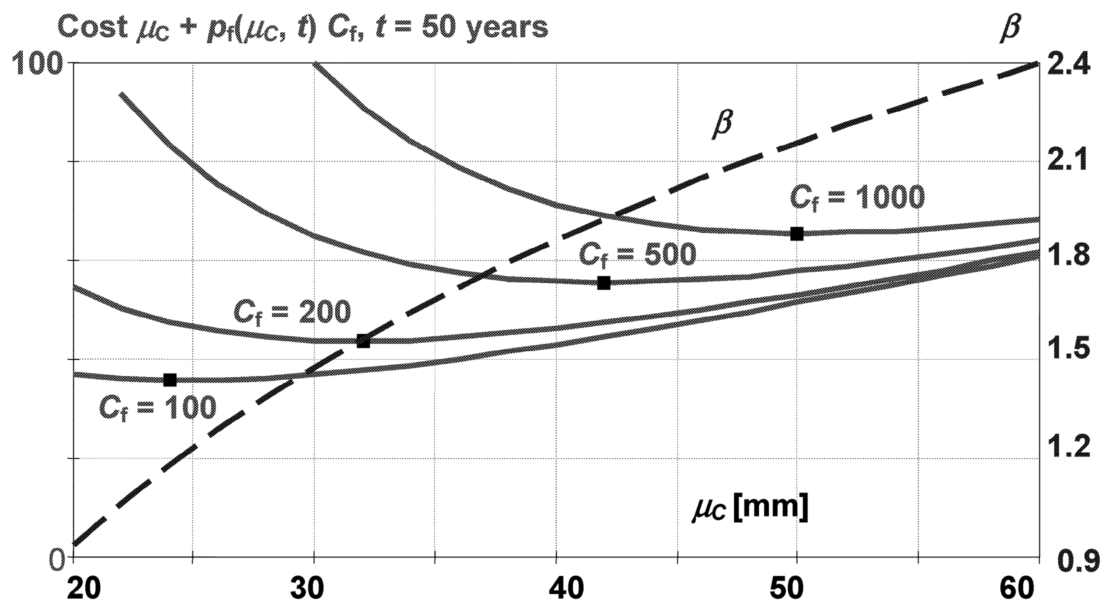

Figure 3. The total cost.

Note, that these results are valid for a design life of 50 years and the time invariant coefficient of variation $v_d = 0.1\ t^{0.5}$. However, this prediction was derived from measurements on structures being in average 16 years old, and thus may not be adequate (may be rather high) for the design life of 50 years considered in *Figure 3*. It should be also noted that the type of assumed probabilistic models might affect the optimum cover (see for example the results presented for lognormal distribution of both the carbonation depth and concrete cover in [9]).

In addition to the cost, *Figure 3* also shows the reliability index β as a function of the mean concrete cover (dashed line). This curve makes it possible to find the reliability index β and the probability $p_f(50,c)$ (t = 50 years) for a given optimum concrete cover corresponding to the cost C_f. For example for $C_f = 1000$ the optimum μ_c = 50 mm, the reliability index $\beta \approx 2.15$ and the probability $p_f(50,50) = 0.016$. In general, with an increasing cost C_f the optimum β increases and the optimum probability $p_f(t,c)$ decreases.

6. Discussion

Considering the experimental data obtained when investigating cooling towers it appears that the carbonation depth of unprotected concrete can be well described by the Gamma distribution having the time dependent coefficient of variation $0.1 \times t^{0.5}$ (and the skewness about a double) equal to 0.35 (skewness 0.70). An average mean value of the coefficient A is about 5.3, the average of the coefficient K is about 2.4. However, these coefficients should be checked by measurements.

The concrete cover of cooling towers seems to be well described by Beta distribution with the coefficient of variation of about 0.35. The skewness has in average the same value. If the lower bound of the Beta distribution is zero, then the upper bound b is approximately $3 \times \mu$, where μ is the mean.

There is an indication that the carbonation depth may be dependent on the concrete cover. This is an alarming finding, which may have a significant effect on the prediction of the carbonation effect in time. Unfortunately the available data do not include all information necessary for a comprehensive analysis of all the possible hypotheses and an unambiguous identification of the reasons for this phenomena. Note that in particular a negative dependence may have an unfavourable effect on the durability of the reinforced concrete structure.

All the above findings should be, however, interpreted as prior information only. Practical applications should be supplemented by additional measurements that should be used for updating the prior information. In addition further experimental data and appropriate theoretical models for the carbonation process including a description of the wetting and drying effects in outdoor conditions and, for more realistic reliability conditions which would take into account expected corrosion of reinforcement are obviously needed. Also expected costs, including the marginal cost and the cost due to protection failure, should be thoroughly investigated in order to formulate appropriate objective functions and to develop required operational design rules.

7. Concluding remarks

The following conclusions may be drawn from the presented study:

- Gamma distribution having the time dependent coefficient of variation $0.1\ t^{0.5}$, where t denotes time in years, may be assumed as a prior distribution of the carbonation depth; the mean is strongly dependent on local conditions.
- Beta distribution having the coefficient of variation approximately equal to the skewness, the lower bound at zero and the upper bound $3\ \mu$, where μ denotes the mean, may be assumed as a prior distribution of the concrete cover.
- The optimum thickness of the concrete cover of reinforced concrete structures is significantly dependent on both the cost ratio C_f / C_1 and the required design life.
- Commonly used concrete covers of reinforced concrete structures correspond to the low cost ratios C_f / C_1 and seem to be, in actual conditions, uneconomical.
- For the required design life of 50 years and the cost ratio $C_f / C_1 = 100$, the optimum cover is about 25 mm. If the cost ratio $C_f / C_1 = 1000$, then the optimum thickness is about 50 mm.

Further experimental data and appropriate theoretical models for the carbonation process including a description of the wetting and drying effects in outdoor conditions and, for more realistic conditions, taking into account the expected corrosion of reinforcement, are needed.

Further studies on a model for the total expected cost and its components including the marginal cost and the costs due to protection failure are needed in order to formulate more realistic objective functions.

8. Acknowledgement

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9. References

[1] CEB, Bulletin d’ Information No. 238 (1997) *New Approach to Durability Design. An example for carbonation induced corrosion.*

[2] Funk G. (1996) Contec ES, *Computer Aided Consulting für Betonober-flächenschäden*, Beuth, Berlin.

[3] Funk G., Reinhard H.-W. (1997) Realistic Computation for the Depth of Carbonation by Taking Material Parameters and Weathering Exposure into Consideration, *Betonwerk+fertigteil-Technik*, heft 3/1997, pp. 83-88.

[4] RILEM REPORT 14 (1997) *Durability Design of Concrete Structures*, Edited by Sarja A. and Vesikari E., E&FN SPON, London.

[5] Bohdanecký V., Fidranský P., Holický M. (1998) Statistical Data on Carbonation Depth. In *New Requirements for Materials and Structures*, The Czech Technical University in Prague, Faculty of Civil Engineering, pp. 39-44.

[6] Holický M. (1995) Reliability measures for deteriorating structures. In: *Application of Statistics and Probability*, ICASP7, Lemaire M, Favre J.-L. & Mebarki A. (eds), Balkema, Rotterdam, pp. 957-962.

[7] Holický M., Hrabánek M., Vorlí□ek M. (1994) Fuzzy Probabilistic Analysis of Carbonation Effects. In: *New Requirements for Structures and their Reliability*, Czech Technical University in Prague, Faculty of Civil Engineering, pp. 121-126.

[8] Holický M., Mihashi H. (1998) The Optimum Thickness of Concrete Cover. In: P. Schwesinger and F.H.Wittmann (ed.), *Durable Reinforced Concrete Structures.* AEDIFICATIO Publishers, Frieburg, pp. 141-150.

[9] Holický M., Mihashi H. (2000) Stochastic Optimisation of Concrete Cover Exposed to Carbonation. In: R.E. Melchers & Mark G. Steward (ed.), *Application of Statistics and Probability.* A. A. Balkema Rotterdam 2000, pp. 279-284.

[10] COMREL 7.10 (1999) *Component of A Structural Reliability Analysis System STRUREL*, RCP Munich.

Risk versus Reliability Based Seismic Assessment of Individual Buildings

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Keywords: seismic evaluation criterion, reliability, risk

Summary

While the awareness of the seismic risk has grown considerably over the last decades in many countries, only a few buildings have been designed to withstand seismic actions. The vulnerability of most buildings is therefore unknown.

Many Swiss communities now want to act against the seismic risk affecting their buildings. For that reason, the seismically vulnerable structures must be identified. To determine if a structure is vulnerable, a detailed seismic evaluation is necessary. As a part of the seismic evaluation, an acceptance criterion must be selected. This contribution discusses the suitability of the risk criterion versus the reliability criterion for detailed seismic evaluations of individual structures.

1. Introduction: controlling the seismic risk of group of buildings

For regions with medium to low seismicity like Switzerland, the frequency of damaging earthquakes is rather low. Nevertheless, the occurrence of strong earthquakes is probable and their consequences will be serious. Over the last few decades, the awareness of seismic risk has grown considerably. Forty years ago Swiss building codes contained no seismic regulations. They were introduced in 1970 only and replaced by more demanding and modern regulations in 1989. Since most existing buildings were built before 1970 (1989), these buildings have not been designed to resist seismic actions and their seismic performance is unknown. Therefore, the number of buildings in Switzerland that are seismically vulnerable has the potential to be very large.

Recently, many Swiss communities (several cantons and the confederation) have shown the intent to control seismic risk [1]. This includes the investigation of the seismic vulnerability of their buildings and other important structures. The seismically vulnerable structures are then retrofitted.

When investigating the seismic vulnerability of a group of buildings in order to identify the structures that require retrofitting, the chosen approach usually includes the following steps:

1. rapid visual screenings to identify buildings that are probably sufficiently performant in a seismic event.

2. detailed seismic evaluation of buildings that according to step 1 are possibly seismically vulnerable and represent a relatively high seismic risk.

The detailed seismic evaluation will allow for accurate distinction between buildings that will perform sufficiently and those that will probably perform insufficiently during an earthquake. A detailed seismic evaluation is necessary to decide if the building must be retrofitted, and its results are relevant to the design of eventual retrofit measures.

3. design and realization of retrofit measures for buildings with insufficient seismic performance, according to results from step 2.

In the approach to control seismic risk, as given above, it is generally agreed that in step 1, "risk" should be the criterion for seismic evaluations [2]. It depends on the specific interests of the building owner if the risk considered is the risk of loss of life and / or the risk of financial losses.

Why is the risk criterion chosen ?

When countermeasures are decided, (financial) resources must be allocated. These are generally limited and the allocation of resources should be done in the most efficient way. For the case of risk to human life, the following citation illustrates drastically the need for efficient allocation of resources: "... if our priorities in managing risks are not cost-effective, we are, in effect, killing people whose premature deaths could be prevented ..." [3]. Therefore, it is widely recognized that hazard mitigation should be realized under the guidance of risk criterion.

Subsequently to evaluation step 1, based on risk, the following evaluation steps may be conducted using the same acceptance criterion. That means, that for an individual structure, the seismic risk must be assessed and judged. Then, the following situation is likely to occur: the seismic risk of a structure is evaluated and judged to be acceptable, while, at the same time, the structure does not fulfil code requirements (reliability criterion). Questions will arise as to why this structure does not have to adhere to the safety standards given by the code (i.e. reliability criterion) unlike a new structure that must take into consideration these standards. In fact, the code implies a safety standard that is simple to communicate and that is widely recognized. For example, the code requirements are in Switzerland the standard of reference when there is a lawsuit against the design engineer of a structure.

The selection of an acceptance criteria and of the level of accepted risk / reliability is a difficult task. This is especially the case when dealing with hazards that have little probability of occurrence and serious damage potential. The challenge to establish an adequate acceptance criterion invloves many aspects. Some of them are the lack of experience with infrequent, but catastrophic events [1], the aversion to life-threatening hazards and practical aspects of the application of the acceptance criterion.

In the selection of an acceptance criterion and in the level of the accepted risk or reliability, the non-technical (psychological) aspect is very important.[2]

This contribution aims at discussing the advantages and disadvantages of risk vs. reliability criteria in the context of detailed seismic evaluations of individual structures. Theoretical and practical considerations are considered.

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[1] As an example, the statement of an estate agent after the Turkish earthquake 1999 may be given: "... people are no longer interested in the colour [the house / apartments] are painted or whether or not they have parquet floors. All they care about is if the columns and the beams are strong and if the ground beneath the building is geologically suitable for the structure. It's very unfortunate that people had to experience a disaster of this magnitude to find out what the priorities should be in selecting a residence." - Turkish Daily News, Sept. 10, 1999.

[2] For example, the discussions on nuclear power plant safety can be given. In such discussions not only experts but all interested people are involved. It is characteristic by discussions where the results of engineering risk studies are contested. Results of computations convince especially those who made the computations, but not necessarily those who are exposed to a danger.

2. General discussion of risk and reliability criterion

2.1 Risk

2.1.1 What is risk ?

Many scientists and engineers dealing with natural hazards use the following definition [4]:

$$\text{Risk} = \sum_{\text{all events}} (\text{probability of occurence}) \cdot (\text{vulnerability}) \cdot (\text{potential damage}) \qquad (1)$$

Simplified, one can say that risk is the probability of occurrence of an event multiplied with its (negative) consequences.

The term "risk" is not only used in the sense given by the equation above. Insurance and risk expert M. Haller defines risk as "the sum of all possibilities that expectations of a system are not fulfilled because of disturbing processes" [5]. The expectation of a system can be the expected production of a company and the disturbing processes might be any "unexpected" event, for example a production interruption by a short circuit in the electric system of a production unit. This rather wide description of the term risk is useful when dealing with risks of business activities.

The given definitions show, first, that scientists, managers or decision makers do not necessarily stick to the same definition of risk. In the first definition the quantitative aspect prevails while in the second definition a quantification is not explicitly included. One can imagine that in a public discussion the variable perception of the term *risk* is a cause for misunderstandings.

2.1.2 How big is the acceptable risk ?

The accepted risk varies enormously and depends on many factors. Two of them are:

- possibility of benefits by taking risks
- risk imposed / risk accepted voluntarily

Seismic risk offers no possibilities for benefits and the risk is imposed, hence the risk acceptance is relatively low.

When risk acceptance criteria are fixed, sometimes risk aversion is introduced, as for example in [6], a guide to evaluate the risk of handling hazardous substances. Risk aversion is an expression of the feeling that 100 accidents with one fatality each are not to be treated equal to one accident causing 100 fatalities. This aspect is of particular importance when dealing with the risk of infrequent, but catastrophic events—such as strong earthquakes. How the aversion is weighed in an attempt at quantification is a matter of (individual) perception.

The assessment of the acceptance level of the seismic risk in Switzerland is a difficult task, mainly because of the lack of experience with severe earthquakes. To the author’s knowledge, no thorough discussion of the acceptance level for seismic risk has been conducted. Consequently, it is difficult at present to give a generally accepted answer.

Perhaps the seismic acceptance level will be estimated on the basis of the known risk acceptance of other hazards. Once the acceptable risk has been determined based on risks due to other hazards, there is a need for an un-biased estimation of the seismic risk. Thus far, probabilistic seismic hazard studies have only been conducted in terms of seismic intensities. A probabilistic description of strong motion parameters (accelerations, displacements) is not yet available in most countries.

Consequently, for a detailed seismic evaluation based on engineering analyses of a building, it will be difficult to get an un-biased seismic risk estimation.

2.2 Reliability

2.2.1 What is reliability ?

The term reliability is—unlike "risk"—mainly used by the engineering community. For structural engineers, reliability is equal to unity minus the probability of failure of a structure during its lifetime. Reliability analyses usually take advantage of the framework of a code that specifies the limit state function and the accepted probability of failure. Not included in the definition of reliability are the consequences in the case of failure, and not included are the consequences of a "near accident". In the case of an earthquake, this means that severe damages are accepted, as long as the structural capacity is not exceeded. However, modern building codes take into account the building function by controlling the seismic damage of very important buildings (such as, hospitals, firestations, schools etc.)

Many reliability analyses take advantage of the relatively precise framework of codes and use (mathematically) sophisticated solutions. This results in reliability analyses that distinguish precisely between structures fulfilling the acceptance criterion and structures that do not.

2.2.2 What is the minimum reliability ?

Building codes (or other standards) define a reliability level that is widely accepted. When a structure performs as well or better than required by the code, people are generally convinced that this structure is "safe". The code forms a kind of written acceptance criterion.

If the minimum reliability is not given explicitly, then it must be determined for each case with probabilistic analyses. The reliability required by codes for a structure to withstand an earthquake is probably not equal to the reliability required to withstand live and dead loads.

3. Application of risk and reliability criterion in detailed seismic evaluations of buildings

To study practical aspects of evaluations based on risk and evaluations based on reliability, the two evaluation criteria were applied to example buildings. These buildings were modelled and seismically analysed using non-linear static and dynamic analysis methods. The applied methods allow for the realistic estimation of increasing damages with increasing earthquake intensity up to the ultimate response [7].

In the following, the results of the analyses are given along with a short description of the computation, with the intent of conveying the practical aspects of the problem.

3.1 Reliability based seismic evaluation

In the following, a short description of the updating of the seismic structural response will be given. No updating of the seismic actions was conducted. While for the updating of the structural response measures on laboratory tests are available, little information on strong motions (in regions of low to medium seismicity) is available.

To conduct a reliability analysis within the given framework of the Swiss building code, it was necessary to study the requirements of the code dealing with seismic design and evaluation [8, 9, 10]. It was concluded, that the code requires a minimum seismic capacity given by the product of the resistance against horizontal seismic actions and the energy dissipation capacity under these actions. A "seismic fitness parameter" was defined that describes the building's capacity to resist earthquakes:

$$sak = S_{a,ultimate} \cdot K \quad [m/s^2] \tag{2}$$

sak	seismic fitness parameter
$S_{a,ultimate}$	spectral acceleration of the building's equivalent SDOF system at ultimate response
K	factor to consider the energy dissipation capacity

The seismic fitness parameter was computed for the building models. And, using Monte-Carlo simulations, the distribution function of the seismic fitness parameter was computed by varying the structural properties. As the building models are formulated in a macro-model approach, the varied input parameters are the properties of the non-linear springs of the macro models. To obtain varying spring properties, their mean properties were multiplied with a random number following a lognormal distribution. The coefficients of variation for spring properties were obtained as a result from calibration of the macro models. They are given in table 1.

structural element property at ultimate response	coefficient of variation	
	force	deformation
RC wall flexural response	0.05	0.20
RC wall shear response	0.20	0.10

Table 1 Coefficients of variations to generate random numbers used for the Monte-Carlo simulations

The resulting cumulative distribution function for the seismic fitness parameter, sak, for the building model KJA, is represented in Figure 1. The design value $(sak)_d$ required for this building is obtained from the design code by considering the seismic zone and the fundamental frequency of the building: $(sak)_d = 2.0$ [m/s^2]. From the design value $(sak)_d = 2.0$ and the cumulative distribution function, a probability of failure of 13% (or a reliability of 87%) is calculated. The required reliability is 84% for the event of an earthquake with a return period of 400 years. It can be concluded that the building model KJA fulfils the safety requirements of the Swiss building code.

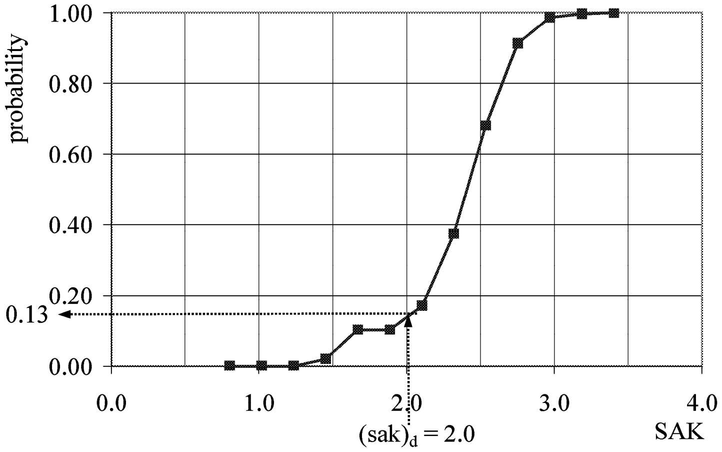

Figure 1 *Cumulative distribution function of "seismic fitness parameter" sak for the building model KJA.*

3.2 Risk based seismic evaluation

How can seismic risk be computed ? For this analysis, the risk definition given by formula (1) will be applied. The risk analysis conducted comprises the following steps:

For each earthquake intensity (MSK), the probability of occurrence (Figure 2) and a representative response spectrum are determined. The representative response spectrum defines the seismic actions needed to compute the seismic building response. The same building models were used as for the reliability analysis described above. The building response is computed in terms of limit states, as for example the "ultimate limit state". The (economical) losses due to building damages are computed based on a damage function. The damage function indicates the relation between the structural limit states and the expected overall damages (Figure 3). The probability of occurrence of the earthquake with the given intensity multiplied with the computed damage to the building yields the seismic risk. The entire seismic risk is computed as the sum of the risk contributions of all earthquake intensities.

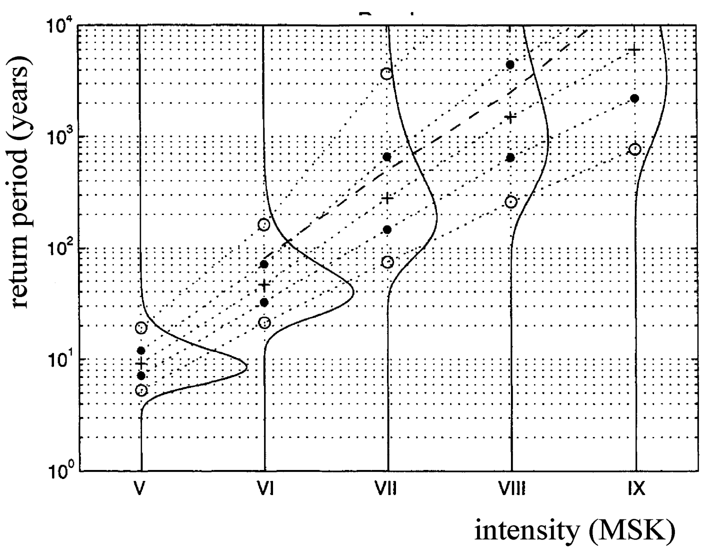

Figure 2 Probability of occurrence (return periods) for earthquakes of a given intensity in the town of Basel according to [11]. The hollow circles indicate the 5% and 95%-fractiles, the solid circles give the 25% and 75%-fractiles, and the crosses the 50%-fractiles.

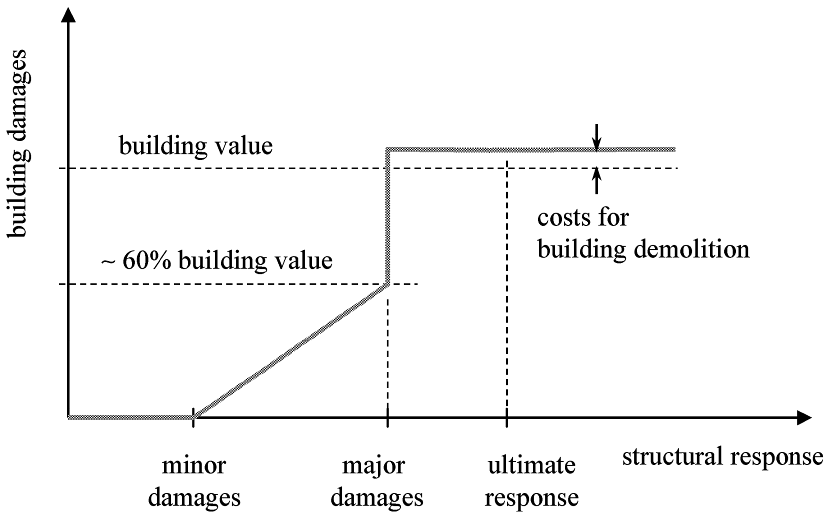

Figure 3 Expected overall damages in function of the building's seismic structural response

The described discrete procedure for the computation of the seismic risk was conducted three times for each building using three fractile values of the probability of occurrence of an earthquake. This yielded three different risk levels (Table 2).

The risk (in terms of financial losses per year as a fraction of the building value) given in Table 2 shows considerable variations, only due to the choice of the fractile value of the probability of occurrence of an earthquake. Other parameters with considerable uncertainties (that were not considered here) include the damage function, and the determination of the response spectrum.

It is clear from the results above that an indication of the seismic risk (of an individual structure) without a detailed description of the assumptions made (fractile values, damage function, etc.) is not very meaningful.

building	risk of damage to buildings [fraction of the building value per year]		
	hazard according to [11]		
	5%-fractile of probability of occurrence	50%-fractile of probability of occurrence	95%-fractile of probability of occurrence
VID	$1{\cdot}10^{-7}$	$1{\cdot}10^{-4}$	$1{\cdot}10^{-3}$
KJA	$5{\cdot}10^{-4}$	$7{\cdot}10^{-3}$	$2.5{\cdot}10^{-2}$

Table 2 Seismic risk analysis using different fractile values of probability of occurrence of the earthquakes

4. Selection of a seismic evaluation criterion

An evaluation criterion that is a basis for retrofit decisions should, according the author's opinion, fulfil the following requirements:

- the level of the accepted risk / minimum reliability should be widely recognized
- understandable not only by specialists, but by all people concerned
- the same evaluation conclusions should be reached regardless of the person performing the evaluation

Do risk and reliability criterion fulfil the requirements given above ?

The quantitative assessment of an accepted risk or reliability level of infrequent, but catastrophic events can not be established from experience, but needs often abstract reflections. Due to the lack of experience, the risk of such events is difficult to assess by "the man from the street" (see footnote 1 on page 2).

The same lack of experience prevails when an individual person must determine an acceptable reliability level. It is not common for one to decide if the accepted probability of failure should be 10^{-4} or 10^{-5}. Usually, not even building codes do indicate these types of accepted probabilities of failure. But, in construction, decisions on safety & risk always have had to be taken. Building codes that have been developed over several decades, from edition to edition, can be considered as a kind of memory of the experiences on construction sites and with the structural safety of buildings. Therefore, a building code is a document that indicates the minimum reliability level or the accepted probability of failure that is accepted widely.

The seismic regulations in Swiss building codes are relatively new. It is true that there are no local and recent experiences with seismically loaded structures in Switzerland, but the regulations have been developed taking into account experiences in other countries [12].

The evaluation criterion should be understandable for everyone. When looking at the definitions, both evaluation criteria discussed can be understood quite easily. Everyone who invests money in the stock markets knows about risk. And the reference "building code" is a standard that is recognized generally.

The third question posed above: will two appropriately educated persons who analyse the seismic risk / reliability draw the same conclusions for a given structure ? - If there are detailed guidelines for the application of a criterion, this requirement can be fulfilled more easily. For the reliability analysis, the seismic regulations in building codes can be used as guidelines. The analysis of the seismic risk can - at present - not take advantage of such regulations. From the application example above, it can be concluded that seismic risk analysis can not be conducted (at present) without some assumptions of the analyst. The results of the risk analysis depend heavily on such assumptions. This is illustrated by the example of section 3.

For the selection of a criterion for detailed seismic evaluations, it can be concluded that the risk criterion is - theoretically - the more appealing option. However, the discussion above shows that from a practical point of view, the reliability criterion offers important advantages.

5. References

1. SGEB, 2000 (Swiss Society for Earthquake Engineering and Structural Dynamics); *Erdbebenvorsorge in der Schweiz, Massnahmen bei neuen und bestehenden Bauwerken,* Referate der SGEB-Tagung vom 7./8. September 2000, ETH Zürich, Dokumentation SIA D 0162
2. Bürge M., Kölz E., Neujahr M., Schneider J. , 1997; *Festlegung von Handlungsprioritäten für die Erdbebensicherung bestehender Bauten,* Referat an der D-A-CH-Tagung vom 25./26.9.1997 in Zürich, SIA-Dokumentation D 0145
3. Okrent D., 1980; *Comment on Societal Risk,* Science, Vol. 208, April 1980
4. UNDRO, 1979; *Natural Disasters and Vulnerability Analysis: Report of Expert Group Meeting,* Office of United Nations Disaster Relief Co-ordinator (UNDRO), Palais des Nations, CH-1211 Geneva 10, Switzerland
5. Haller M., 1999; *Risk-Management,* ETH post-graduate studies "Risk and Safety of technical systems", Module WW1, Institute für Versicherungswirtschaft, Universität St. Gallen
6. Bundesamt für Umwelt, Wald und Landschaft (BUWAL), 1991; *Handbuch I zur Störfallverordnung StFV,* Bern, Juni 1991
7. Peter K., 2000; *Erdbeben-Überprüfung bestehender Stahlbeton-Gebäude,* Thesis EPFL no 2285
8. Schweizerischer Ingenieur- und Architekten-Verein (SIA), 1989; *Einwirkungen auf Tragwerke, Norm 160,* Ausgabe 1989
9. Schweizerischer Ingenieur- und Architekten-Verein (SIA), 1989; *Les prescriptions parasismiques de la norme SIA 160,* Documentation D045
10. Schweizerischer Ingenieur- und Architekten-Verein (SIA), 1994; *Beurteilung der Tragsicherheit bestehender Bauwerke,* Richtlinie 462, Ausgabe 1994
11. Rüttener E., 1995; *Earthquake Hazard Evaluation for Switzerland,* Dissertation Nr. 11048, ETH Zürich
12. Bachmann H., 1986; *The Philosophy of the Swiss Earthquake Regulations,* 8th World Conference on Earthquake Engineering, Lisboa, Portugal

Integrated Approach for RBI of Offshore Installations

Jean Goyet, Bureau Veritas (France)

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1. Introduction

The present paper presents and describes the Bureau Veritas approach to Risk Based Inspection planning for offshore production facilities.

The approach presented rests on the expertise and experience gained by Bureau Veritas and its personnel as a result of projects conducted for various clients in the petrochemical industry offshore and onshore.

Furthermore Bureau Veritas has conducted and participated in a large number of national and international research and development activities relating to Risk Based Inspection Planning including :

- ICON (**I**nter**C**alibration of **O**ffshore **N**DT)
- SHIPREL (RELiability methods for SHIP structural design)
- RASOS (**R**eliability **A**nalysis **S**ystem for **O**ffshore **S**tructures)
- RACH (**R**eliability **A**ssessment for **C**ontainers of **H**azardous material)
- IMREL (**I**nspection and **M**aintenance using **REL**iability methods)
- FMD (guidance on the use of Flooded Member detection for assuring the integrity of offshore platform substructure)

2. Objective of RBI

Engineering systems such as offshore structures, bridges, ship hulls, pipelines and process systems are ideally designed to ensure an economical operation throughout the anticipated service life in compliance with given requirements and acceptance criteria. Such acceptance criteria are typically related to the safety of personnel and risk to environment.

Deterioration processes such as fatigue crack growth and corrosion will always be present to some degree and depending on the adapted design philosophy in terms of degradation allowance and protective measures the deterioration processes may reduce the performance of the system beyond what is acceptable.

In order to ensure that the given acceptance criteria are fulfilled throughout the service life of the engineering systems it may thus be necessary to control the development of deterioration and if required to install corrective maintenance measures. In usual practical applications inspection is the most relevant and effective means of deterioration control.

The objectives of the inspection are thus to :

- Ensure that the risks to personnel arising from consequences of loss of pressure containment or structural failure are as ***low as reasonably practicable.***
- Ensure that the risks to the environment arising from consequences of leakage of hydrocarbons and chemicals are maintained below given specified limits during the lifetime of the installation
- Ensure that the physical condition of the installation remains within design limits that will allow continued safe operation in accordance with the requirements of the relevant legislation during the lifetime of the installation.
- Ensure that the target production availability is maintained or exceeded for the design life of the installation.

Planning of inspections concerns the identification of ***what*** to inspect, ***how*** to inspect***, where*** to inspect and ***how often*** to inspect.

Even though inspections may be used as an effective means for controlling the degradation of the considered engineering system and thus imply a potential benefit they may also have considerable impact on the operation of the system and other economical consequences themselves. For this reason it is necessary to plan the inspections such that a balance is achieved between the expected benefit of the inspections and the corresponding economical consequences implied by the inspections themselves. In inspection planning problems decisions must be made in regard to

- when to perform inspections (this decision regards the number and times of the future inspections).
- where to perform inspections (this decision regards the specific locations and extent of the planned inspections)
- how to perform inspections (this decision regards the inspections methods to apply at the different inspections at the different locations)
- what actions to take based on the results of the inspections (this regards the remedial actions to take on the basis of the results of the inspections)

Finally Risk Based Inspection plans should be performed in general compliance with the rules specified by the relevant certifying society for the specific installation.

3. System description of offshore installations

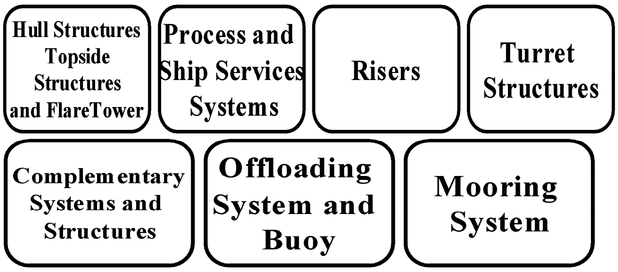

Figure 1 FPSO : list of equipment and systems

Risk acceptance criteria (see section 5) of any offshore unit have to be set for the unit as a whole and therefore have to take into account all systems involved in operation phase (For a FPSO : hull structure, process system, turret mooring system…see figure 1). This preoccupation is common to operators and classification societies. On one side, main objective of operators is to know continuously the global safety level of their installation. On the other side, objective of classification societies is to develop rigorous methodologies for inspection that is to say methodologies including all systems of the installation.

This new line of conduct – *which appears clearly in recent tenders from oil and gas companies* – implies that specific approaches developed in the past for structures and process are harmonised and/or unified. It is not so obvious insofar as philosophical inspection background characterising these approaches is not always the same.

4. Historical background

Review of R&D works achieved this last decade in domains closely related to inspection and coming from various industries allows to bring thematic, concepts, methodologies and theories to the fore. A summary dealing with offshore industry is given figure 2.

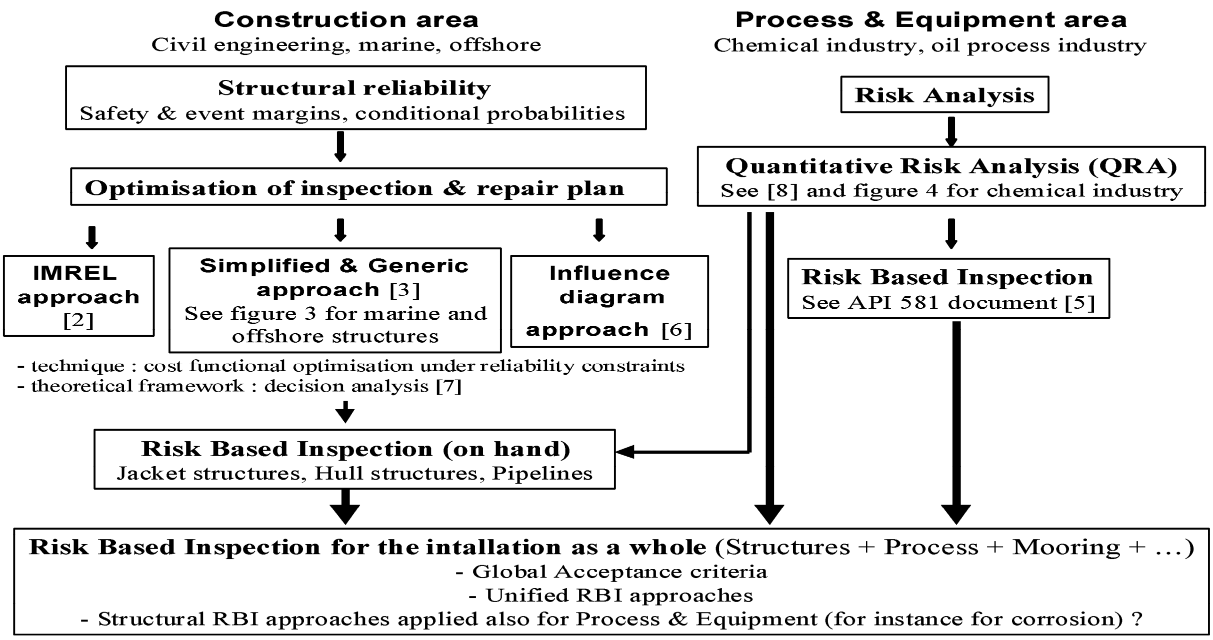

Figure 2 Risk Based Inspection thematic : historical background and context

5. General RBI Working Approach

Offshore petrochemical production facilities are complex within the context of RBI not least due to the large diversity of equipment they consist of but also due to the enormous amount of different components needed to be considered. It is therefore required that the working process when conducting RBI analysis for such facilities is very structured and targeted in view of ***the objective – assessing and controlling risk***.

Conducting a RBI project involves the exchange of knowledge and information at many different levels both in the organisation of the consultant but also in the organisation of the owner of the facility.

It is therefore of utmost importance that the flow of a RBI project is well structured not least in respect to interfaces of exchange of information.

Furthermore RBI projects should benefit from a carefully planned involvement of the client – ensuring that a firm basis for the project is achieved – and that the valuable knowledge of the client about the considered facility is utilised most efficiently.

An efficient approach to RBI analysis is to follow the principle of risk analysis in the identification of components and degradation process to consider and which of these need to be assessed in detail. Thereby the effort spend for the RBI analysis is kept to the necessary minimum and at the same time it is ensured that the important components are included.

The work procedure, client interfaces and quality control milestones are illustrated in figure 3.

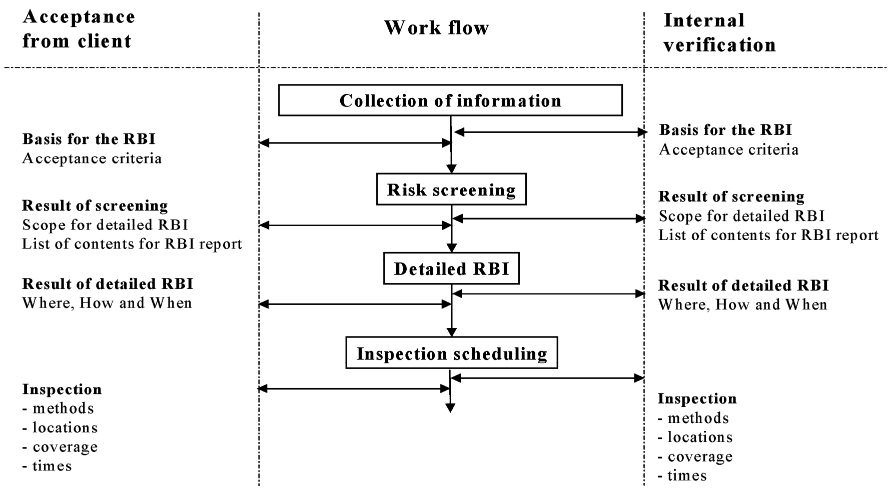

Figure 3 : Illustration of the proposed working RBI process, client interfaces and internal quality control flow.

A RBI project is initiated by identifying the overall target to be achieved by the inspection plans, namely the acceptance criteria. Thereafter or simultaneously the basic information about the considered facility is collected.

Having collected the basic information a qualitative and partly semi-quantitative risk analysis is performed in order to undertake a preliminary risk assessment of in principle all components which could have influence on the risks to be considered in accordance with the acceptance criteria. The risk screening requires a close collaboration between the client, the operator and the consultant in order to ensure that all relevant information and experience is fully utilised. The preliminary risk assessment is called a risk screening as the result is only an ordering of components into two groups, namely components which either should be included in regular maintenance or monitoring activities and components which should be considered for further and more detailed quantitative inspection planning assessments. The risk screening process reduces the components to be considered further drastically.

Following the risk screening the detailed inspection planning is conducted for all components selected for detailed RBI during the risk screening. The result of the detailed RBI is where to inspect, how to inspect and when to inspect. Of course the detailed inspection planning takes provision for the necessary updating required when deteriorating already has been observed or when in the future such observations will be made. Finally also a ranking of components according to risks is performed and the significance of different degradation mechanisms and systems is evaluated.

After the detailed RBI the inspection plans have to optimised logistically resulting in inspection schedules in accordance with the availability of the inspection crews and the operational procedures of the facility. This again is an exercise requiring an intimate team work between the client and the consultant. The inspection schedules are finally also adapted to the requirements and rules of the relevant class society.

Finally the inspection schedules are incorporated into inspection handbooks or into a maintenance management system in accordance with the preferences of the client.

The RBI steps are summarised in figure 4 below.

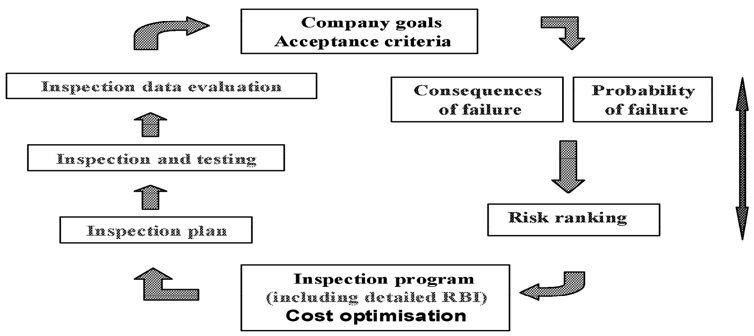

Figure 4 : RBI methodology is a stepwise methodology

6. Acceptance Criteria

Requirements to the safety of offshore petrochemical facilities are commonly specified in terms of acceptance criteria related to the maximum acceptable risks to

- Personnel
- Environment
- Economy

In addition to the acceptance criteria directly related to risks as described above also direct and indirect requirements to the safety of process components and structural components are given in the relevant codes of practice for design.

The acceptance criteria form the basis on which the decisions are made for future inspection and maintenance activities. Given the overall risk acceptance criteria for the installation the acceptance criteria are derived for the various components in the installation.

7. Data Collection - Facility/Systems Breakdown

One of the first tasks in a RBI project is to collect all available information concerning the systems and components to be considered. It is assumed that the available information include :

<u>Process systems :</u> line lists (electronic format), equipment lists (electronic format), product service codes, PFD's, P&ID's, baseline inspections, operations inspection and maintenance history

<u>Structural systems :</u> design resume, construction and installation resume, baseline inspections, operations and maintenance history

It is also necessary to organise this information in a format suitable for the incorporation into an Inspection Management Database System. For the components of the process system this organisation may be facilitated using the already defined tag numbering systems included in the electronic line lists. For the components of the structural systems, a tag numbering system has to be developed. The tag numbering system for the structural components should always be compatible with the "area" sub-division applied by the relevant certification society. Thereby it is ensured that both process and structural components may be incorporated into the Maintenance Management system completely integrated and that the achieved inspection plans may easily be compared with the requirements of the rules of the certification society.

8. Risk Screening

The risk screening is performed on the basis of the established information basis. The overall purpose of the risk screening is to establish an overview of the facility, its systems and components in regard to their contribution to the risks and in regard to the prevailing degradation mechanisms. Furthermore the risk screening facilitates an identification of those systems and components which need no further detailed RBI assessment and allocates these for either regular corrective maintenance activities or for monitoring depending on the characteristics of the degradation process and the consequences of failure.

The considered facility is divided into subsystems and components and for each of these the consequences of failure together with the probability of failure are assessed qualitatively or semi-quantitatively depending on the information available.

The considered degradation processes include

<u>Degradation models for process type components</u>

- CO_2 corrosion
- Sand erosion
- Microbially-Induced Corrosion (MIC)
- Corrosion in water
- Corrosion under insulation (uniform & local) – carbon & stainless steels
- Stress corrosion cracking – stainless steels

<u>Degradation models for structural components</u>

- Fatigue
- corrosion

The risk screening is mainly conducted through HAZID-scenario procedures with participation of both the client/operator and the consultant.

On the basis of the expertise, hands on experience and knowledge about the systems, all components are considered at a sub-system level and for each relevant materials degradation mechanisms the consequences of failure and the probability of failure are assessed qualitatively or semi-quantitatively.

The principle of the risk screening and categorization is illustrated in the risk matrix in figure 5.

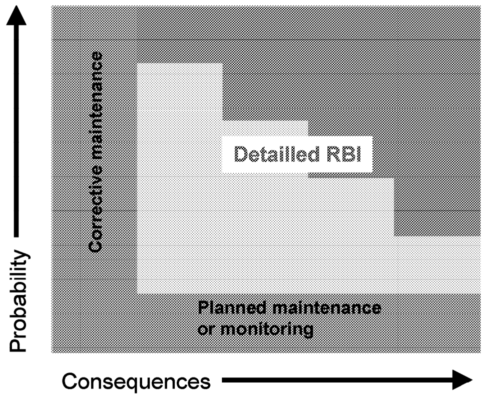

Figure 5 : Risk screening matrix.

- If the probability of failure is low, inspection will have no effect in further reducing the risk; if the consequence is also low, then the recommended action is minimum surveillance.
- If the probability of failure is low but consequence is high, preventative maintenance should be considered to control the risk
- If the probability is high but consequence is low risk corrective maintenance is recommended.
- Where both probability and consequence are high detailed RBI is required.

For systems, sub-systems and areas which require detailed RBI the following information is collected at the screening meeting.

- criticality (definition of failure)
- consequence of failure (effect on operation and production, cost)
- inspectability (possibility to inspect and how, costs of inspection)
- repair strategies (how /how soon to repair, effect on production/operation cost of repair)

For structural systems, normally only fatigue is considered but for some areas also corrosion may be an issue. Details with a fatigue life higher than 10 times the design life of the facility in general do not require detailed RBI analysis. Areas, with potentially critical structural details but where no information is available regarding the design fatigue lives, are identified for further fatigue analysis during the detailed RBI.

9. Generic Detailed RBI Analysis

❑ Acceptance criteria

The first step in the detailed RBI assessment is to derive the risk acceptance criteria for all components. First the acceptance criteria related to the risk of personnel and to the risk of the environment are considered. Acceptance criteria related to the economical risk is treated later.

Formally the basis for this is the concept evaluation QRA performed in the early design stages for the facility. If such a QRA analysis is not available the required information may be established with a QRA specifically tailored for the purpose.

Through the QRA analysis the allocation of the different risks to the different systems of the facility may be identified and the overall contribution from degradation of the process system, marine systems, structural system, etc. may be established.

If the total risk for the facility is below the overall facility acceptance criteria the acceptable risk for the various systems may increased by a corresponding scaling.

<u>Process type systems</u>

Having identified the maximum acceptable risk for a given system this risk must be distributed to the individual components/corrosion loops of the system. It is a typical approach to distribute the risk equally to all the components of the system which require detailed RBI. However, a more consistent approach is to distribute the acceptable risk in accordance with the risk distribution achieved through the original design. This ensures that the acceptable risk for the different components is consistent with the design philosophy underlying the original design.

Structural systems

For the structural system a similar approach as for the process systems is applied. The total acceptable risk due to structural failures is distributed to the various components considered for detailed RBI in accordance with the original design. However, for structural components it is usually the case that failure only can lead to the loss of lives or environmental consequences in a very limited number of areas. Therefore the economical risks are the dominating for structural components.

❑ Collection of additional information

Both for the process type systems and structural systems the risk screening may have identified a need to collect additional information. For process type systems this typically concerns the complementation of line lists, equipment lists, etc..

For structural systems it may be the case that fundamental information regarding e.g. the design fatigue lives of various details are not available or that the available information is insufficient or inadequate for the purpose. In those cases it can be required that additional structural analysis are performed for the assessment of fatigue lives.

❑ General framework for Risk Based Inspection planning

For the most critical components, a detailed RBI analysis is performed. Methods from offshore industry consist generally in minimising a cost functional under reliability constraints : The process for determining the optimal inspection & repair plan is the following (figure6) :

- Selection of a mitigation strategy in case of detection : selection of a set of mitigation alternatives and discrimination criteria between these alternatives
- Scenarios tree enumeration (failure scenarios and survival scenarios)
- Calculation of occurrence probability for each scenario
- Calculation of total cost attached to each scenario (total cost = cost of inspection + cost of mitigation + cost of failure)
- Evaluation of the expected total cost $E[C_T]$
- Identification of optimisation parameters and optimisation constraints
- Minimisation of the total expected cost $E[C_T]$

$$\mathbf{min\ E[C_T] = min\ \Sigma\ C(S_i)\ P(S_i)}$$

With :

$P(S_i)$ = occurrence probability of scenario n° i

$C(S_i)$ = total cost of scenario n° i

Optimisation parameters are :

- The number of inspections N in the design lifetime T_L
- The times $(T_1,T_2,..,T_N)$ of the N inspections
- The inspection qualities $(Q_1,Q_2,..Q_N)$
- The mitigation strategy d in case of detection
- The performance characteristics of repair techniques

Constraints are either reliability constraints or constraints dealing with optimisation parameters (range of variation restrictions)

Optimisation problem can be written :

$$\mathbf{min\ E[C_T(N,T,Q,d)] = min\ \{\ E[C_{INS}(N,T,Q,d)] + E[C_{REP}(N,T,Q,d)] + E[C_{FAIL}(N,T,Q,d)]\ \}}$$

with :

1) $\Delta P_f(t-1,t) < P_f(t-1,t)^{max}$ $\quad t = 1,2,3,\ldots\ldots,T_L$

2) constraints on optimisation parameters

$T=(T_1,T_2,\ldots,T_N)$

$Q=(Q_1,Q_2,\ldots Q_N)$

C_T= total cost

C_{INSP}=cost of inspection

C_{REP}= cost of repair

C_{FAIL}= cost of failure

$\Delta P_f(t-1,t)$ is the annual failure probability between (t-1) and t

$P_f(t-1,t)^{max}$ is the threshold value for $\Delta P_f(t-1,t)$

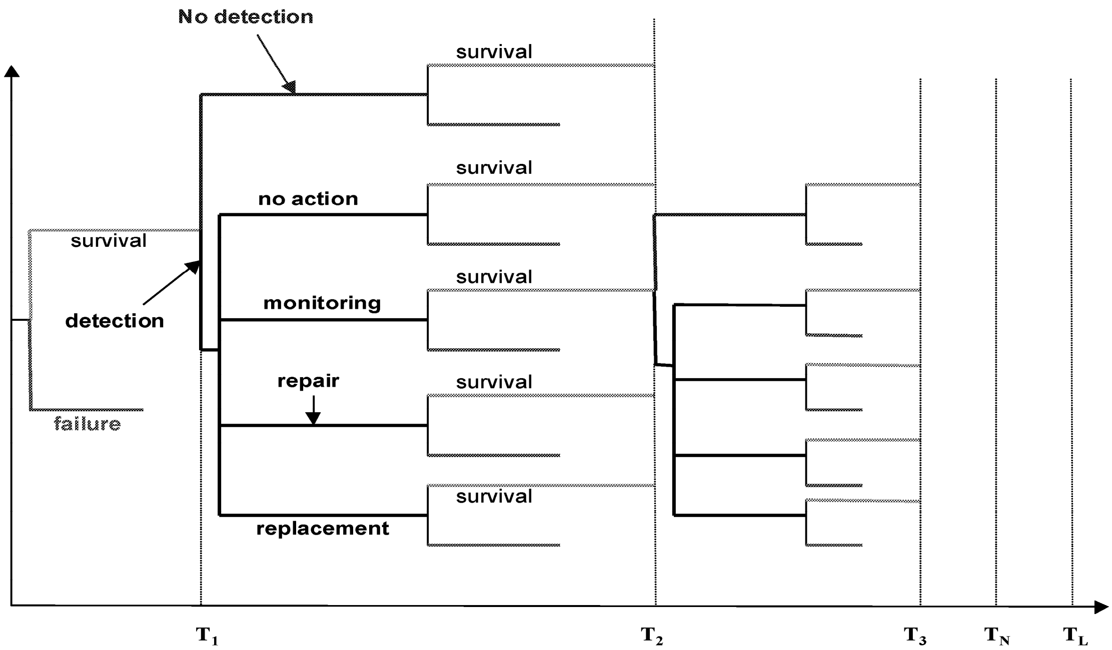

Figure 6 : General support for defining inspection and repair schemes

Figure 6 gives a general support for defining specific inspection–maintenance schemes. In J. Goyet, M.H. Faber et al [1], a simplified scheme with only one inspection and also only one mitigation alternative (automatic repair) is presented. In M. H. Faber et al [2], a more complex scheme (figure 7) is selected with several inspections and two mitigation alternatives in case of detection (detection and no repair, detection and repair). Other schemes could be defined from framework given in figure 6 (Scenarios tree enumeration process).

Solving optimisation problem lead to inspection program :

- number of inspections
- time intervals between inspections
- inspection methods
- discrimination criteria between mitigation alternatives….

The evaluation of the probabilities required to establish the inspection plans are performed using standard methods of structural reliability theory (FORM/SORM methods or simulation techniques). Inspection plans are generated automatically using a generic probability evaluation module, given the characteristics of the degradation process. In principle the probability evaluation quantifies the probability for all states in the event trees shown in figures 6 and/or 7.

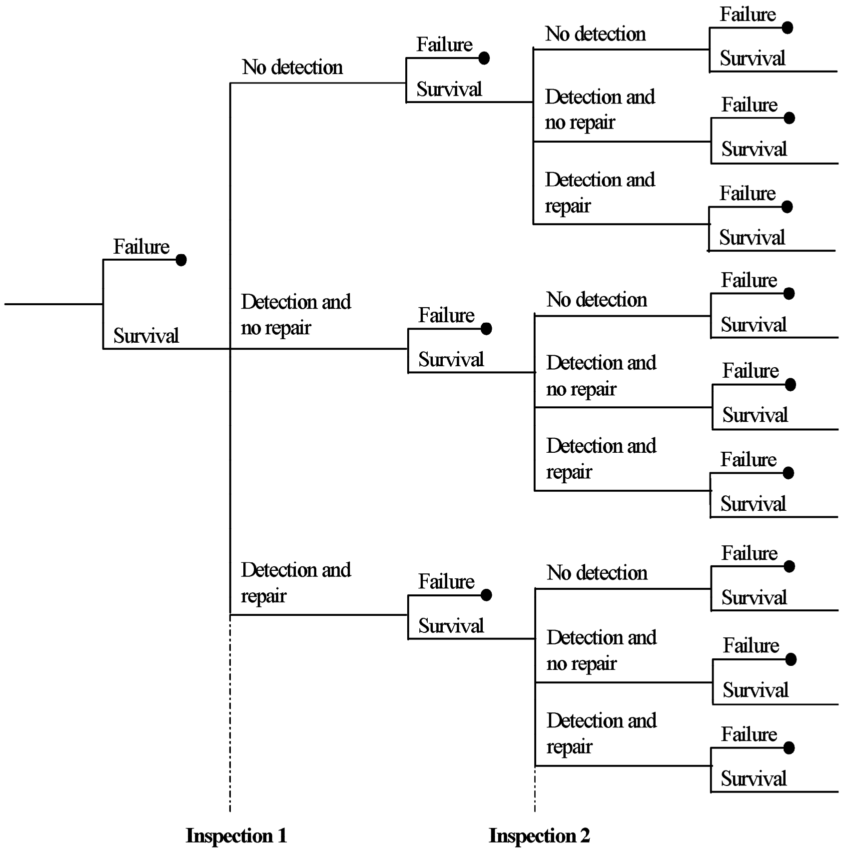

Figure 7 : One example of event tree defined from general support given in figure 6

❑ Generic Inspection Planning

Given the required information in order to perform probabilistic assessment of the future degradation of the various components of the facility, a generic approach to the planning of inspection can be performed [2].

The idea is that for all degradation processes a characteristic of the design life is established. This could be e.g. the ratio between the design wall thinning life and the design service life for process type components or the ratio between the design fatigue life and the design service life for structural components.

Given a typical range of values for this characteristic and a given type of component and assumptions of the initial degradation characteristics at the beginning of the service life, inspection plans are produced under the assumption of no detection of degradation at the time of the inspections. Inspection plans are produced for different levels of the acceptable annual probability of failure . By performing such inspection plans for all the relevant different types of components in the facility a library of generic inspection plans are established.

For all components considered for detailed RBI a cost consequence is performed in order to identify the level of acceptable annual failure probability which minimise overall service life costs, considering costs due to inspections, costs due to repairs and costs due to failure.

Finally it is checked which of the three different acceptance criteria, risk to personnel, risk to environment and economical risk give the strongest requirement to the acceptable annual failure probability and this is selected as basis for the selection of the inspection plan which complies optimally with the given component acceptance criteria. In figure 8 the principle is illustrated for RBI for structural components characterised generically by the Fatigue Design Factor

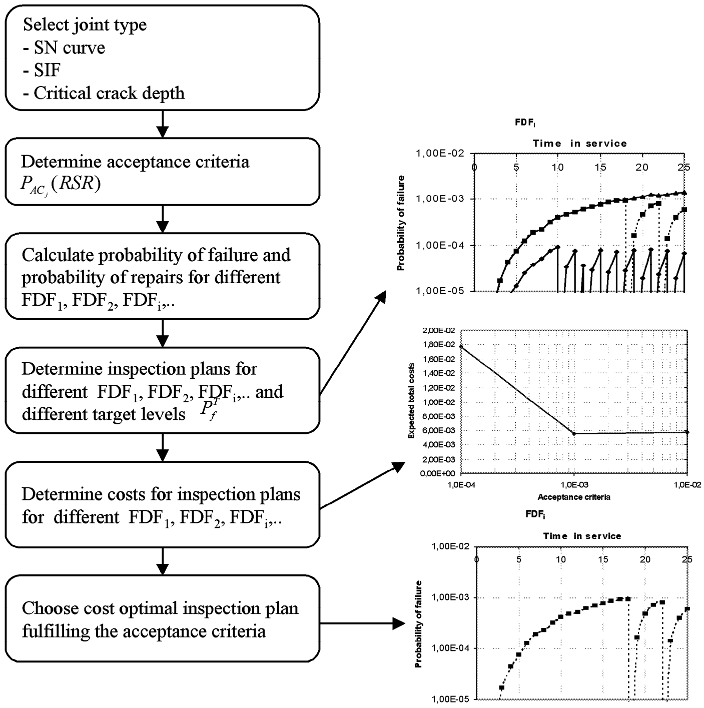

Figure 8 : Illustration of the principle for generic RBI [2]

❑ Updating of inspection plans

Given that it has been observed by inspection that degradation has taken effect for a component the corresponding inspection plans must be updated accordingly.

However, using the generic approach this may readily be achieved by an appropriate updating of the characteristic used to describe, in generic terms, the degradation life of the considered component. Modules for the automatic updating of the relevant characteristics for the degradation life of the considered types of components, as a function of the observed degradation and the time in service will be prepared in the future. Thus facilitating a continued use of the generic inspection plans.

10. Inspection Scheduling

Having identified the cost optimal inspection plans for all considered components of the facility these inspection plans are co-ordinated and organised in order to achieve an overall optimal inspection schedule under consideration of the

- Availability and capacity of the inspection contractor
- Impact of inspections on the operation of the facility
- Logistics
- The rule based specifications for inspection plans as given by the relevant certification society

These four points may readily be taken in to consideration by a rearrangement of the component inspection plans as illustrated in the first step indicated in figure 9.

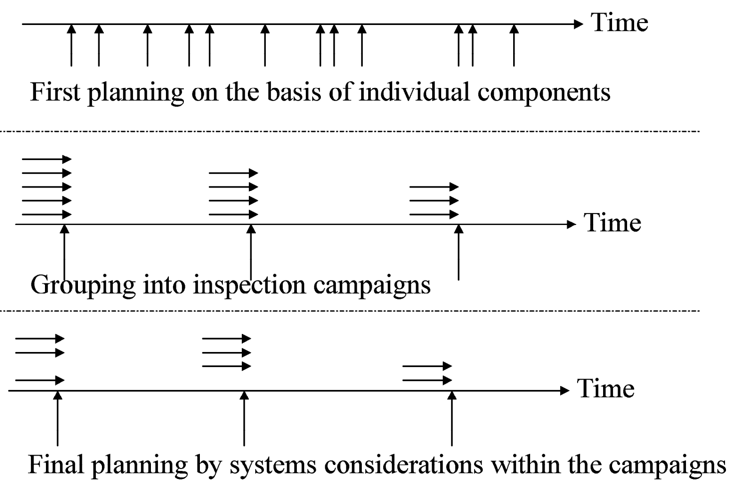

Figure 9 : Illustration of the approach to optimal inspection scheduling
(arrows indicate times of inspection)

Finally, the planning may be rearranged in order to take into account the impact of information obtained from inspection on one component on the risk of others.

11. Fatigue analysis

The main degradation process involved in structural systems is the fatigue process. In order to apply the RBI proposed approach for fatigue process, information about design lives is required (Fatigue design factors). Usually design fatigue analysis is performed using S-N approach. Below, as an example, the methodology for the fatigue analysis of the hull structure of a FPSO is presented. The purpose of such an analysis is to address the fatigue of hull under environmental conditions at the specific vessel site where the vessel will operate. In case of a converted vessel, the same analysis could be used to assess fatigue under sailing conditions previously encountered, if known to a reasonable level of confidence

Fatigue in a FPSO Hull is evaluated following the procedure of « Spectral Fatigue Analysis » , that can be summarised as follows :

- The fatigue damage at given locations in the structure is evaluated from the distribution of environment (wave) induced stress ranges, by the Miner Sum.
- The « short term » stress distributions are obtained by the technique of Spectral Analysis, based on frequency domain evaluation of Vessel sea-keeping and structural response.
- The long term stress distribution is obtained by summation, over the wave scatter diagram at vessel site, of the « short term » distributions corresponding each to a given sea state and condition of the vessel.(Alternatively, the total damage is obtain by summation of the « short term » damages).

This implies the following four steps

- ♦ ***Hydrodynamic analysis*** : this analysis will determine the external loads induced by the waves on the FPSO, and the resulting motions.
- ♦ ***Structural analysis*** : Loads are applied on a structural model of the FPSO. The structural analysis will provide the RAO's of stresses at location of interest, within the model.
- ♦ ***Evaluation of fatigue strength*** of structural details.
- ♦ ***Statistics*** of stress ranges and ***fatigue damage*** calculation.

12. References

1) Goyet J., Faber M.H., Paygnard J.C. and Maroini A., "Optimal inspection and repair planning case studies using IMREL software", OMAE 1994, February 27 – March 3, 1994, Houston, Texas

2) Faber, M.H., Engelund, S., Sorensen, J.D. and Bloch, A.; *Simplified and Generic Risk Based Inspection Planning*; Proceedings OMAE 2000, New Orleans.

Generic Risk Based Inspection Planning for Components Subject to Corrosion

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Keywords: Inspection, planning, maintenance, cost optimisation, safety, pitting, corrosion, generic inspection plans.

Abstract

Risk Based Inspection planning for engineering systems and components subject to corrosion such as pipes and pressure vessels is considered. Based on the theoretical framework originally developed within the field offshore structural engineering and some very recent developments concerning generic inspection planning techniques for components subject to fatigue degradation (Faber et al. [1]) it is investigated if similar approaches may be followed for consideration of pipes and pressure vessels. The paper gives and introduction to the methodology, discusses some of the special aspects of systems and components subject to corrosion and gives an example on the application of generic inspection planning for components subject to corrosion degradation.

1. Introduction

Reliability and risk based inspection planning for offshore facilities has been an issue of high interest for the offshore engineering profession over the last two decades. The first initiatives and developments were directed toward inspection planning for welded connections subject to fatigue in fixed steel offshore structures see e.g. Madsen et al. [2], Faber [3], Moan et al. [4] and Sindel and Rackwitz [5]. Later the same methodology was adapted to other applications such as e.g. tankers, FPSO's and semi-sub's and also inspection planning of structures subject to corrosion such as concrete offshore platforms has been considered within the same methodological framework. Structural reliability methods have played an important role in these developments.

Considering inspection planning for process equipment and marine systems the inspection planning methodology has evolved not from the methodology of structural reliability but rather from the methodology of traditional QRA. Still inspection planning of such systems is typically performed on the basis of QRA methodology but within the recent years structural reliability methods have also been introduced with success in this field see e.g Ahammed and Melchers [6], Strutt et al [7], Harlow and Wei [8] and lately by Shi and Mahadevan [9].

For the owner or the operator responsible for the safe and economic operation of an entire engineering facility it is important that the overall facility specific requirements to the risk to personnel, environment and economy can be verified and documented to the relevant authorities. A prerequisite to this is that the risks for all types of equipment and systems are assessed on a compatible and consistent basis. This implies that the fundamental modeling of failure modes, treatment of uncertainties and applied methodology for the quantification of risks should be uniform in this respect. However, typically this is not the case in present practice and therefore the inspection plans developed fail to achieve a compatible quantification of the risk contribution from various different components and systems. Therefore it is also not possible directly to quantify and document the impact of inspection plans on the overall risks for the facility.

Based on new methodological developments in the area of risk based inspection planning for structural components subject to fatigue degradation the present paper presents a approach to risk based inspection planning for process type components subject to corrosion degradation. The methodology facilitates a common generic modeling of corrosion degradation for process type components following the same approach as has been followed for structural components subject to fatigue and thus facilitates a complete integration of the inspection planning for an engineering facility where both types of components are present.

First a general introduction to the RBI framework is presented, thereafter some considerations to the specific issues concerning inspection and maintenance of systems and components subject to corrosion are given and finally a generic approach to inspection planning for process type components is illustrated by an example.

2. General formulations

The performance of engineering systems over time is subject to a number of uncertainties. These include operational conditions, material characteristics and environmental exposure. The uncertainties have origin in inherent physical randomness and in uncertainties associated with the models used to assess the performance of the systems. If furthermore the statistical basis for the assessment of the uncertainties is limited then also statistical uncertainties may be important.

When inspection planning for engineering systems is considered it is important to take all these uncertainties into consideration as they may strongly influence the future performance of the systems. It is also important to realize that the degree of control of the engineering systems achieved by the inspections is strongly influenced by the reliability of the inspections, i.e. their ability to detect and size degradation. The reliability of inspections themselves may be subject to very significant uncertainty and this must be taken into account in the planning of inspections.

The decision problem of identifying the cost optimal inspection plan may be solved within the framework of pre-posterior analysis from the classical decision theory see e.g. Raiffa and Schlaifer [10] and Benjamin and Cornell [11]. Here a short summary is given closely following Faber et al. [12]. The inspection decision problem may be represented as shown in Figure 1.

With reference to Figure 1, the parameters defining the inspection plan may be collected in $\mathbf{i} = (\Delta\mathbf{t}, \mathbf{l}, \mathbf{r})^T$ where $\Delta\mathbf{t} = (\Delta t_1,..,\Delta t_N)^T$ are the intervals between the times of N inspections $\mathbf{t} = (t_1,..,t_N)^T$, $\mathbf{l} = (\mathbf{l}(t_1),..,\mathbf{l}(t_N))^T$ are the locations to inspect at the inspection times with $\mathbf{l}(t_i) = (l_1,..,l_{M(t_i)})^T$. Finally $\mathbf{r} = (r_1,..,r_N)^T$ defines the reliability (quality) of the planned inspections.

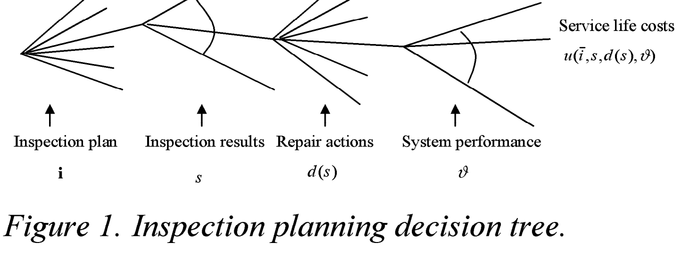

Figure 1. Inspection planning decision tree.

The inspection results are uncertain due to the fact that they depend not only on the uncertain performance of the inspection itself but also on the uncertain state of degradation. The uncertain inspection results, see Figure 1, are modeled by the random vector $\mathbf{S} = (\mathbf{S}(t_1),..,\mathbf{S}(t_N))^T$ in which the individual components refer to the results obtained from the inspections at the different locations $\mathbf{l}(t_i)$. $M(t_i)$ is the total number of inspection locations at time t_i. $d(s)$ is a decision rule defining the repair action to take depending on the inspection result. Finally ϑ is the realization of the uncertainties θ influencing the state of the system.

The utility associated with the inspection plan and the repair decision rule is denoted $u(\mathbf{i},\mathbf{s},d(\mathbf{s}),\vartheta)$ and the optimal inspection may be determined as the plan which maximizes the expected utility

$$u^* = \max_{\mathbf{i}} \max_{d} u(\mathbf{i},d) \tag{1}$$

where the expected utility u is determined as

$$u(\mathbf{i},d) = E_{\theta,\mathbf{S}|\mathbf{i}}\left[u(\mathbf{i},\mathbf{S},d(\mathbf{S}),\theta\right] \tag{2}$$

If inspection plans are performed simultaneously with the design of the system the design variables may easily be included in the decision problem as together with the inspection decision variables.

Usually the utility function may be readily associated with the service life costs and the optimization problem in equation (1) reformulated as a costs minimization problem as

$$\min_{\mathbf{i}} \min_{d} E_{\theta,\mathbf{S}|\mathbf{i},d}\left[C_{Total}(\mathbf{i},\mathbf{S},d(\mathbf{S}),\theta\right] \tag{3}$$

where $C_{Total}(\mathbf{i},\mathbf{S},d(\mathbf{S}),\theta)$ is the total service life costs. If the total expected costs are divided into inspection, repair and failure costs and a constraint related to the minimum level of service life reliability is added the optimisation problem is

$$\begin{aligned} &\min_{\mathbf{i},d}(C_I(\mathbf{i},d) + C_R(\mathbf{i},d) + C_F(\mathbf{i},d)) \\ &\quad s.t. \quad \beta(T,\mathbf{i},d) \geq \beta_{\min} \end{aligned} \tag{4}$$

where C_I are the expected inspection costs, C_R the expected repair costs and C_F the expected failure costs. $\beta(T)$ is the generalized safety index defined by

$$\beta(T) = -\Phi^{-1}(P_F(T)) \tag{5}$$

and $P_F(T)$ is the system failure probability in a specified reference period T such as one year or the service life.

2.1. Assessment of expected costs

The capitalized expected costs of inspection can be evaluated as

$$C_I = \sum_{i=1}^{N} M(i)C_I(r_i)(1 - P_F(t_i))\frac{1}{(1+\gamma)^{t_i}} \tag{6}$$

where it is assumed that all inspections at a given inspection time are performed with the same reliability. γ is the real rate of interest. The capitalized expected repair costs are evaluated as

$$C_R = \sum_{i=1}^{N} \sum_{j=1}^{M(i)} C_{R,i,j} P_{R,i,j} (1 - P_F(t_i)) \frac{1}{(1+\gamma)^{t_i}} \tag{7}$$

where $C_{R,i,j}$ are the costs and the probability of repair and $P_{R,i,j}$ respectively and where index *i,j* refers to the *j*'th inspected location at the *i*'th inspection time. The capitalized expected failure costs are

$$C_F \cong \sum_{i=1}^{N+1} C_F(t_i)(P_F(t_i) - P_F(t_{i-1})) \frac{1}{(1+\gamma)^{t_i}} \tag{8}$$

where $C_F(t_i)$ are the costs of system failure at time t_i.

2.2. Assessment of probabilities

The probabilities of failure can be written as

$$P_F(T) = \begin{cases} P\left(S(0) \cap \overline{S}(T)\right) & , \; 0 \le T < t_1 \\ P_F(t_i) + P\left(S(0) \cap \bigcup\limits_j \left\{ B_j \cap \overline{S}_j^U(T) \right\} \right) & , \; t_i \le T < t_{i+1} \end{cases} \tag{9}$$

where S refers to the survival event for the system and $\overline{S}$ to the failure event. B_j represents all combinations of inspection and repair events leading to a system configuration *j* after the *i*'th inspection . $\overline{S}_j^U$ is the event of failure in the interval between the *i*'th and the *i+1*'th inspection. *U* indicates that the system characteristics may have changed due to repairs and that the event probability is updated on the basis of the previous inspection results.
The probability of repair may be found in a similar way by

$$P_R(t_{i+1}) = P(S(0) \cap \bigcup_j \{B_j \cap R(t_{i+1})\}) \tag{10}$$

where $R(t_{i+1})$ is the repair event at inspection time *i+1*.
The system survival events, the inspection event and the repair event in equation (8)-(10) depend very much on the configuration of the considered system and it is thus not possible to write these more precisely for a general system.

3. RBI for components subject to corrosion

For components subject to fatigue degradation it has been shown, see e.g. Engelund et al. [13] that inspection times may be planned under the assumption that inspections will reveal no defects or equivalently that only the not detected defects will contribute to the failure probability. For corroded elements, however, this assumption would not be fulfilled as corrosion normally is expected and the inspection efforts for such components are planned in order to ensure that the actual corrosion degradation develops within the limits foreseen at the time of the design. In order to develop generic schemes for the inspection planning for components subject to corrosion degradation some considerations are thus made concerning the scenarios of inspections and repairs, which may lead to the event of failure.

3.1. Inspection Reliability

For components subject to corrosion, inspections are in general performed in order to detect locations where corrosion is taking place and secondly in order to assess the extent (area and shape) of corrosion and the depth of corrosion. It is important to distinguish between components where hot spots for corrosion may be identified a-priory and components where in principle all spots are hot. For the latter case the inspection reliability, i.e. measured in terms of the probability that the inspection will find an area with corrosion will depend on the inspection coverage, i.e. the fraction of the inspected area to the total area where corrosion may occur. Furthermore the inspection reliability will depend on the correlation structure of the corrosion process, see e.g. Faber and Sorensen [14].

In the following, however, for the purpose of illustration it is assumed that a well-defined hot spot has been identified, or equivalently that the corrosion process, is spatially fully correlated. In this case it may be assumed that the probability of detecting corrosion is 1 and that the probability of sizing may be modelled by a normally distributed random variable with zero mean and standard deviation corresponding to the standard deviation of the thickness of the considered component.

3.2. Repair modelling

The inspection planning will depend to a high degree on the repair or exchange strategy implemented by the operator. Typically decisions in regard to the repair or exchange of components subject to corrosion are based on the measured corrosion depth. This approach is also followed where it is assumed that a repair or equivalently an exchange of the considered component will take place if the measured depth of corrosion exceeds a certain critical depth d_{REP} . This depth may be expressed as a percentage of the wall thickness of the considered member. The material characteristics of the original component and the repaired or exchanged component are assumed to be stochastically independent.

3.3. The inspection, repair and failure event tree

Assuming a perfect inspection, i.e. a POD equal to 1 means that corrosion beyond the repair limit d_{REP} will always be detected. The event tree may thus be represented as shown in Figure 2.

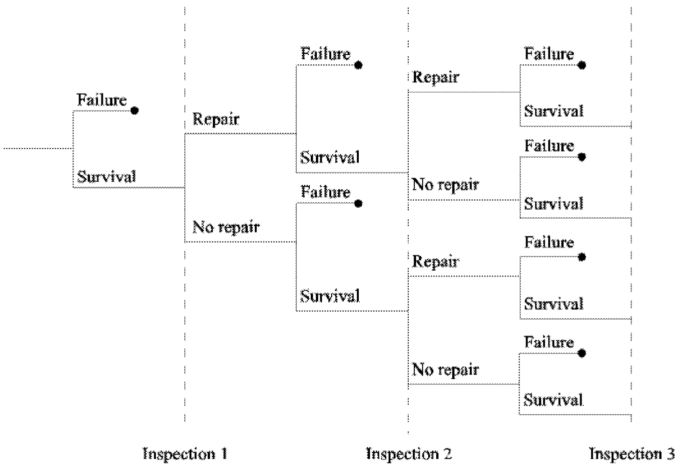

Figure 2 The event tree for one component

3.4. Generic corrosion degradation modelling

For the purpose of illustration, the following widely applied model for the time evolution of the corrosion pit depth w is used, see e.g. Muto et al. [15]

$$w = A \cdot t_p^{\ B} \tag{11}$$

In this model, t_p is the corrosion propagation time given as $t_p = t - t_i$ where t_i is the corrosion initiation time.

In Muto et al. [15], where empirical studies were performed, A was found to represent the material properties (A is a function of the alloy content) whereas the value of B depends on the type of atmosphere, i.e. B represents the environmental characteristics. The initiation time will depend on the particulars of the surface of the considered component including the corrosion inhibiting effect of coatings.

In the literature, more redefined models for corrosion pit growth can be found, even though based on the same principles, especially for corrosion fatigue, see Kondo [16]. A review on different approaches can be found in Gabrielli et al. [17].

In codes for the design of pipes and pressure vessels the effect of corrosion is usually modelled in terms of a constant characteristic value (assumed to be the upper 95 % fractile value) for the corrosion rate r. This model which is used for the design of pipes and pressure vessels is simple in use but is in general not useful for assessment purposes where models like the one given by equation (11) are more appropriate.

A characteristic for the component is the so-called wall thinning life L_W i.e. the time till the wall thinning exceeds the wall thickness d

$$L_W = \frac{d}{r} \tag{12}$$

The proposed generic approach takes basis in the wall thinning life L_W by calculating a corrosion Design Factor (CDF), defined in terms of the ratio between the wall thinning life L_W and the design service of component L_D, i.e.

$$CDF = \frac{L_W}{L_D} \tag{13}$$

The refined model for the development of the corrosion depth (equation (11)) is calibrated such that the 5 % fractile value of the probability density function for the time till corrosion has grown through the wall thickness of the component is equal to L_W.

This calibration is undertaken for different values of the CDF by means of fitting the probabilistic parameters of the constant A, i.e. the corrosion relevant material characteristics for the considered component. The calibration is, however, dependent on the wall thickness of the considered component and in principle the calibration should be performed for different wall thickness.

3.5. Numerical example

In the following example a hot spot of a steel component subject to pitting corrosion is considered. The considered limit state function for failure is given as

$$g(x) = d_{crit} - w \tag{14}$$

where the critical (or tolerable) defect depth d_{crit} could be a function of other parameters, e.g. the maximum allowable operating pressure in a pipeline [6]. However, in the present example, a deterministic value for d_{crit} was chosen.

In table 1, parameters of the calculated example are given:

Parameter	Dimension	Distribution	Mean	Standard deviation
A	Mm / yr	LogNormal	µA	0.25 µA
B	-	Normal	0.6	0.1
T_I	Yr	LogNormal	5	3
d_{crit}	Mm	Deterministic	30	

Table 1 Parameters of the calculated example

In table 2, the calibration parameter A is given as a function of the CDF, determined as explained in the previous section.

CDF [-]	μA [mm/yr]	σA [mm/yr]
1	2.70	0.675
1.5	1.92	0.48
2	1.55	0.387
3	1.12	0.28

Table 2 Parameter A

The exponent B is chosen as 0.6, which is representative for a severe marine atmospheric environment, see e.g. [15].

For illustration purposes it is furthermore assumed that the measurement errors may be neglected.

The cost models, as given in table 3, represent the previously determined relative costs of the different events, as well as an interest factor. They provide the basis for calculating the total expected costs of different inspection plans. Different cost models were considered in order to examine the influence of these parameters on the optimal inspection plan.

	Cost model 1	Cost model 2
Cost of failure	1	1
Cost of repair	0.02	0.001
Cost of inspection	0.002	0.002
Interest	0.06	0.10

Table 3 Cost models

In figure 3, the influence of an inspection after 15 years is depicted. Comparison is made between the case of no inspection, the case of an inspection assuming no corrosion and the case of full analysis of the possible inspection results, given the predefined repair strategy and assuming that the material properties after repair are equal to the initial qualities.

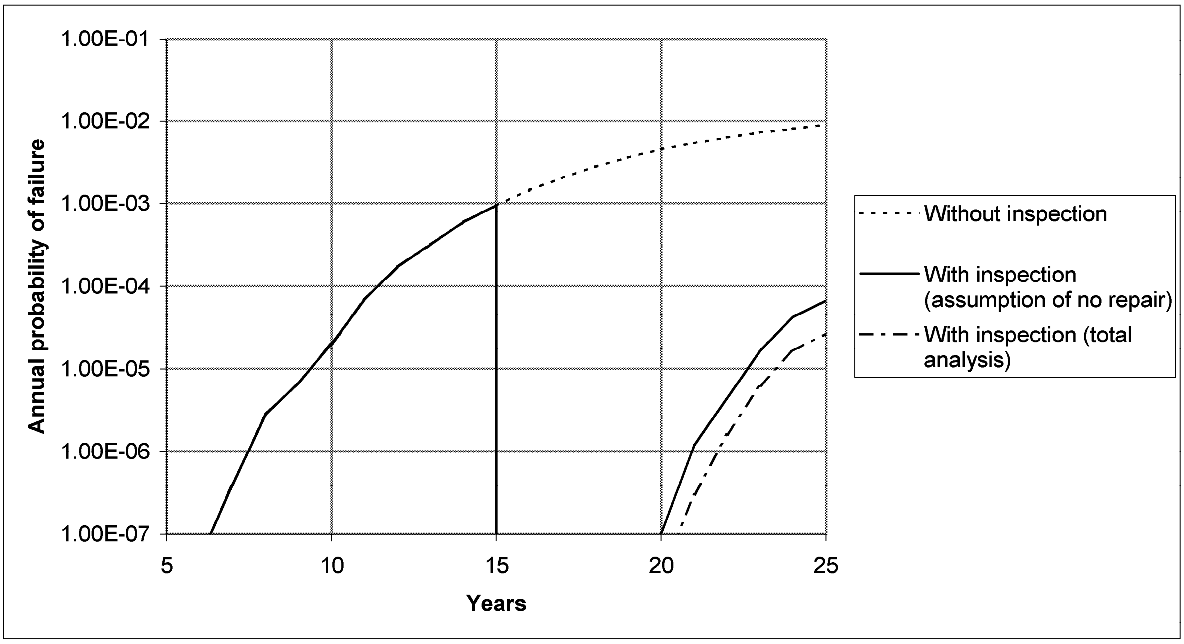

Figure 3 Influence of an inspection after 15 years (CDF = 1)

It is seen that the inspection has a great influence on the reliability of the element due to the slow progression of the deterioration and the effectiveness of the inspections.

In figure 4 the total costs of different inspection plans are illustrated as a function of the threshold probability of failure for which the inspection will be performed. Figure 3 as an example gives the annual probability of failure for an inspection plan with a threshold of 10^{-3}.

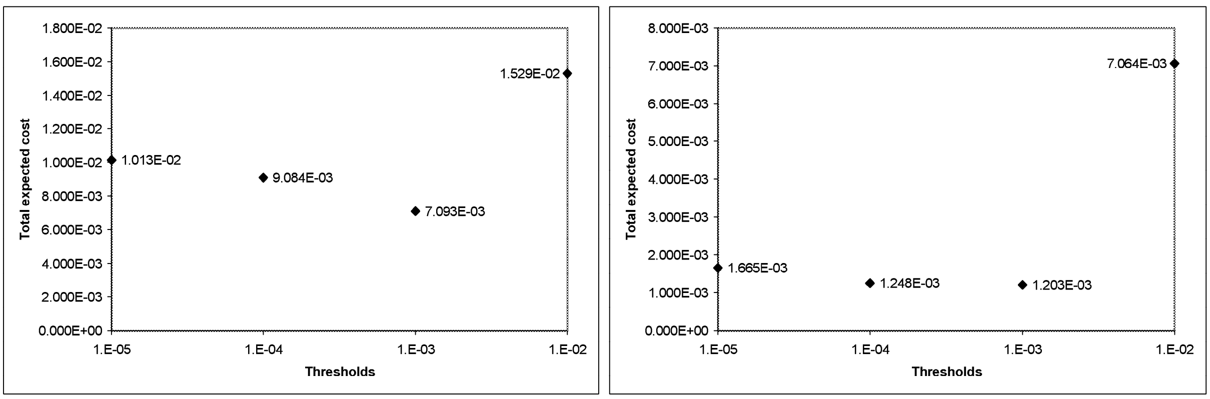

Figure 4 Total expected cost of different threshold inspection plans (CDF=1); cost model 1 left, cost model 2 right

The cost are calculated according to the equations (6)–(8) and the relative cost given in table 3.

The inspection times for a threshold of 10^{-4} per annum are given in figure 5 as a function of the CDF. This provides the basis for establishing the inspection times of components, which are represented by the generic model, after determination of the cost optimal threshold.

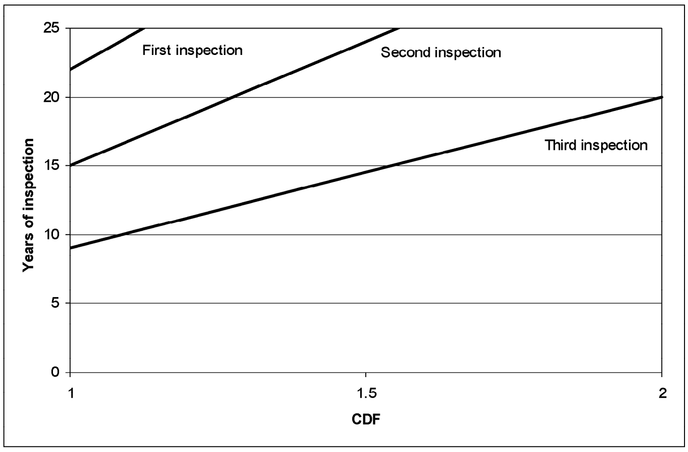

Figure 5 Inspection times for a threshold of 10^{-5} per annum as a function of the CDF

The inspection times of subsequent inspections are evaluated under the assumption that no repair has been performed at the earlier inspections, thus considering only one branch of the event tree. This is reasonable because in case of a repair a new inspection plan will be established. However, when calculating the total expected costs of an inspection plan these branches of the event tree are of course taken into account.

4. Discussions and conclusions

The procedure for the generic inspection planning for components subject to corrosion may be summarised as follows: First, the CDF for the detail must be evaluated. Then a generic inspection plan must be identified which is specific for the considered component wall thickness and for the costs of inspections, repairs and failure.

The inspections are then carried out according to the plan as long as no repair is carried out. After a repair, a new inspection plan can be achieved simply by calculating the new CDF. Updating is not necessary due to the fact that defect parts will be replaced.

For systems of components subject to corrosion, as e.g. a pipeline, the model has to be extended. In particular the modelling of corrosion and the modelling of the POD has to be reviewed.

When considering systems where the hotspot is not known or were there are a great amount of hotspots (equal likelihood), in general only a particular part of the system (or the assumed hotspots) will be inspected. The reliability of the total system is thus an estimation based on this partial information. The POD will then be a function of the inspection coverage (what part of the system is inspected) and of the correlation length of the corrosion process. Clearly, the assumption made in the first part, that the POD is equal to one, cannot longer hold. Therefore the event tree as shown in figure 2 has to be extended, including the possibility of not detecting defects larger than the repair limit. Further investigations concerning the modelling of partial inspections is, however, necessary.

Normally the maximum defect depth of the total system at any time t is the random variable of interest. An approach often followed is to model the maximum pit depth with extreme value statistics for systems where the first perforation is crucial, e.g. for pressurised structures.

Another approach is to model the spatial stochastic behaviour of pitting corrosion, a somewhat more sophisticated description of the corrosion characteristics. Corrosion deterioration would then be modelled as stochastic processes. It has to be examined whether such an approach is reasonable for establishing generic inspection plans, but it could provide the basis for a better understanding of the phenomena that underlies the extreme value statistics used.

5. References

[1] Faber, M.H., Engelund, S., Sorensen, J.D. and Bloch, A.; *Simplified and Generic Risk Based Inspection Planning*; Proceeding OMAE 2000, New Orleans

[2] Madsen, H.O., Sorensen, J.D., Olesen, R., Optimal Inspection Planning for Fatigue Damage of Offshore Structures, Procc. ICOSSAR 1989, San Francisco, pp. 1619-1626

[3] Faber, M.H., *Risk Based Inspection – The Framework*; Procc. Workshop on Risk and Reliability Based Inspection Planning, Zürich, 2000

[4] Moan, T., Vardal, O.T., Hellvig, N.-C.. Skjoldli, K; *In-Service Observations of Cracks in North Sea Jackets. A Study on Initial Crack Depth and POD values*; Procc. OMAE 1996, Vol. II, pp 189 – 197

[5] Sindel, R., Rackwitz, R.; *Calculation of the Failure Probability and Inspection Optimisation for Cracks in Ships*; Technische Universität München, Institut für Tragwerksbau, 1996

[6] Ahammed, M., Melchers, R. E.; *Reliability Estimation of Pressurised Pipelines Subject to Localised Corrosion Defects*; Int. Journal of Pressure Vessels and Piping, Vol. 69 (1996), pp. 267-272

[7] Strutt, J.E., Allsopp, A., Newmann, D., Trille, C.; *Reliability Prediction of Corroding Pipelines*; Procc. OMAE 1996, Vol. V, pp 495 – 509, Florence, Italy

[8] Harlow, D.G., Wei, R.P. ; *Probability Approach for Prediction of Corrosion and Corrosion Fatigue Life*; AIAA Journal, Vol. 32/10, 1994, pp 2073-2079

[9] Shi, P., Mahadevan, S.; Probabilistic Corrosion Fatigue Life Prediction; Procc. 8'th ASCE Speciality Conference on probabilistic mechanics and Structural Reliability, Notre Dame, July, 2000,

[10] Raiffa, H., Schlaifer, R.; *Applied Statistical Decision Theory*; Cambridge University Press, Cambridge; Mass., 1961

[11] Benjamin, J.R. and Cornell, C.A.; *Probability Statistics and Decision for Civil Engineers*; McGraw-Hill; New York, 1970

[12] Faber, M.H., Kroon, I.B., Sorensen, J.D.; *Sensitivies in Structural Maintenance Plannin*g; J. of Reliability Engineeing and Systems Safety, 51, 1996, Elsevier Science Limited, pp. 317-329

[13] Engelund, S.E., Faber, M.H., Sorensen, J.D. and Bloch, A.; *Approximations in Inspection Planning*; Procc. 8'th ASCE Speciality Conference on probabilistic mechanics and Structural Reliability, Notre Dame, July, 2000

[14] Faber, M.H., Sørensen, J.D; *Aspects of Inspection Planning - Quality and Quantity*; ICASP8, Sydney, December, 1999

[15] Muto, I., Sato, E., Kihira, H.; *Pitting Corrosion Behaviour of Stainless Steel in Atmospheric Environments*; Proc. Plant Aging and Prediction of Corrodible Structures; Sapporo, 1995

[16] Kondo, Y., *Prediction of Fatigue Crack Initiation Life Based on Pit Growth*, Corrosion, Vol. 45/1; 1989

[17] Gabrielli, C., Huet, F., Keddam, M., Oltra, R.; *A Review of the Probabilistic Aspects of Localized Corrosion; Corrosion*; Vol. 46/4; 1990

Inspection planning for offshore jacket structures using Bayesian Networks

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Keywords: Inspection planning; Offshore jacket structures; Probability; Bayesian networks; Influence diagrams.

Summary

Over the last two decades Bayesian networks and influence diagrams have received notable attention within the field of artificial intelligence and expert systems. During the last few years the technology has been further developed for problem solving within other engineering fields. The objective of this study is to present a conceptual Bayesian network model for probabilistic prediction of fatigue crack growth in welded steel tubes. It is shown that despite discretisation of the variable domain, the prediction is in good agreement with results obtained by the well-established structural reliability methods FORM/SORM. The Bayesian network model is augmented by decision and utility nodes, thus forming a full decision model for inspection planning. With the applied program package the optimal inspection plan is easily obtained. Moreover, the updating facilities allow for fast changes of the inspection plan when new knowledge becomes available.

1. Introduction

This study addresses the problem of cost optimal inspection planning of an offshore structure subjected to uncertain loads. The objective of all inspection and repair planning is to maintain a sufficient level of reliability while minimising the total costs over the lifetime of the structure. This problem may be modelled stochastically and solved by maximising the expected utility in the probabilistic sense. In addition the present study takes into account the dynamic decision problem arising from the fact that the model must be updated after every new inspection.

Offshore structures generally consist of too many elements to perform a full inspection. Therefore samples are taken out by selecting elements that empirically or computationally are known to be more likely to fail than others. These fatigue prone details are the so-called hotspots. Because of inherent flaws and stress concentration, welded seams are much more likely to fail than the steel tubes themselves. Inspection of a fatigue prone detail will either result in finding a crack of a given length or no finding. Independent of the inspection result, more information about the hotspot will be available after the inspection than was before. Based on the result of the inspection the decision of when the next inspection is going to take place must be taken. Different criteria have been established to guide this decision. Some inspection planning schemes exclusively regard the reliability of the structure and perform a new inspection as soon as the reliability index drops below a certain limit. Studies such as Madsen et al. [1] or Sørensen et al. [2] take into account the costs associated with inspection, repair and failure of the structure. They use expected utility measures to optimise the inspection plan under the constraints of a given repair strategy. The present study is of the latter type where the new feature is to exploit the properties of influence diagrams to build a comprehensive model and perform the calculations of expected utility.

The paper is compiled as follows: Section 2 briefly describes theoretical aspects related to the model building. We introduce Bayesian networks and influence diagrams and outline fatigue crack analysis as well as structural reliability methods. Section 3 presents a Bayesian network model for probabilistic fatigue crack prediction. The assumptions on which the model is based are described and aspects of discretisation and probability assignment are discussed. In order to validate the established model, the results are compared to results obtained by more traditional structural reliability models. In section 4 the validation model is extended to a full decision model for inspection and repair planning. Aspects of inspection quality and cost optimisation are included in this section. Section 5 accounts for the results obtained by the established decision model. It is shown how the optimal inspection plan may be easily identified and updated, how the reliability index may be monitored and sequence effects taken into account. Section 6 discusses the applicability of influence diagrams to the problem of inspection planning and section 7 concludes the study.

2. Theoretical aspects

This section briefly summarises theoretical aspects related to the model building performed in the subsequent sections. Bayesian networks and influence diagrams are introduced. Fatigue crack analysis is briefly outlined and the applied theory on structural reliability methods is mentioned.

2.1 Bayesian Networks

A Bayesian network is a graphical representation of a set of uncertain quantities. It consists of a set of probabilistic nodes (ovals) and a set of directed arcs connecting the nodes. See figure 1.

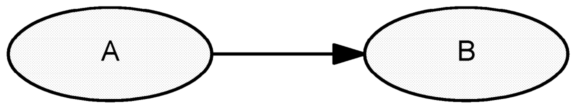


Figure 1: Simple Bayesian network consisting of two variables

The nodes represent stochastic variables, represented as a set of discrete states where each state is associated with a probability measure. Arcs into variables indicate conditional probabilistic dependence so that the probability of a dependent variable B (child node) being in a particular state is given for each combination of the states of the preceding variables A (parent nodes). In a Bayesian network loops are not allowed.

Since the variables X_i of the model are defined in terms of conditional probability distributions, the joint probability distribution over the entire variable domain $X_1, X_2, \ldots, X_n$ can be obtained using the chain rule:

$$P(X_1, X_2, \ldots, X_n) = P(X_1 | X_2, \ldots, X_n)\; P(X_2 | X_3, \ldots, X_n) \cdots P(X_n)$$

where $P(X_1,X_2)$ is the joint distribution of X_1 and X_2 and $P(X_1 \mid X_2)$ is the conditional distribution of X_1 given X_2. Although the diagram is compact and intuitive it represents a complete probabilistic description of the problem.

A central feature of Bayesian networks is that they allow inference based on observed evidence. The model is updated in accordance with observations using Bayes rule. Consider again figure 1. For the random variables A and B, Bayes rule states:

$$P(A \mid B) = \frac{P(B \mid A)P(A)}{\sum_{all\ i} P(B \mid A = a_i)P(A = a_i)}$$

Assume for instance that variable B is observed to be in state b_1. This information is inserted in the model by assigning the probability 1 to this particular state in the observed variable so that $P(B=b_1)=1$. In a larger network the inserted information is propagated to the other nodes such that all variables in the model are updated in accordance with Bayes rule. This allows us to exploit the model for answering queries and to investigate different scenarios.

2.2 Influence diagrams

An influence diagram is a Bayesian network that is extended to solve decision problems. In an influence diagram two additional types of nodes are included in the network, namely decision nodes (rectangles) and utility nodes (diamonds). A decision node defines the action alternatives that the user is considering. The preceding nodes define the available information at the time of the decision. Utility nodes have no successors but are conditioned on probabilistic and/or decision nodes. They hold tables of utility for all configurations of the parent nodes. A configuration is a set consisting of exactly one state from each variable. The utilities are measures of the decision-makers preference for each configuration.

The rational basis for decision-making is established by computation of the expected utility (EU) of each of the action alternatives. In order to perform this calculation, it must be apparent what information is available at the time of each decision. The computation is thus sensitive to the sequence (temporal order) in which the decisions are taken, and an influence diagram therefore requires a directed path connecting all the decision nodes sequentially. The path may pass probabilistic nodes also. In this way information once inserted into the model remains available for all subsequent decisions. This is known as the "no-forgetting principle".

Bayesian networks and influence diagrams can be built and manipulated using a program package such as HUGIN (see Andersen et al. [3]). The outcome of compiling a model is the marginal probability distributions of all variables in the domain as well as the expected utility of all decision alternatives. The computational complexity of the model analysis grows exponentially with the number of nodes and the number of states in each node. If evidence is inserted and propagated, the updated expected utilities for all decision variables are computed. Hence, the influence diagram serves as a dynamic decision model always showing the optimal strategy, conditional on a set of observations. When an influence diagram is compiled the network structure is transformed into a tree structure (strong junction tree) for more efficient computations and updating. Details about the associated algorithms can be found in e.g. Jensen [4] and Jensen et al. [5].

2.3 Fatigue cracks

The prediction of fatigue crack growth is based on a description of the functional relationship between the crack depth (length) and the size and number of applied loads. Paris and Erdogan [6] investigated the stress and strain relationships at the crack tip of a center cracked panel and obtained the following differential equation:

$$\frac{da}{dN} = C\left(\Delta\sigma Y(a)\sqrt{\pi a}\right)^m \tag{1}$$

where a is the crack depth, N is the number of applied load cycles, C and m are material constants, $\Delta\sigma$ is the stress range and Y(a) is a geometry function. Basically the formula states that the crack growth per load cycle is a power function of the crack length, stress range and geometry function. Explicit solution of this equation with respect to the crack depth a is in general not possible. However, assuming the geometry function Y to be independent of the crack depth a and the stress range $\Delta\sigma$ to be adequately described by a Weibull distribution, the following analytical solution can be found (see e.g. Madsen et al. [7]):

$$a = \left[a_0^{1-m/2} + C\ N\ A^m\, \Gamma\left(1+\frac{m}{B}\right) Y^m \left(\sqrt{\pi}\right)^m \left(1-\frac{m}{2}\right) \right]^{\frac{2}{2-m}} \qquad ,m \neq 2 \tag{2}$$

In this equation a_0 is the crack depth at time T=0. A and B are parameters (scale, shape respectively) of the Weibull distribution describing the stress range $\Delta\sigma$.

2.4 Structural reliability methods

Dealing with natural loads and materials the parameters entering equation 2 are of uncertain nature. This uncertainty may be taken into account by regarding the parameters as stochastic variables and assign probability distributions to each of them. The dependent variable (the crack depth a) is thus a stochastic variable itself. The reliability of a structural member may be evaluated in terms of the probability of functional failure i.e. the probability of the crack depth exceeding the material thickness. The failure probability may be expressed as:

$$P_f = P(t - a \leq 0) \tag{3}$$

where t is the material thickness and a is the crack depth. The limit state function g(**X**)=t-a thus separates the safe and the failure set in a multidimensional probability space. The failure probability may thus be evaluated as an integral over the failure event in this space. Efficient algorithms to approximate such multidimensional probability integrals have been developed within the discipline of structural reliability. More information about these methods can be found in e.g. Ditlevsen and Madsen [8]. As such methods are widely applied and well-established in industry they are taken as the standard that alternate methods may be compared to. The Bayesian network model presented in section 3 is therefore compared to a model based on FORM/SORM. The model is evaluated using the general probabilistic analysis tool PROBAN, see [9].

Since the failure probability is usually very small, it is convenient to express it as a reliability index defined as:

$$\beta = -\Phi^{-1}(P_F) \tag{4}$$

where $\mathbf{\Phi}$ is the standardised normal distribution.

3. Network model for validation

In this section a Bayesian network model for prediction of fatigue crack growth is presented, see figure 2. The intention is to investigate if a Bayesian network model is able to predict the crack growth over time as well as structural reliability methods. The point of departure for the model building is Paris' equation in the explicit form of equation 2. In the following subsections the assumptions leading to the model will be discussed.

3.1 Structure and distributions

It is convenient to be able to monitor the failure probability in fixed time steps. In order to avoid too many nodes in the network, time steps of 2 years have been chosen, see figure 2. The variables a(0) through a(10) constitute a time axis from T = 0 up to T = 10 years.

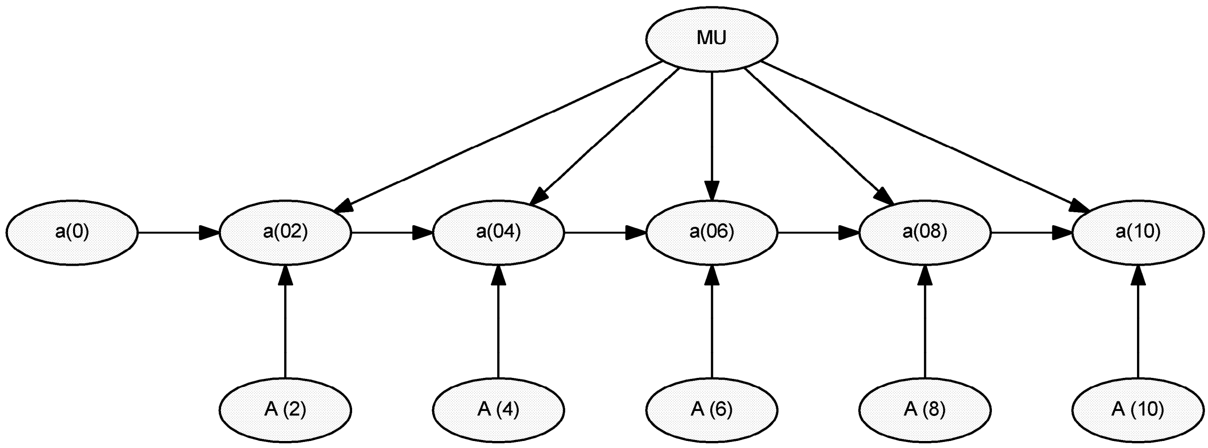


Figure 2: Validation model.

The initial distribution of crack depths is held by the variable a(0) which is assumed to follow an exponential distribution with a mean value of 1 mm. The following a-variables hold conditional probability distributions that will be discussed further in section 3.3.

The variables A(2) through A(10) model the loads from the sea. As mentioned in section 2.3 the loads are described by a Weibull distribution. The scale parameter A is assumed to follow a normal distribution with the mean value μ_A=5.35 MPa and standard deviation σ_A=0.96 MPa. This parameter is modelled in the network variables A(2) through A(10). The shape parameter B is assumed to have negligible variance and is therefore modelled as a constant B=0.66. As the load in one time step does not depend on the load in other time steps, the model comprises 5 independent variables with the same distribution. In the PROBAN model the temporal development of a crack is modelled in terms of the number of wave loads, N, in the considered time period. This means that incremental temporal development of a crack can only be captured by repeated runs of the model with a new N-value for each run. In each run the load is modelled as one single variable (the Weibull scale parameter A) with the same distribution as in the HUGIN model. The influence of the load being described by a Weibull distribution was already integrated out in the establishment of equation 2. Hence, the main difference between the two models is that in the HUGIN model the crack depth is developed incrementally in time whereas the PROBAN model considers a fixed time span.

The material variables C and m are taken as deterministic values with $C=2.17 \cdot 10^{13}$ and m=3. The number of load cycles N per year does not fluctuate much, and is therefore modelled as a fixed number $N=10^6$ per year. The geometry function Y describes the boundary conditions of the considered structural member. Empirical expressions have been found for the dependence on the crack depth for different geometries. As mentioned in section 2.3, the geometry function is taken to be independent of the crack size in this study. Taking the variables C, N, m, B and Y as deterministic values, equation 2 may be rearranged to yield:

$$a = \left[a_0^{\frac{2-m}{2}} + K A^m \right]^{\frac{2}{2-m}} \qquad ,m \neq 2 \qquad (5)$$

where

$$K = C N \Gamma\left(1+\frac{m}{B}\right) Y^m \pi^{m/2} \left(1-\frac{m}{2}\right) \qquad (6)$$

The uncertainty on the deterministic variables may be accounted for by introducing a model uncertainty M_U thus yielding the model:

$$a = \left[a_0^{\frac{2-m}{2}} + K M_U A^m \right]^{\frac{2}{2-m}} \qquad , m \neq 2 \tag{7}$$

We further assume that the model uncertainty M_U can be described by a normal distribution with mean μ_{MU}=1 and σ_{MU}=0.18. The coefficient of variation CoV=18 % is computed from the assumptions that 10 % of the uncertainty stems from the FEM-analysis (not a part of this study) and 15 % from the model.

A summary of the stochastic model is shown in table 1. The PROBAN and the HUGIN models use the same variables and distributions.

Parameter	Description	Distribution	Mean	Std. Dev.	Unit
m	Material parameter	Deterministic	3	-	-
C	Material parameter	Deterministic	$2.17 \cdot 10^{-13}$	-	-
B	Shape parameter Weibull	Deterministic	0.66	-	-
N	Number of load cycles	Deterministic	10^6	-	years^{-1}
a0	Initial crack depth	Exponential	1	1	mm
A	Scale parameter Weibull	Normal	5.35	0.96	MPa
MU	Model Uncertainty	Normal	1	0.18	-

Table 1:Probability distributions.

3.2 Discretisation

Fatigue crack growth is a physical phenomenon where the variables are inherently continuous. As it is not generally possible to perform exact inference with Bayesian networks in the case where the variables hold continuous distributions. Many ways of circumventing this problem have been proposed, yet the most straightforward (and the one chosen here) is to discretise the distribution on each of the variables.

The node a(0) models the assumed initial probability distribution of the crack depth. Since the variable is modelled as exponentially distributed, a uniform length of the 20 discretisation intervals would not capture the distribution in an adequate way, as the very low probabilities in the tail are subject to rounding errors. We choose to let the interval length of the i'th interval be given as a constant times the length of the (i-1)'th interval. In this way the interval length becomes a power function of a basic length δ. An additional restriction is that the interval lengths (excluding the last tail interval) have to sum up to the material thickness (here chosen to t = 20 mm). The probability mass in the tail of the distribution is lumped in the state "Fail" corresponding to the event that the crack is deeper than the thickness of the material.

The variables M_U and A(2) through A(10) are normally distributed and discretised into 10 intervals of equal length. In order to capture the whole distribution in a finite number of intervals we need to truncate the distribution at both ends. The tail probabilities are lumped into the probabilities in the first and last intervals. It has been found that truncating at three standard deviations from the mean yields negligible difference from the PROBAN model.

3.3 Conditional probability tables

At this point the structure of the network, the distribution assignment and the discretisation of the root nodes (no predecessors) are completed. The next step is to establish the conditional probability distributions in the a-variables.

Since in each time step these tables advance the probability vector of the crack depth, they may be compared to transition matrices in a Markov process. As the time steps are equal in length, the "transition matrices" may for convenience be chosen to be identical in each time slice. This implies that we only need to establish it once, say for a(2) and then copy it to the rest of the a-variables. In this way the discretisation of the a-variables becomes identical in each time step. It is noted that the model is not a Markov model because the a-variables with a distance of more than one time step are only *conditionally* independent. This is a consequence of the M_U-node being a common parent to all of the a-variables, see figure 2.

Equation 2 yields a deterministic value for the crack depth for any value of each input variable. In the Bayesian network model only the most significant, namely a_0, A and M_U are included as input variables. It is recalled that a configuration is a set of states with exactly one state from each variable.

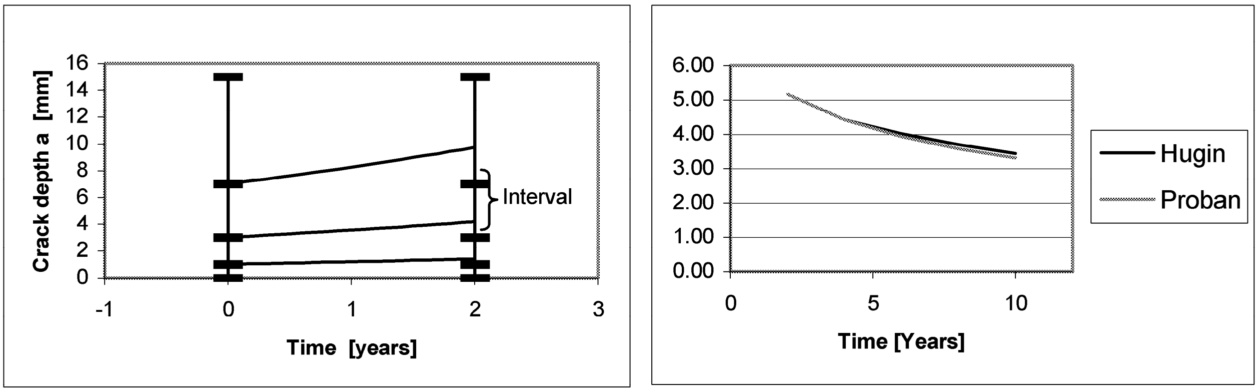


Figure 3: Left: Assignment of conditional probability in the a-variables. Right: Reliability index as a function of time.

The conditional probability table of the node a(2) is established in the following way:

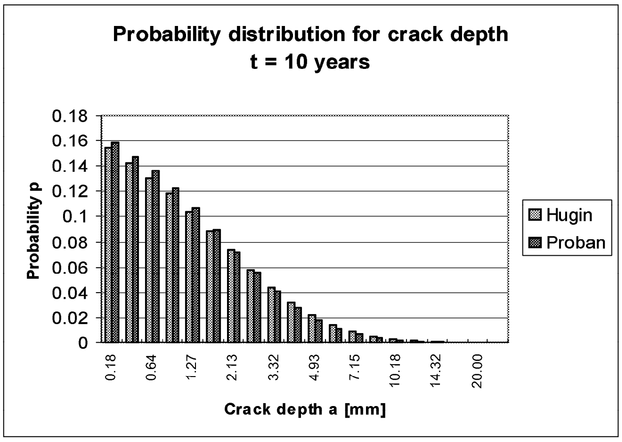
For a given configuration of the input variables a(0), A(2) and M_U, the endpoint values of the corresponding intervals are inserted into equation 7. This yields an upper and a lower bound on the crack depth at time T=2. These bounds comprise an interval I that overlaps at least two intervals (states) of the a(2)-variable. See figure 3 (left). Remember that the discretisation of a(2) is identical to the discretisation of a(0). Therefore, by assigning probability to the states of the a(2) variable according to the relative overlap from the calculated interval I, we get the conditional probability of the a(2) states given the current configuration of the input variables. Repeating this calculation for all configurations of the input variables, the conditional probability table of the node a(2) are obtained. This table can now be copied to all the other a-variables.

Since the prior distribution of the a(0) node is exponentially decreasing with the crack depth, the uniform assignment of probabilities to the overlapped intervals of node a(2) causes an overestimation of the probability in the upper interval. In general this will imply a slight overestimation of the crack growth rate.

3.4 Results of Bayesian network validation model

The model described above was built in HUGIN and compiled such that the marginal probability distribution of all variables were obtained. This allows the user to follow the distribution of the crack depth through the time steps.

In the time step corresponding to 10 years (node a(10) in figure 2) the probability distribution found in figure 4 is obtained. The rightmost figure shows a more detailed view of the probability distribution for crack depths larger than 8 mm. It is noted that the distribution obtained by HUGIN is very close to the one obtained by PROBAN. The slight difference is attributed partly to the overestimation of the crack growth rate due to the probability assignment method. However, modelling the load (A-variables) as independent variables instead of one single variable results in lower failure probabilities and thus counteracts this effect.



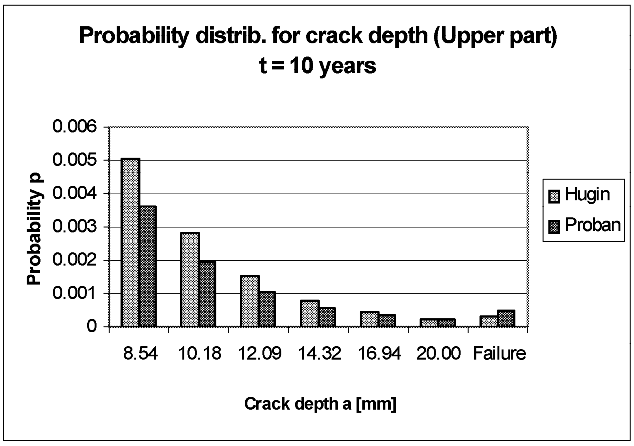


Figure 4: Comparison of probability distributions for Hugin and Proban.

The reliability index corresponding to the failure probability can be found by using the inverse normal distribution function, see equation 4. The reliability index as a function of time is shown in figure 3 (right). Again the agreement between HUGIN and PROBAN is very good.

We therefore conclude this section by noting that Bayesian networks are sufficiently accurate in predicting fatigue crack growth despite the need for discretisation and elicitation of conditional probability distributions.

4. Decision model for optimal maintenance strategy

By adding appropriate probabilistic, decision and utility nodes, the Bayesian network model for crack growth prediction is extended to a full decision model for inspection and repair planning, see figure 6. For clarity only the first 6 years are shown, although any number of time steps may be added.

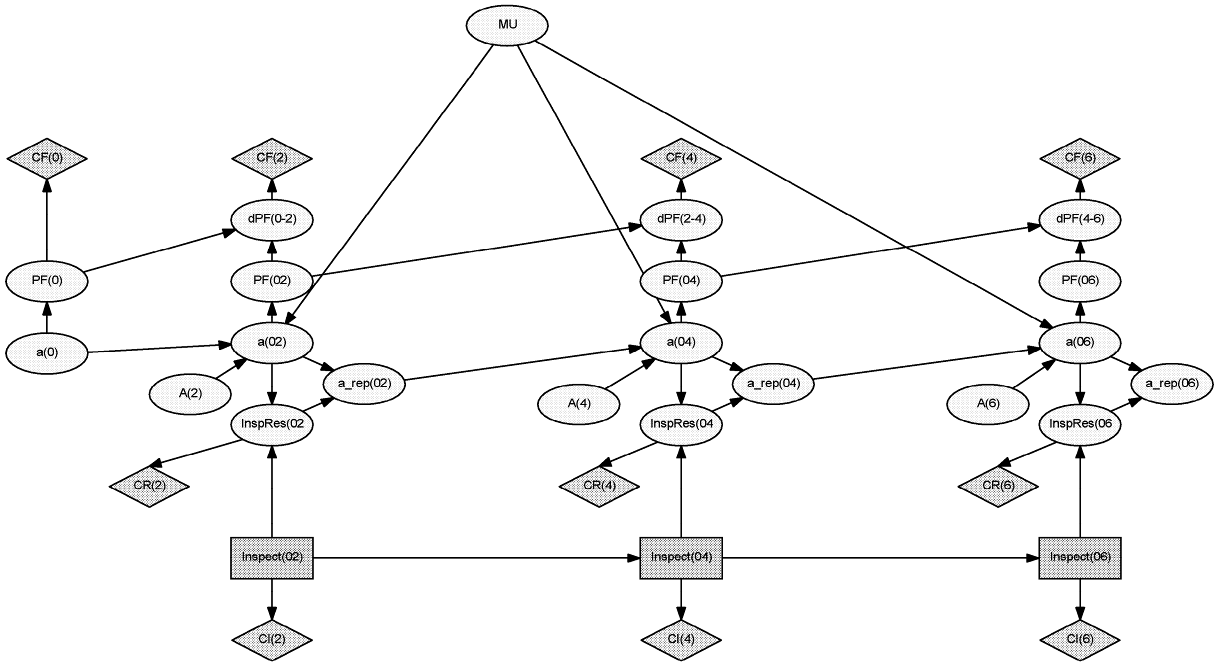


Figure 6: Decision model for inspection planning.

Since the applied repair strategy has a significant impact on the modelling, it is convenient to present it before further description of the additional nodes.

Repair strategy

When a hotspot is inspected, the possible outcomes are:

1. Finding a failure
2. Finding a crack (that is not a failure)
3. No finding

All detected cracks or failures are repaired immediately. In the present model there is only one type of repair and after the repair the hotspot is assumed to have the same quality as a new weld. If a failure has occurred, then it is assumed to be found for sure. Since the states in a variable are mutually exclusive, this also implies that if the inspection result is "Finding a crack" or "No finding", then there is no failure.

4.1 Node descriptions

The network model is augmented by a number of identical nodes in each time step. Therefore only nodes of one time step are described in the following using the general term T_i to refer to the time at an arbitrary time step i.

The decision node *Inspect(T_i)* has two states, "Inspection" and "Nothing" corresponding to inspecting the hotspot or leaving it as it is. The **C**osts of these two **I**nspection alternatives are modelled by the utility node *CI(T_i)*. It is assumed that inspection costs DKK 0.1 million per hotspot.

The decision nodes are connected sequentially thus defining the order in which the decisions are taken.

The *InspRes(T_i)* node models the outcome of the inspection. It is conditioned on both the probabilistic node *a(T_i)* and the decision node *Inspect(T_i)* as the outcome of the inspection depends on the probability distribution of the crack and whether or not an inspection has been performed. It has 4 states: "Failure", "Find", "No find" and "No inspection".

Inspection equipment and personnel are not infallible, so even in the case where a crack is present, it is not given that it will be found. However, the probability of finding a crack increases with its size. The probability of detection is thus conditional on the actual depth, a, of the crack. It may be modelled as an exponential distribution function:

$$POD(a) = P(\text{det}\,ection|a) = 1 - \exp(-ba) \tag{8}$$

where a is the real crack depth and b is a parameter describing the quality of the inspection method. In this study b is selected to 0.77 mm^{-1}.

The probabilistic node *a_rep(T_i)* holds the distribution of the crack depth after (possible) inspection and repair. It is dependent on both the real crack distribution *a(T_i)* and the inspection result *InspRes(T_i)*. The applied repair strategy dictates that a detected crack or failure should be repaired immediately. After a repair the probability distribution of the crack depth is thus reset to the initial crack depth distribution (as in the node *a(0)*). In the case where no inspection has been performed or no crack has been found, no additional information about the crack sizes is obtained and consequently the current distribution of the crack depth is transferred to the next time step.

The cost of a repair is modelled by the utility node *CR(T_i)* which is solely dependent on the inspection result as repair is only performed when a crack or failure has been found. It is assumed that repair costs DKK 1 million per hotspot.

The node *PF(Ti)* holds the failure probability in the current time step. It has two states, namely "Fail" and "No fail".

Failure cost modelling

The expected failure costs $E(C_F(T_L))$ over the lifetime T_L of the structure may be calculated as:

$$E(C_F(T_L)) = C_F(0)P_F(0) + \sum_{i=1}^{T_L} C_F(T_i)(P_F(T_i) - P_F(T_{i-1}))\frac{1}{(1+r)^{T_i}} \tag{9}$$

where r is the real rate of interest, $C_F(T_i)$ and $P_F(T_i)$ are the failure costs and failure probability, respectively, at time T_i.

The failure costs $C_F(T_i)$ are modelled by the utility nodes of the same name. In the present study the failure costs are set to DKK 3000 million in all time steps, although they may easily be capitalised as net present value.

Subtraction of probabilities

Subtracting probabilities as in equation 9 is not generally possible with Bayesian networks as probabilities must be nonnegative. In this case, however, the nature of the problem allows us to capture the behaviour of the model and to obtain the correct incremental probabilities. It can be shown that the incremental probability can be calculated as the joint probability of failure at T_i and no failure at T_{i-1}:

$$dP_F(T_i) = P_F(T_i) - P_F(T_{i-1}) = P(Fail(T_i) \cap Nofail(T_{i-1})) \tag{10}$$

The joint probability in equation 10 may be computed in the node $dP_F(T_i)$ which is dependent on both of the nodes $P_F(T_i)$ and $P_F(T_{i-1})$. It holds a probability table with 4 entries, namely the combinations of "Fail" and "No fail" in the two time steps. If the entry corresponding to the event $[Fail(T_i) \cap Nofail(T_{i-1})]$ is set to one and the rest of the entries are set to zero, the desired probability increment dP_F is obtained for each time step.

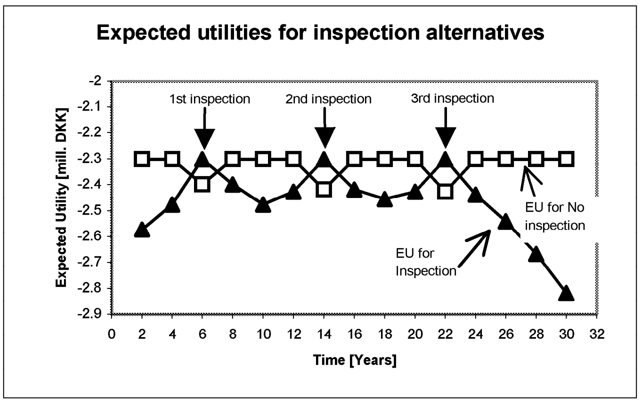
It is noted that the model does not intend to model the case of multiple failures in the same hotspot. If the hotspot fails, extensive repair work must be carried out and the probabilistic model is no longer valid. However, the probabilities and utilities associated with this event must be included in order to be able to perform cost optimisation based on expected utilities.

5. Results of Decision model

In this section the results of the established model are described and analysed. The model is extended to encompass a lifetime of 30 years and it is shown that the optimal inspection plan may be directly read off the user interface. Furthermore it is shown how the optimal plan may be updated on the basis of information obtained by inspection. Finally the inclusion of sequence effects is treated.

5.1 Optimal plan

The model computes the expected utilities of the decision alternatives (inspection, no inspection) for each time step. If the model shown in figure 6 (left) is extended to encompass 30 years, the result



shown in figure 6 (right) is obtained.

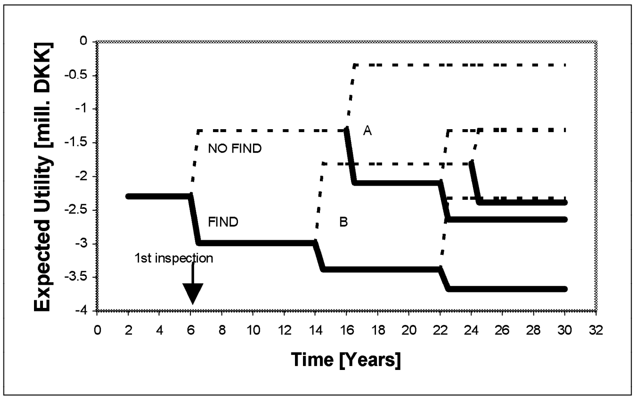


Figure 6: Left: Expected utility of decision alternatives. Right: All combinations of optimal plans. Inspections are performed at the branching points.

Each time step has one value for the expected utility of inspection (triangle) and one value for no inspection (square). Note that the program computes the expected utility at time T_i conditioned on the assumption that the optimal choice has been taken in all previous steps. In other words the figure shows which combination of decision alternatives to choose in order to obtain the minimal expected costs (maximal EU). In the time steps T = 6 years, T = 14 years and T = 22 years the expected utility of inspecting the hotspot is greater than for not inspecting it. Inspection should therefore be carried out in these 3 time steps. The optimal inspection plan thus appears directly, simply by choosing the highest expected utility in each time step.

If no cracks or failures are found at the first inspection at T = 6 years we might argue that the loads have not been as severe as we expected or that the structure is stronger than we thought. In any case the reliability of the structure is increased given this inspection result, and the expected failure costs are reduced accordingly. It might therefore be rational to postpone the next inspection. The calculations necessary for evaluating this decision are automatically carried out within the model after insertion of the inspection result. It is relatively easy to check all combinations of "Find" and "No find" for each inspection. Figure 6 (right) shows all optimal inspection plans given observations. The figure may be regarded as an event tree where the branches emanate from the points where inspection has been performed. The branches that correspond to a finding are shown as full lines, the no-find branches are dashed. The tree is plotted in a coordinate system such that the level of its branches corresponds to the expected utility of the considered inspection plan. For the inspection result "No find" at T=6 years the updated model suggests the next inspection to be performed at T=16 instead of T=14 years as initially suggested. Hence, we have immediately saved two years of interests on the inspection costs (and possibly on the repair costs). But the model also takes into account that the structure is now safer than before the inspection. This is reflected in the decrease of expected costs over the lifetime from DKK 2.30 mill. to DKK 1.32 mill. If, by the second inspection at T = 16 years, we do not find a crack either, then the model does not suggest any further inspections during the 30 years lifetime. We see that if no cracks are found, we not only postpone the second inspection (and possible repair) but also save the third inspection. If, on the other hand we find a crack during the inspection at T=6 years, we have to inspect again at T=14 years. If we find a crack on this occasion too, a third inspection is required at T= 22 years. We see that the influence diagram model is able to identify the optimal plan for any sequence of observations.

Most studies of cost-optimal inspection planning deal with a set of inspection times that are common to both the "Find"-branch and the "No find"-branch. This means for instance that both of the branching points A and B in figure 6 (right) would appear at the same time (either at T=14 or at T=16 years) independent of the inspection result at T=6 years. Hence, the optimisation would be performed on the basis of a tree that is symmetrical with respect to the branching points. As seen in figure 6 (right) the tree does not have to be symmetrical. It is noted that the inspection plan corresponding to the lower branch is identical to the initial plan as identified by figure 6 (left). The initial plan is therefore dominated by this event sequence although substantial savings must be expected if "No find" results are obtained.

Note also that the model is capable of analysing situations where the optimal plan is not followed. If for some reason it is chosen not to inspect the hotspot at T=6 years (as initially suggested by the model), all you need to do is to insert evidence of no inspection, and let the program compute the optimal plan given this new situation as well as the associated expected costs. It will suggest an inspection at T=8 years and compute the expected costs DKK 2.40 mill., which is 0.1 mill. more than inspecting at T=6 years. This increase is due to the increased failure probability.

5.2 Reliability indices

The above use of the model yields the optimal plans based on a cost optimisation without side constraints. Building codes often require a lower threshold value for the reliability index to be maintained throughout the lifetime of the structure. We thus have to monitor the reliability index over time in order to secure that the structure complies with the rules. As the failure probability is readily available in all time steps, so is the reliability index. In figure 7 the reliability index is plotted for two different inspection plans corresponding to the upper and lower branch of figure 6 (right).

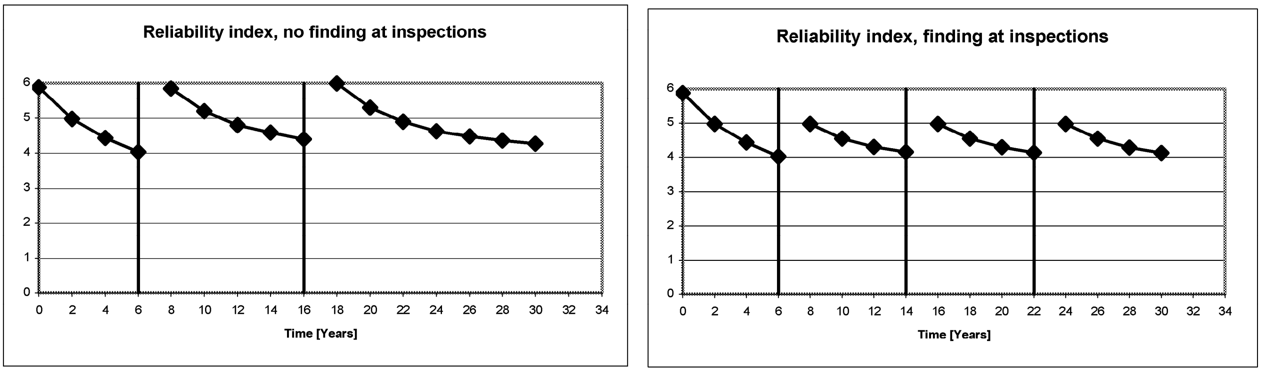


Figure 7: Reliability index as a function of time. Left: In case of NO finding at each inspection. Right: In case of finding at each inspection.

In both cases the reliability index remains larger than $\beta = 4$, although the optimisation was performed without side constraints. The apparent limit is thus formed by the given relationship between inspection, repair and failure costs. If a higher safety level is desired then the failure costs or inspection/repair costs must be modified.

5.3 Sequence effects

As mentioned earlier the main difference between the HUGIN and the PROBAN models is the way the time aspect is modelled. With HUGIN the crack depth is developed incrementally in time whereas a fixed time span is considered using PROBAN. In the influence diagram model the loads are modelled as independent variables in each individual time step. Therefore any sequence of large and small loads may be assigned to the A-variables and the impact evaluated by the program. Such sequence effects are rarely accounted for in traditional fatigue models.

6. Discussion

The decision model is based on a very simple crack growth model. Many improvements in fatigue modelling have been made since Paris' equation was suggested, many of which have been implemented in reliability analyses. Examples are two-dimensional modelling of the cracks, crack closure and threshold ranges. In this study the crack growth formula is used in its deterministic form solely to establish the conditional probability table for the a-variables. Once these tables are created, the crack growth formula is no longer used in the probabilistic model. This implies that any formulation and complexity of the crack growth formula may be chosen without impact on the complexity of the Bayesian network model. More elaborate formulas may therefore easily replace the applied crack growth formula.

The probabilistic model is rather simple as only 3 of the 7 variables are modelled stochastically. This is a choice taken under consideration of the computational complexity of the Bayesian network model. The state space grows exponentially in the number of nodes and in the number of states in each node. With 15 time steps the total table size would be too large if all 7 variables were modelled stochastically with a reasonable discretisation.

Discretisation of the variable domain is necessary as continuous variables can only be handled for very restricted classes of distributions and graphs. The most straightforward approach is to discretise each variable uniformly only taking into account the trade-off between accuracy and computational complexity. However, in some cases better accuracy may be achieved by non-uniform discretisation in which case a more sophisticated algorithm is needed.

In this study the conditional probability distributions were established by means of a deterministic formula and an algorithm assigning the correct probabilities to the right intervals. A more sophisticated sampling algorithm has recently been implemented in Hugin. In this study, however, this feature has not been investigated.

If a crack is found during an inspection, its length may be measured to give an impression of the structures condition. In the presented model it is not possible to enter quantitative information of this kind. However, by simply adding a number of states to the *InspRes*(T_i) -node as well as a mapping from crack length to crack depth, this feature may be included.

Despite the above-mentioned problems, the presented influence diagram model offers the following advantages:

- It readily reproduces results obtained by structural reliability methods.
- The network representation allows easy conveying of the concepts to third parties and makes it easy to validate the probabilistic model.
- The optimal plan may be directly identified once the model has been established and compiled.
- In the event of available inspection results the plan may be updated.
- All optimal plans may be identified.
- Sequence effects may be evaluated.

7. Conclusion

A model for probabilistic fatigue crack growth has been established using Bayesian networks. On this basis a full decision model was built and its results and behaviour have been described and analysed. It was found that the advantages of this method are significant thus demonstrating the potential of the modelling technique. It was shown that influence diagrams are able to correctly model most important aspects of structural reliability methods as well as a number of additional features not obtainable by existing methods. There is room for improvement and the model must be extended in some respects in order to be applicable in practice.

8. References

[1] Madsen H. O., Sørensen J. D., and Olesen R. D., “Optimal inspection planning for fatigue damage of offshore structures”, *Proceedings of ICOSSAR*, San Francisco, USA 1989, pp. 2099-2106.

[2] Sørensen J. D., Faber M. H., Rackwitz R., and Thoft-Christensen P. T., "Modeling in optimal inspection and repair", *Proceedings of OMAE*, Stavanger, Norway, 1991, pp. 281 - 288.

[3] Andersen S. K., Olesen K. G.., Jensen F. V., and Jensen F., "HUGIN - A shell for building Bayesian belief universes for expert systems", *Proceedings of the 11th International Joint Conference on Artificial Intelligence*, San Mateo, California 1989

[4] Jensen F. V., *An Introduction to Bayesian Networks*, Springer-Verlag, New York, 1996.

[5] Jensen F., Jensen F. V. and Dittmer S. L., "From Influence Diagrams to Junction Trees", *Proceedings of the 10th conference on Uncertainty in Artificial Intelligence*, Morgan Kaufmann 1994.

[6] Paris P. C. and Erdogan F., "A critical analysis of crack propagation laws", *Journal of Basic Engineering,* 1963

[7] Madsen H. O., Krenk S., and Lind N. C., *Methods of Structural Safety*, Prentice-Hall Inc. Englewood Cliffs, New Jersey 1986

[8] Ditlevsen O. D. and Madsen H. O., *Structural Reliability Methods*, John Wiley & Sons, 1996

[9] Veritas Research, "PROBAN, The PROBabilistic ANalysis Program", Version 2, Høvik, Norway 1989.

Field Experience with RBI in Oil & Gas Exploration & Production

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Keywords:

Risk based inspection
Risk assessment
Offshore

Abstract:

A specific Risk Based Inspection approach (RBI) was started since 1997 within North Sea branches of TFE Exploration and Production and is now fully operational. As a result of optimised inspection programmes, significant OPEX savings have been achieved in the same time a better management of risks associated with facilities integrity was also improved. Considering potential benefits of such an approach in other contexts, TFE are now expanding the field of application of RBI to other E&P subsidiaries worldwide and to new development projects.

The Regulatory Background

In Europe, the implementation of RBI approach in TFE Started with North Sea facilities and was originally dictated by a move of local regulations from prescriptive requirements to goal settings and self certification concepts. British offshore Design & Construction Regulations (DCR) is the most significant example of the new era [1,2]. DCR came into force in June 1998 and constitute the achievement of the Safety policy progressively developed in the UK after the Piper Alpha disaster in 1988 and following Lord Cullen Report. The rationale of the policy is that Operators are becoming fully responsible as regards how they would develop their integrity verification programmes and demonstrate permanent fitness for purpose of installations. Risk based inspection has then become a natural component of Integrity Verification. Since then, such approaches have been widely disseminated, and are now being applied for a fast growing number of production sites, world-wide.

Risk Based Inspection Principles

RBI being geared by safety considerations, the first essential step is for the Operator is to assess safety criticality of the installations. This can done through the implementation of Quantitative Risk Assessments (QRA), Hazard in operation studies (HAZOPS) and Safety Cases. However, most often, RBI methodologies currently deployed embed simplified QRA's more closely dedicated to inspection activities. Written schemes of examination are then developed, the effort of verification being relative to the risks the element is exposed to. This process, the verification scheme, has to be fully structured so that it remains fully auditable at any time.

Risk based inspection proceeds on these grounds, looking at also the various risks of degradation. The inspection effort in terms of nature, extent and frequencies is adapted to the risks that may affect the installations in the course of operations, corrosion for example, and to the likelihood for incidents or failures. Not only safety critical components are assessed, but also all equipment for which a failure would compromise availability of the facility and induce significant losses of revenue. Inspection shall then be concentrated on critical components and relaxed on others. Low criticality components being in general in relative large number, significant cost savings can be generated, including reduced direct inspection costs, reduced maintenance costs and more importantly increased availability of production installations. Particular attention shall be paid to pressure vessels, inspection of which being an important contributor to production shut downs.

Application to Development Projects

Moving from past practice which used to involve inspection activities only from the start of operations and sometimes from the first occurrence of failures, Operators are now conducted to prepare inspection plans much earlier, during the design phase of new developments. Although this is not always compulsory, TFE promotes this policy for new projects such as the Girassol Project offshore Angola, or the Elgin Franklin Project offshore UK.

This is geared by a number of potential benefits:

Parallel to Maintenance philosophies, Inspection is becoming a genuine part of a modern vision of project management. Optimised inspection programmes will conduct to minimise maintenance plans and plant shut-downs and in some instances, significantly contribute to the economics of a new development. As an example, deep and ultra deep sea production installations will be very sensitive to shut downs as flow assurance from the wells shall be difficult to maintain. Reducing the frequency of topsides facilities shut-downs through optimised inspection is seen as one of the ways to improve the operation of the field.

Risk based inspection is a structuring approach, leading to the clear definition of inspections systems and corrosion loops. In the present context of massive subcontracting of operation and maintenance activities, RBI provides a good way for the Operator to exert his responsibilities and to clearly define and transmit essential requirements to sub-contractors as regards integrity verification.

The History of the TFE approach

This approach originates from a previous base frame implemented in 1996 by the Norwegian branch of Elf EP, Elf Petroleum Norge. On the basis of the new regulatory background in the UK, the implementation of a structured RBI approach was started by Elf Exploration UK plc, operating the Claymore, Piper Bravo and Saltire platforms. After existing commercial systems were evaluated, the decision to develop a proprietary approach and software was taken in 1997. Reasons leading to this choice were of different nature:

- The absolute requirement to integrate the vision of an Operator, vision which was found lacking to commercial systems at the time,

- The necessity to establish and validate inspection regimes compiled by the system on the basis of Company's experience and operational practice,

- The requirement to integrate TFE know-how in some key parameters influencing the approach, such as internal corrosion in CO_2 and H_2S bearing fluids.

- The requirement to operate a system that would minimise data collection and specialist technical support

The methodology was further formalised through the dedicated software FAME (Facility Assessment Model in Exploration) briefly described further on. On completion of the first versions developed for EEUK plc assets, subsequent improvements were brought to comply with incoming Elgin & Franklin major project. Since then the field of application has been significantly widen to other TFE subsidiaries such as Holland, Nigeria, Angola, Congo, Brunei and Abu Dhabi. During this implementation phase FAME and the underlying philosophies have been presented to certifying bodies and Authorities in several countries and have been favourably received. The approach is now fully integrated in Corporate Integrity policy and application to a growing number of production sites contributes to improve coherence of our inspection practice world-wide and reinforces the robustness of the approach.

The principles of the approach

The methodology is basically complying with the reference document API P 581 [3]. However, it has been significantly lighten in a number of areas in order to take full consideration of specific process systems in Oil & Gas production and to minimise the collection of data. The establishment of inspection regimes is also specific and based on our field experience. Compared to refining process systems which are the main field of application of API 581, production process systems are significantly simpler. Potential degradation modes are also comparatively in limited number. As a result, computerised degradation models were developed and integrated into the software so that implementation to new facilities could be achieved in a simple manner and at minimum cost. In the same time, the level of subjectivity when assessing criticality parameters could be significantly reduced. The main steps of an assessment can be described as follows:

① Determine the criticality of a system or component

② Consider the actual condition of the equipment using inspection history

③ Determine optimised inspection regimes

This process is automatically generated by FAME, the logic of it being further detailed below:

⇒ **Safety criticality is established from three fundamental parameters:**

- Consequence factors are calculated from the assessment of potential hazard associated to the fluids themselves, to operating conditions and to the type of equipment involved. When Quantitative Risk Assessments (QRA) are available the software offers the option to utilise QRA numerical results. In this case QRA results take priority over calculated factors. Coherence between both methods has been checked and found satisfactory. Noticeably, the QRA Option has been used for the Elgin & Franklin Facilities.

- Degradation modes in operation (e.g. corrosion, mechanical failures, erosion). Numerical probability factors are compiled by specific computerised models established for a wide variety of conditions, fluids and materials. These models are based on TFE published know how, published data and accepted standards. They compose the heart of the software and largely contribute to the cost effective implementation of the approach.

- Consequences in terms of equipment availability. Although a number of process systems do not present any significant safety concern, they often remain essential for the operator if a failure can potentially conduct to shut down production and induce direct loss of revenue. As a result, the approach will not only consider safety critical elements but all elements for which a failure would impact on facilities availability.

These principles are illustrated in fig.1. In reality, it is a large number of parameters which will be assessed when performing a criticality assessment. Some of them will also interact between them. Depending on the nature of the process they represent, they will influence the probability or the consequences of an event. All these factors will then be compiled following varied processes of averaging. The schematic process leading to the determination of the criticality is shown in the simplified diagram in fig.2.

⇒ The consideration of equipment condition

The condition of a piece of equipment, known through inspection history is obviously a key parameter when determining inspection regimes. In contrast to the preceding criticality assessment which is of a predicting nature, inspection findings are the real expression of the behaviour of a component over its life and within its operating environment. In order to rationalise the quantitative integration of this parameter in the evaluation, tables have been established, showing the severity grading of typical defects or anomalies (12 levels from A to L). These tables are based on inspection procedures in use in some subsidiaries. They are not only based on the description of a defect but also on the nature of the expected corrective action and the level of decision making (the site inspector, the materials/corrosion specialist or the operational management). Despite extreme simplicity, they are actually found as a good tool to quantitatively express the condition status of a component. Being integrated in the FAME software, these tables allow to automatically generate conditions factors. These factors have a major influence on calculated inspection regimes (fig.2).

⇒ The determination of optimised inspection regimes

A relatively simple approach has finally been established on the basis of the long term inspection experience gathered on TFE operating fields. Inspection requirements are automatically generated by the software for external inspection and NDT as well as for internal inspection of pressure vessels. Simulations and reference to actual experience have been utilised to balance the outputs of the software. A simplified overview of the process conducting to the determination of inspection frequencies is shown in fig 3 and 4. Frequencies are obtained by crossing calculated criticality (from 1 to 6, 1 being the more severe) and condition status (A to L). These diagrams are exclusively based on TFE experience and policy in the North Sea. They should not be directly utilised to other environments would they be regulatory, technical or operational.

The diagrams are limited to so called “generic” inspection regimes leading to the preparation of annual inspection programmes. When specific concerns have been identified in the course of criticality assessments (a high risk of internal corrosion for instance) specific requirements are automatically proposed by FAME. These requirements superimpose to generic regimes.

Finally, on completion of the evaluation work it is absolutely required to have the results validated by an experienced inspection/materials specialist. Although the various computerised models included in the software allow to systematically screen potential problems, the full control of the approach can only be obtained if the correct level of expertise is exerted. Our position is presently to have RBI based inspection plans prepared in TFE EP Headquarters if the level of experience/competence is not sufficient locally. After a specific training of the future users, updating can generally be done on site, and only requires regular reviews.

The Practical deployment of the approach

Present methodology is aiming at generating first level inspection plans, so that the Operator's essential requirements are transmitted is a clear manner to operational teams within the Company or to the Contractor in charge of inspection activities. These first level plans must then be further developed into detailed inspection programmes, including the location of inspection points, inspection methods and techniques to be used to correctly monitor suspected degradation. Inspection programmes are co-ordinated with maintenance plans so that surveys, maintenance campaigns and plant shut-downs can be optimised (fig.5).

After three years of operation, the approach has proved to be reliable and effective. After an initial period dedicated to balancing the outputs of the system against operational experience, implementation has been relatively easy to achieve. Thanks to the user friendliness of the program and context sensitive data entry, the induction period for personnel having the required competence in inspection and materials engineering is limited to a couple of days. The total duration required for generating first level plans for a medium size platform is usually limited to a couple of man months. Obviously, the amount of work will largely depend on the size and complexity of the installations. A longer period is required for complex Platforms such as the Elgin PUQ in the North Sea. The implementation time will also depend on the availability and on the quality of the data, especially for ageing installations for which design and construction data may be incomplete or missing.

The other advantage of the approach is also to take benefit of the full integration of degradation models such as the risk for internal corrosion or erosion. Company's technical positions could therefore be immediately applied, which was considered of prime importance when inspection activities were sub-contracted. When applied to projects, the approach has allowed in some instances to identify incorrect material selection. In another case, it has allowed to rapidly implement selective inspection surveys after an in service failure due to internal corrosion was observed on a site.

The benefits

Potential benefits associated to the implementation of an RBI approach are of varied nature. Thanks to a fast growing experience, they can be summarised as follows:

- **A better control of the risks** associated to integrity of installations. This aspect is of particular importance in the North Sea, considering also that the Company's image will be reinforced.
- **Optimised inspection programmes**. Such an approach is basically conducting to reduce the inspection effort on low risk components. These components are usually present in large number and represent a large proportion of inspection programmes and maintenance plans. Direct cost savings of 0.5 M$ corresponding to the reduction of annual inspection budgets have been achieved by implementing the RBI approach in some of the Group North Sea facilities.
- **Significant indirect cost savings** associated to the improved availability of process systems. Unit and platform shut-downs are often driven by the requirement to internally inspect pressure vessels. By offering the opportunity to extend inspection frequencies, RBI can significantly reduce production losses associated to plant shut-down. On some

platforms we found that for more than 200 vessels the inspection interval could be moved from 5 to 10 years. Financial savings are expected to be significant, although they will depend on the Operator's usual practice. For one major African subsidiary it has been estimated that savings were potentially as high as 30 M$ over 5 years, as production losses associated to shut-downs could be significantly reduced.

- **A clear way to transmit essential inspection requirements** to Contractors who will further develop and implement detailed inspection programmes, allowing the Company to maintain the correct level of operatorship and implement integrity verification on an indisputable basis.
- **A structuring and systematic approach** offering long term traceability. All operating data and associated inspection requirements are stored in a data base embedded in FAME. Successive changes over the field life will therefore be easily visible especially at the occasion of audits.
- **Better practices.** Besides visible benefits, a major improvement found to appear through the approach is undoubtedly the commitment of those involved in integrity verification to improve their working practice by considering more closely certain aspects such as safety, process and operations.

Conclusions

Risk Based Inspection has been introduced since 1997 in the Norwegian and Exploration & Production branches of TFE. A specific approach was developed on the basis of the specific nature of Oil & Gas production installations. Long term operational experience of TFE in the North Sea was also fully integrated in the approach. Although being compatible with RBI principles developed in API 581, the methodology could be significantly simplified and made easy to implement. The approach is based on the FAME software developed in house which embeds computerised degradation models and allows to quickly assess the various risks associated to potential failure modes. Safety criticality of process components can be quantified and inspection regimes determined accordingly.

The deployment of the approach has conducted to optimise inspection programmes in the same time a better management of operational risk was achieved. Substantial savings have been achieved on direct inspection costs but main benefit obtained with the reduction of operating costs, thanks to increased inspection intervals and better availability of production facilities. The approach has also been successfully applied to establish initial inspection regimes during the design phase of major development projects.

Further to initial deployment in the North Sea, other applications are being developed in other TFE sites, such as Africa, the Middle East and Far East. Although objectives may differ according to varied local environments and regulatory contexts, these applications can now be implemented without any technical major difficulty and at minimum cost. Finally, it is considered that the reference to a common approach will reinforce the coherence of inspection practices throughout the various subsidiaries of the TFE Exploration and Production branch.

References

1- HSE - SI 913 The Offshore Installations and wells (Design & Construction, etc.)
2- Guidance on regulations L85 - A guide to integrity, workplace environment and miscellaneous aspects of the offshore installations and wells
3- API publication 581 - Base Resource Document on Risk Based Inspection - Preliminary draft, May 1996

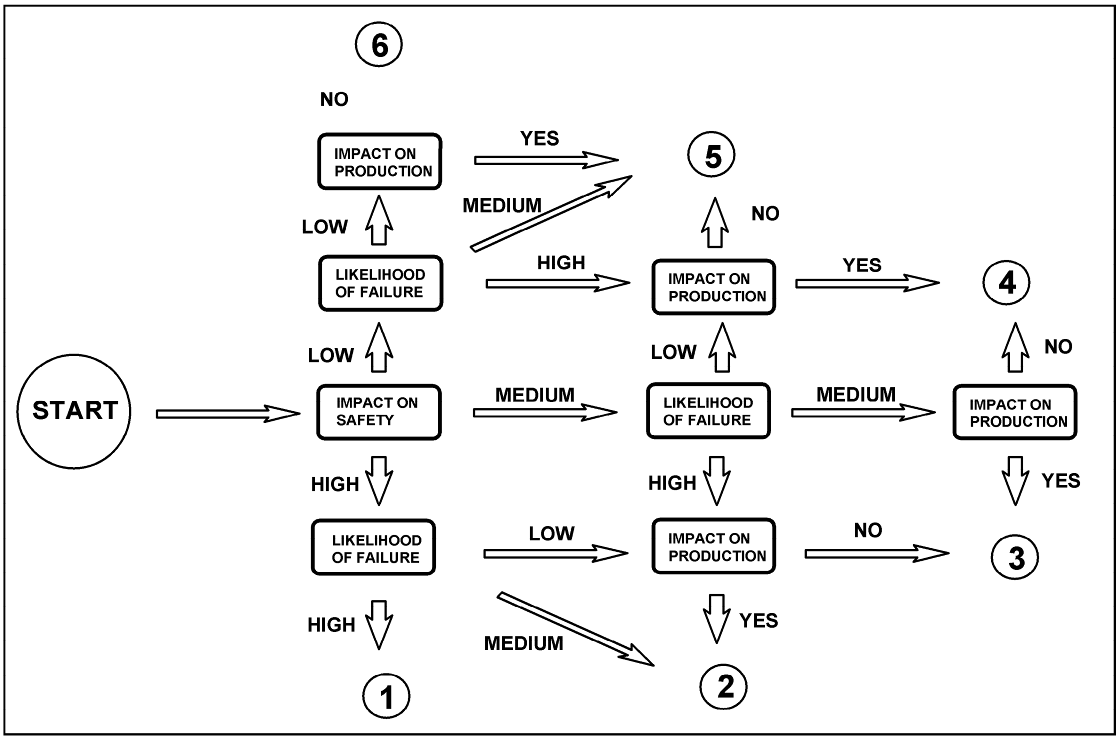


fig.1 – Principles of the determination of a component criticality

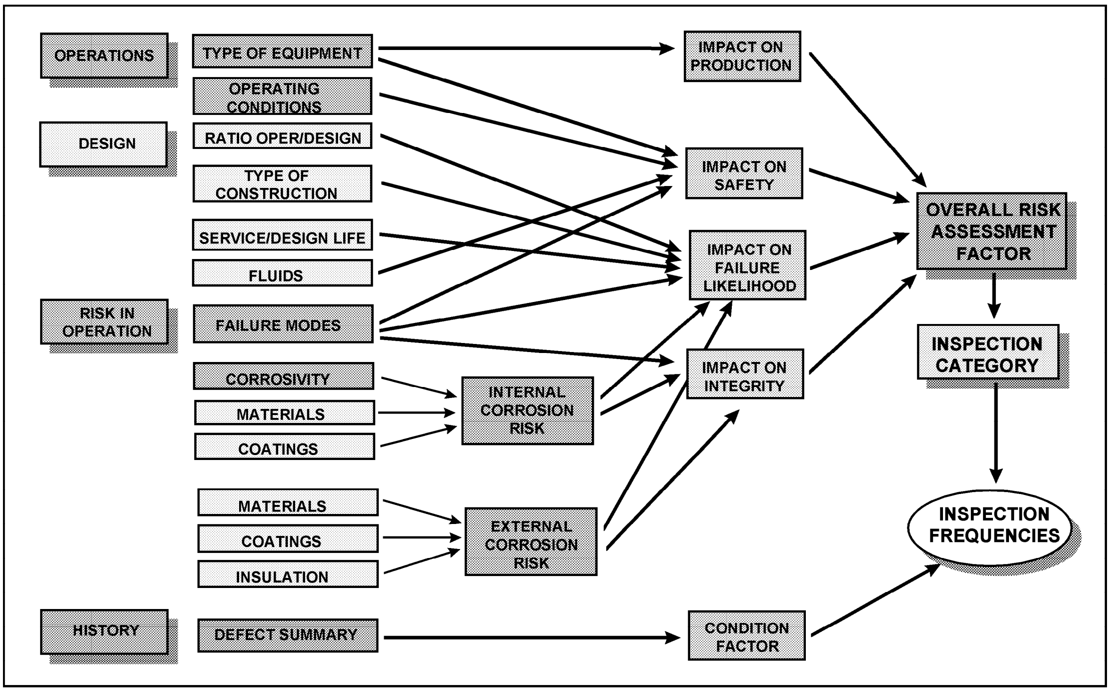


fig.2 – Simplified process showing interaction between criticality parameters

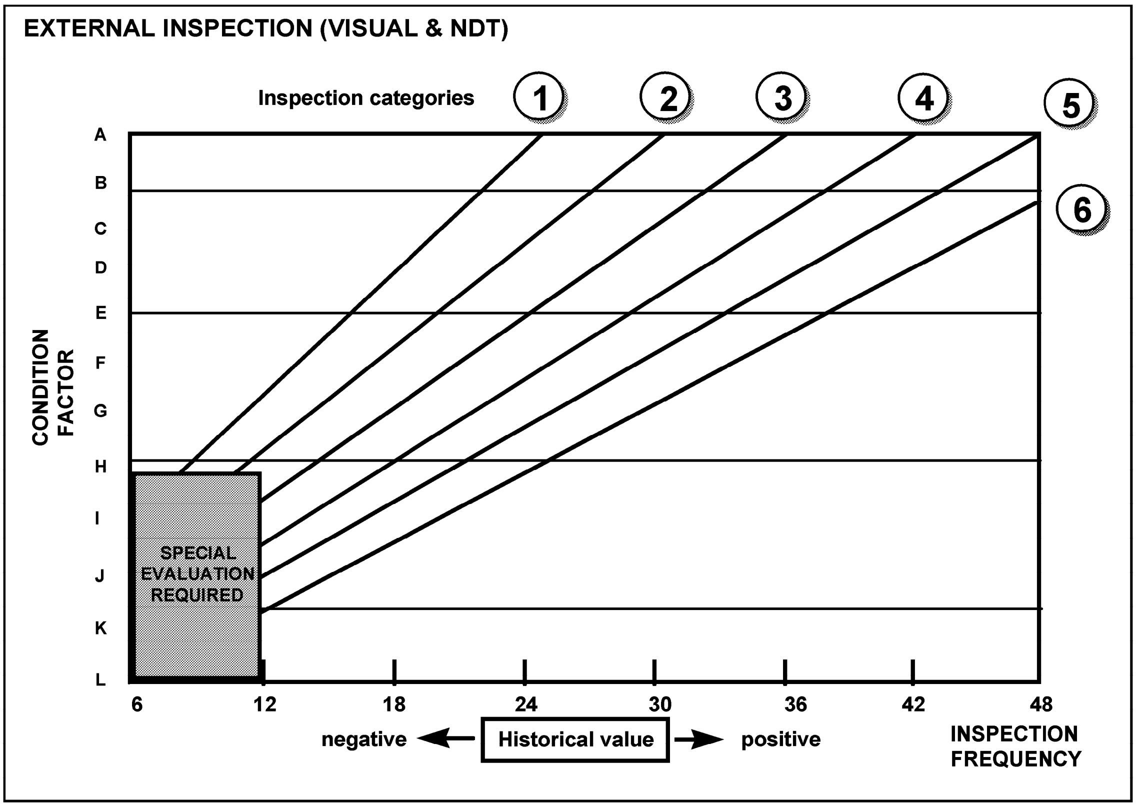


Fig. 3 – Principle for determining external inspection frequencies

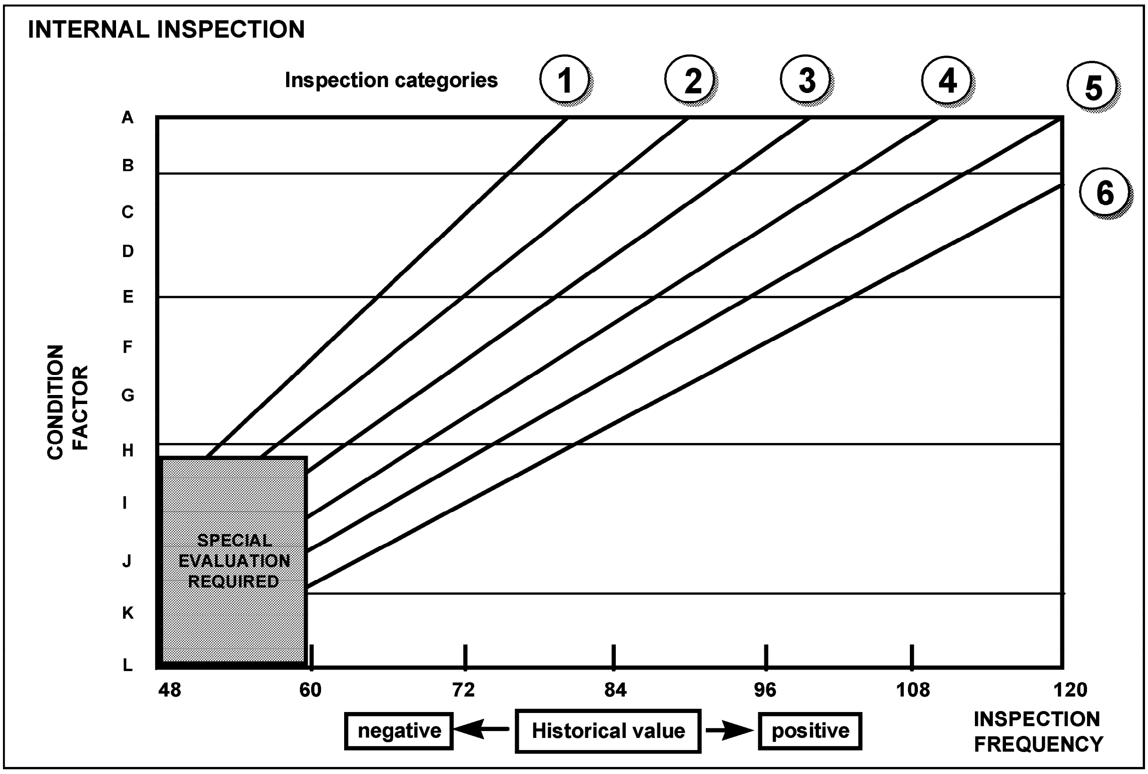
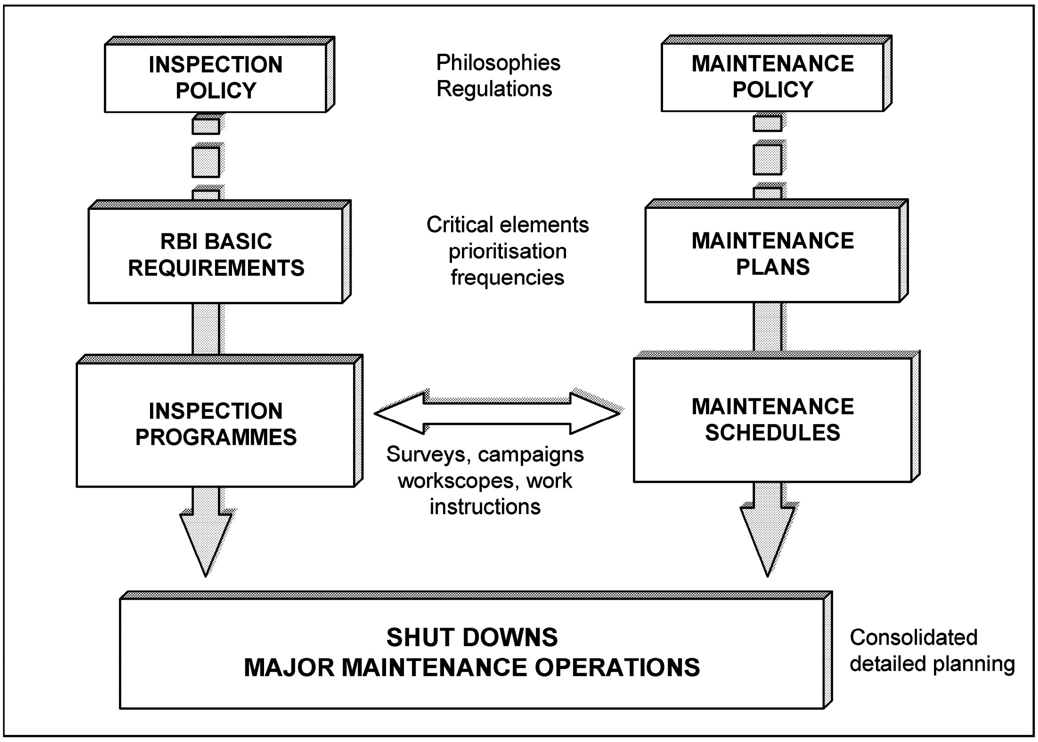


Fig. 4 - Principle for determining external inspection frequencies

Fig. 5 – Maintenance and Inspection planning process



Basis for offshore NDT inspections ranking, in IMR plans

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Keywords : Offshore, Inspections Optimization, Existing Structures, Probability of Detection, IMR plans, ROC curves.

Summary

An operator of large civil engineering structures has to ensure that structural integrity is maintained to a sufficient level during in-service life, or during the total operating life if it is to be prolongated. This can be achieved by Inspection, Maintain and Repairs plans (IMR), as an aid-tool for decisional purposes. Such plans are complex and can be expensive. This leads to the global optimization of these plans, and particularly inspection plans. In this context, we present original results on the use of inspections results in IMR plans.

This approach is based on the decision and detection theory for inspection and include both the probability of false alarm and the probability of detection. It is shown how to use them, and the effect of false alarm and miss detections on the global cost of inspections planning is highlighted through an example. We give then some indications for NDT criteria ranking, using the same cost analysis.

1 Introduction

Existing civil engineering works have to be monitored during their whole in-service lives. The aim is to provide strategy for repairs or replacements of damaged components, to inspect the major structural parts, and finally to maintain the structure for its operating functions and security requirements. This is the so-called IMR (Inspections, Maintain, Repairs) plans. For very large structures (or with a significant number of failure components), it is necessary to optimize those plans, in terms of costs and performance. The optimal plan would be : to inspect at the right place, at the right time, and with the right tool, at lowest cost. The same procedure has to be applied for Maintenance and Repairs. Recently, there is an increasing need for such plans in civil engineering, and particularly for end-of-live structure re-assessment. This is the case for old bridges, dams, power plants or offshore platforms, that where designed for 20 to 50 years, and whose life is to be prolongated for economics purpose.

IMR plan optimization is a typical decision problem. In this paper, we will focus on the inspection aspects, which, through detection theory point of view, is a decision problem too. Inspections will be first described from a theoretical point of view. It is shown that the set (PoD, PFA) (ie Probability of Detection, Probability of False Alarm) is needed to characterize completely an inspection tool, or inspection results. Then, in the IMR plan context, this set is introduced in the optimization scheme. The effects of bad inspections performance in terms of cost are shown through an example, showing the bad effects of PFA. We insist on the fact that PFA has to be taken into account, when introducing inspections results, as it could introduce false scenario failure, whose cost are high.

2 Monitoring strategy of in-service works

2.1 Preliminary aspects

For risk sensitive civil engineering works, IMR actions are planned for the whole life of the structure. This means that one may predict at the design stage:

- dates and/or frequency of inspections and tools used depending on their performances and ability to be used in-situ,
- dates and/or frequency of structural maintenance,

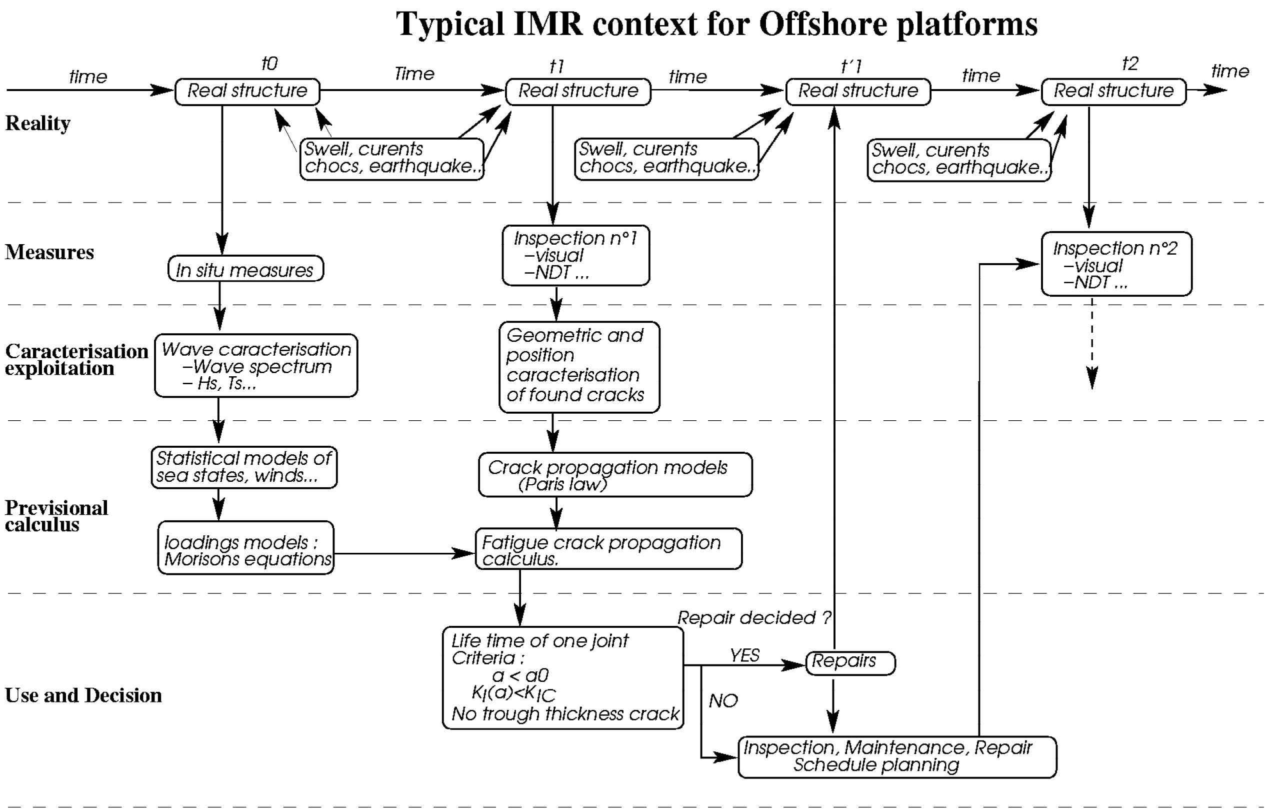

Figure 1: *simplified IMR context for fatigue analysis*

- component repairs and/or change if possible, and criterion of decision associated.

This leads to an overall cost estimation, and global timing operations. For already in-service buildings, the problem is specific, and one may consider two cases:

- numerous damages localizations exists on structures, and one should optimize inspection areas,
- after inspections, unpredicted damages are discovered, and the initial plan cannot be applied : you have to ensure your structure is safe. This means that a general structural safety reassessment is to be done.

In in-service context, figure 1 represents a simplified IMR plan for offshore structures. One of the most important damages for such structures are fatigue of tubular nodes. This figure shows a decision scheme for fatigue analysis. Maintenance is not represented, since it is assumed to be made with inspections.

3 Inspections

Inspection is an essential step in IMR plans, since it's the only way to achieve to a partial view of the structural local integrity. A compete sight can't be reached due to the size of the monitored structures. This underline the importance of the localization to be inspected on large studies. An optimal inspection is located where damages are important, and is done with the right inspection tool. On existing structures, Non Destructive Techniques (abbreviated NDT) are widely used. In this paper, the focus will be on crack detection, as crack is a very common damage in concrete and steel civil engineering works and is representative of a significant lost of local integrity. All NDT tools have limitations, and in complex environment, their capacities are different from those given by laboratory or factory. This is the case for under-water inspections of offshore structures and dams, and for some structural parts of bridges where accessibility is low and the condition of using

NDT tool is not optimal. This leads to lower performances than expected. In the offshore sector, an important effort of inter-calibration was made with the ICON project [2], in order to have an unified over view of different tool performances, in realistic in-situ conditions. All the data performances where introduced in a unique database. This allows an optimal use of different tools using them at their full capacity : the decider has very powerful informations to decide which best NDT tool to use, relatively to his performances, for a particular application. Specifying ranking criteria is very difficult in this complete and multi-disciplinary context.

4 Inspection performances

One challenge in the IMR plan strategy is to use the whole existing information on NDT performances to optimize their use. Most of time, inspections results only deals with probability of detection (abbreviated PoD) , which is the probability to detect an existing crack, and with a_d which is the minimal size crack, under which it is assumed that no detection is done : thus, probability of detection is defined as :

$$PoD(a) = P(a > a_d) \tag{1}$$

where a is the crack size. In a probabilistic scheme, a and a_d are stochastic variables, see [8, 7, 6]. However, in this paper crack size will not be discussed, as only detection will be focussed on. Nevertheless, the use of PoD alone is not suitable. One must consider another variable which is called probability of false alarm (abbreviated PFA). To resume, false alarms corresponds to a detection of a non existent crack and PFA has noise as nature, and different sources : human, nature of phenomenon to be measured, environmental conditions and so far. There is a need to use PFA (see [10]), as for underwater technology in offshore structures for example, finding a non existent crack gives rise to false scenario in the failure tree, resulting in an non negligible over cost. One must remember the harsh operating conditions of these inspections for such structures. As results, false detections are common, and non existing large cracks can be detected. In the next section, both PoD and PFA will be introduced, in the detection theory context.

5 Introduction of inspections in the decision process

5.1 Inspection theory

Lets introduce fundamental developments on inspection theory. To make an inspection, is equivalent to take a decision. To illustrate this, we take a typical crack detection problem. Assume we have to detect an existing crack in a body (here a structural offshore tubular node), and a NDT tool to detect it. After inspection, the NDT tool will give a result, which could be : no crack, or presence of crack. In fact what we have is a decision on the state of the body : cracked or not. The same scheme could be applied, if the body is really uncracked. This is resumed in Fig. 2.

It is then necessary to consider 4 inspections events:

- E_1 : no presence of crack, and no crack detected,
- E_2 : no presence of crack, and one crack detected : this is the False Alarm,
- E_3 : presence of crack, no crack detected : Miss of a crack,
- E_4 : presence of crack, and crack detected.

In those events definitions, the focus is on presence or absence of crack, after an inspection : the aim is finally to know whether or not we've really got an existing crack. To formalize this, we introduce the decision theory. More details on decision and detection theory could be find in [10, 3, 1]. In a probabilistic point of view, we consider the binary random variable 'presence of a crack' X, which value is one if a crack is present, zero otherwise. We note $d()$, the random inspection decision function, which value is one if a crack is detected (ie we decide that one crack in present), zero

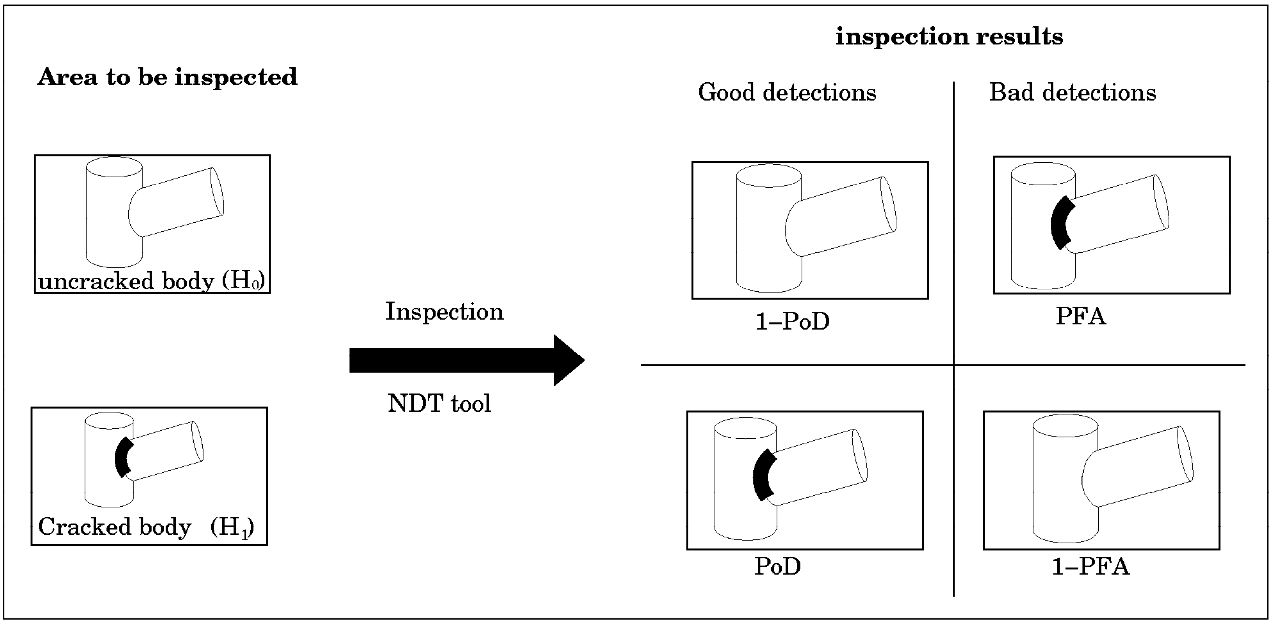

Figure 2: *Illustration of detection theory*

otherwise. Thus, the probability of false alarm (PFA), and the probability of detection (PoD), could be written, according to Bayes' rules (see equation 2):

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \tag{2}$$

$$\begin{aligned} PoD(X) &= P(d(X) = 1 | X = 1) & (3) \\ PFA(X) &= P(d(X) = 1 | X = 0) & (4) \end{aligned}$$

This gives the rights definitions of PoD and PFA:

- PoD is the probability to decide presence of a crack (crack detection), knowing there is effectively an existing crack,
- PFA is the probability to decide presence of a crack (crack detection), knowing there is effectively no crack.

Those definitions are consistent with inspection calibration/inter-calibration aspects. See for instance [2, 5]. Note, and this is very important, that an inspection results can be entirely characterized by a set of PoD/PFA. We can know express $E_{i(i=1,2,3,4)}$ as function of PoD and PFA, using the Bayesian rules. For E_1 we have:

$$\begin{aligned} P(E_1) = P(X = 0 | d(X) = 0) &= \frac{P(X = 0 \cap d(X) = 0)}{P(d(X) = 0)} & (5) \\ &= \frac{P(d(X) = 0 \cap X = 0)}{P(d(X) = 0)} & (6) \\ &= \frac{P(d(X) = 0 | X = 0)}{P(d(X) = 0)} P(X = 0) & (7) \end{aligned}$$

Let γ be the probability of presence of crack at the inspected area, then we have:

$$P(X = 1) = \gamma \,, P(X = 0) = 1 - \gamma$$

$P(d(X)=1)$ could be expressed as:

$$\begin{aligned} P(d(X)=1) &= P(d(X)=1|X=0)P(X=0)+P(d(X)=1|X=1)P(X=1) && (8)\\ &= PFA(X)(1-\gamma)+PoD(X)\gamma && (9)\\ P(d(X)=0) &= (1-PFA(X))(1-\gamma)+(1-PoD(X))\gamma && (10) \end{aligned}$$

This leads to the following probabilities:

$$\begin{aligned} P(E_1) &= P(X=0|d(X)=0)=\frac{(1-PFA(X))(1-\gamma)}{(1-PoD(X))\gamma+(1-PFA(X))(1-\gamma)} && (11)\\ P(E_2) &= P(X=0|d(X)=1)=\frac{PFA(X)(1-\gamma)}{PoD(X)\gamma+PFA(X)(1-\gamma)} && (12)\\ P(E_3) &= P(X=1|d(X)=0)=\frac{(1-PoD(X))\gamma}{(1-PoD(X))\gamma+(1-PFA(X))(1-\gamma)} && (13)\\ P(E_4) &= P(X=1|d(X)=1)=\frac{PoD(X)\gamma}{PoD(X)\gamma+PFA(X)(1-\gamma)} && (14) \end{aligned}$$

First note that there is two way to understand those events:

- the case of inspection results is considered. the two sets respectively (E_2,E_4) and (E_1,E_3) represents respectively crack detection, and no crack detection.
- the crack point of view is considered : the two sets respectively (E_1,E_2) and (E_3,E_4) represents respectively crack absence and crack presence.

Upon what is to be considered, one or other of this sets may be used. Second, some events are complementary. By addition we have:

$$\begin{aligned} P(E_1)+P(E_3) &= 1 && (15)\\ P(E_2)+P(E_4) &= 1 && (16) \end{aligned}$$

This means that only one set of two events can be considered. This is due to the fact that (PoD,PFA) is a typical set of the detection capacity of the NDT tool. Finally, we introduce the following transformation:

$$\left.\begin{array}{c}\gamma\\ PoD\\ PFA\end{array}\right\}\rightarrow\mathcal{T}\left\{\begin{array}{c}1-\gamma\\ 1-PFA\\ 1-PoD\end{array}\right. \qquad (17)$$

Lets demonstrate that $\mathcal{T}(P(E_1))=P(E_4)$ and that $\mathcal{T}(P(E_2))=P(E_3)$:

$$\begin{aligned} \mathcal{T}(P(E_1)) &= \mathcal{T}\left(\frac{(1-PFA(X))(1-\gamma)}{(1-PoD(X))\gamma+(1-PFA(X))(1-\gamma)}\right) && (18)\\ &= \frac{(1-(1-PoD(X)))(1-(1-\gamma))}{(1-(1-PFA(X)))(1-\gamma)+(1-(1-PoD(X)))(1-(1-\gamma))} && (19)\\ &= \frac{PoD(X)\gamma}{PFA(X)(1-\gamma)+PoD(X)\gamma} && (20)\\ &= P(E_4) && (21)\\ \mathcal{T}(P(E_2)) &= \mathcal{T}\left(\frac{PFA(X)(1-\gamma)}{PoD(X)\gamma+PFA(X)(1-\gamma)}\right) && (22)\\ &= \frac{(1-PoD(X))(1-(1-\gamma))}{(1-PoD(X))(1-\gamma)+(1-PFA(X))(1-(1-\gamma))} && (23)\\ &= \frac{(1-PoD(X))\gamma}{(1-PoD(X))\gamma+(1-PFA(X))(1-\gamma)} && (24)\\ &= P(E_3) && (25) \end{aligned}$$

Equations (18),(21) and (22),(25) underline an interesting mathematical property of $\mathcal{T}$: it is an involution. Thus we have:

$$\mathcal{T} \circ \mathcal{T} = \mathrm{I}_d, \text{ and } \mathcal{T} = \mathcal{T}^{-1} \tag{26}$$

$$\left.\begin{matrix} \gamma \\ PoD \\ PFA \end{matrix}\right\} \rightarrow \mathcal{T} \left\{ \begin{matrix} 1-\gamma \\ 1-PFA \\ 1-PoD \end{matrix} \right. \rightarrow \mathcal{T} \left\{ \begin{matrix} 1-(1-\gamma) \\ 1-(1-PoD) \\ 1-(1-PFA) \end{matrix} \right. = \left\{ \begin{matrix} \gamma \\ PoD \\ PFA \end{matrix} \right. \tag{27}$$

This important role of this transformation will be shown further.

5.2 Cost analysis

The optimization will be seem through a global cost analysis, and the main objective will to reduce this cost, for same performances inspections. This basic cost analysis consist in calculating the cost function $E(C)$, defined by the expected total cost:

$$E(C) = \sum_i C(S_i)P(S_i) \tag{28}$$

where :

- $C(S_i)$ is the cost associated with the i$^{\text{th}}$ scenario,
- $P(S_i)$ is the probability that the i$^{\text{th}}$ scenario occurs.

In the following, $\overline{C}_i$ is defined as an over cost resulting of bad consequences due to a bad detection. Let consider the following true scenarios due to NDT inspections results, using the same definition than for the E_i set :

- S_1 : no crack detected, the associated cost is C_1,
- S_4 : crack detected, the associated cost is C_4, comprising repair,

Let consider the dual scenarios, giving false indications :

- S_2 : no crack present, but one detected, cost is $C_2 = C_1 + \overline{C}_4$. In case of repair decision, this is the same cost as in scenario 4 ($C_2 = C_4$), and there is no bad structural failure consequences. However this is not optimal, since the over-cost $\overline{C}_4$ is high, due to high cost repairs. If no repair is made, this leads to false failure scenario increasing highly $\overline{C}_4$. In this particular case, we have $C_1 + \overline{C}_4 > C_4$.
- S_3 : crack present, but missed, cost is $C_3 = C_1 + \overline{C}_1$. Depending on the size of the crack, and its structural criticity, over-cost $\overline{C}_1$ could be : very high, ex if missing a large crack, or very low, ex missing a small or non critic crack.

Now it is possible to evaluate equation (28), trough inspection results. In case of detection (decision of crack presence), using (28) and (16):

$$\begin{aligned} E(C) &= \sum_{2,4} C(S_i)P(S_i) && (29) \\ &= C_2P(E_2) + C_4P(E_4) && (30) \\ &= C_4P(E_4) + (C_1 + \overline{C}_4)(1 - P(E_4)) && (31) \end{aligned}$$

In case of no crack detection (decision of crack absence), using (28) and (15):

$$\begin{aligned} E(C) &= \sum_{1,3} C(S_i)P(S_i) && (32) \\ &= C_1P(E_1) + C_3P(E_3) && (33) \\ &= C_1P(E_1) + (C_1 + \overline{C}_1)(1 - P(E_1)) && (34) \end{aligned}$$

Both (31) and (34) show two terms, one associated with good detection, and the other with bad detection. They are in a certain way complementary, and both are function of PoD, PFA, and γ. It is attractive to understand how costs functions vary with those parameters.

5.3 Illustration

We now study the effect of over costs in the total expected cost. In equations (31),(34) respectively the expected over cost are $(C_1 + \overline{C}_4)(1 - P(E_4))$ and $(C_1 + \overline{C}_1)(1 - P(E_1))$ respectively. By plotting the probabilities of 'bad' events $(1-P(E_4)) = P(E_2)$ and $(1-P(E_1)) = P(E_3)$, as function of PoD and PFA for given values of γ, we can show how PoD, PFA, and γ infer on global cost. Figure 3 shows $P(E_2)$, and figure 4 shows $P(E_3)$, evolutions on the (PoD,PFA) plane, for three different values of γ:

- $\gamma = 0.1$ which represent a low probability presence of crack. It is typical of large cracks,
- $\gamma = 0.5$ which represent a middle probability presence of crack, most representative of common cracks founds,
- $\gamma = 0.9$ which is representative of the small cracks population.

All this probabilities can be inferred from natural crack size probability density functions : [4] assume that the size of preexisting cracks in welded structures is distributed according to an exponential law. The interpretation of figures is as follows:

- for $P(E_2)$, which is associated with false alarms, we have (fig. 3):
  - $\gamma = 0.95$: $P(E_2) \approx 0$ means that for short cracks, the effect of PFA is non significant, even for small PoD values.
  - $\gamma = 0.5$: the effect of PFA increase, even for PoD values near 1.
  - $\gamma = 0.1$: for large cracks, and very large cracks, PFA has a dramatical effect on overall cost as $P(E_2)$ reaches 1 for wide range on the (PoD,PFA) plan, and it is near independent of PoD values. As result, the over cost has a very high probability to occur... This is why PFA has a significant effect on global IMR plans, for large cracks.
- for $P(E_3)$, which is associated with non detection, we have (fig. 4) :
  - $\gamma = 0.95$: for small cracks, the effect of PoD is very high, and is near non dependent of PFA. Only in case of high values of PoD, the effect of non detection is small. Over cost can be high in case of small defect propagation without further inspections, or low otherwise.
  - $\gamma = 0.5$: for intermediate values of γ, ie common cracks, the effect of PoD becomes less significant, when effect of PFA increases.
  - $\gamma = 0.1$: for large cracks, overall effects are insignificant, and only large PFA values acts.

Lets note that the effects of PoD and PFA are inverted, for inverted probability of crack presence γ. This is due to the linear transformation $\mathcal{T}$. One must consider this transformation as a fundamental one in inspection results, and in IMR plans. It governs the expected cost in an IMR context, and for important buildings, where failures leads to serious consequences, as dams, offshore platforms, power plants, it is of importance to consider it.

6 NDT tools performances ranking

This cost analysis can serve as a basis for NDT tools performances ranking. NDT tools performances consists in laboratory tests, in measuring for some defects size the PoD and PFA. Then, PoD is plotted against PFA, which give the Receiver Operating Characteristics curve, abbreviated ROC [1, 9]. This curve shows clearly the performances of an NTD tool, the operator's influence etc... The best point, for which it should tend is the point $(PoD, PFA) = (1, 0)$. On the previous plots, we overprinted two ROC curves, representing two inspections methods, one with very good overall performances (very close to the $(1, 0)$ point), the other less efficient. We know that a qualified operator has performance results one the ROC curve, and on the part which closest to the $(1, 0)$ point, we can estimate the over costs due to bad inspections, and compare hence two NDT tools, in terms of cost performances. A basic ranking criteria could be for example fixing a maximum overcost on the cost associated with the ROC curve.

7 Conclusions

The use of detection and decision theory is very helpful and powerful. Their use permitted to explicit costs inspections functions in an IMR context, on a rational basis, including PFA and PoD. We show that the set of the three variables PoD, γ, PFA characterize completely an inspection. We explicit then a transformation in the three inspection variables space, whose structure is important : it permit to understand why PFA is as important as PoD, depending on the size of existing cracks. Lots developments are available for PoD and at the opposite, few for PFA. Future IMR plans developments should introduce such variables, even if PFA characterization is somewhat coarse. This is particularly true for harsh environmental inspection conditions, as for example in offshore underwater inspections, or in large structures with reduced access. Finally, we clarified a basis for NDT tool criteria ranking, based on a cost function.

The study we presented is a basis for a comparative tool of in-situ inspection performance comparison, allowing a better inspection technique choice. Further works should focus on the following:

- PFA characterization,
- introducing the classification, in the detection theory, allowing the the use of PoD curves, PFA curves and density probability function of natural existing cracks,

References

[1] Arquès P.Y. *Décisions en traitement du signal*. Editions Masson, Paris, 1979.

[2] Barnouin B., Lemoine L., Dover W.D., Rudlin J., Fabbri S., Rebourcet G., Topp D., Kare R., Sangouard D. Underwater inspection reliability trials for offshore structures. In New York ASME, editor, *Proceeding of the 12th International Conference on Offshore Mechanics and Artic Engineering*, volume 3B, pages 883–890, 1993.

[3] Charabit M. *Eléments de théorie du signal : les signaux aléatoires*. Editions Ellipses, Paris, 1990.

[4] Cioclov D. D. A model for fracture risk analysis. *Revue roumaine de sciences techniques. Serie de mécanique appliquée*, 6:613–622, 1986. Fasicule 31.

[5] Goyet J. Evaluation des apports de la technique d'inspection FMD. ATA 580 A - OPI. Rapport interne Bureau Veritas, 1996.

[6] Goyet J., Paygnard J.C.,Maroini A., Faber M.H. Optimal inspection and repair planning : case studies using IMREL software. In New York ASME, editor, *Proceeding of the 13th International Conference on Offshore Mechanics and Artic Engineering*, volume 2, Février 1994.

[7] Guoyang Jiao. Reliability analysis of crack growth with inspection planning. In New York ASME, editor, *Proceeding of the 11th International Conference on Offshore Mechanics and Artic Engineering*, volume 3, Juin 1992.

[8] Madsen H.O., Skjong R.K, Tallin A.G., Kirkemo F. Probabilistic fatigue crack growth analysis of offshore structures, with reliability updating througn inspection. In *Marine Strcucture Reliability Symposium*, 601 pavonia avenue, suite 400, Jersey City, New Jersey 07306, Octobre 1987.

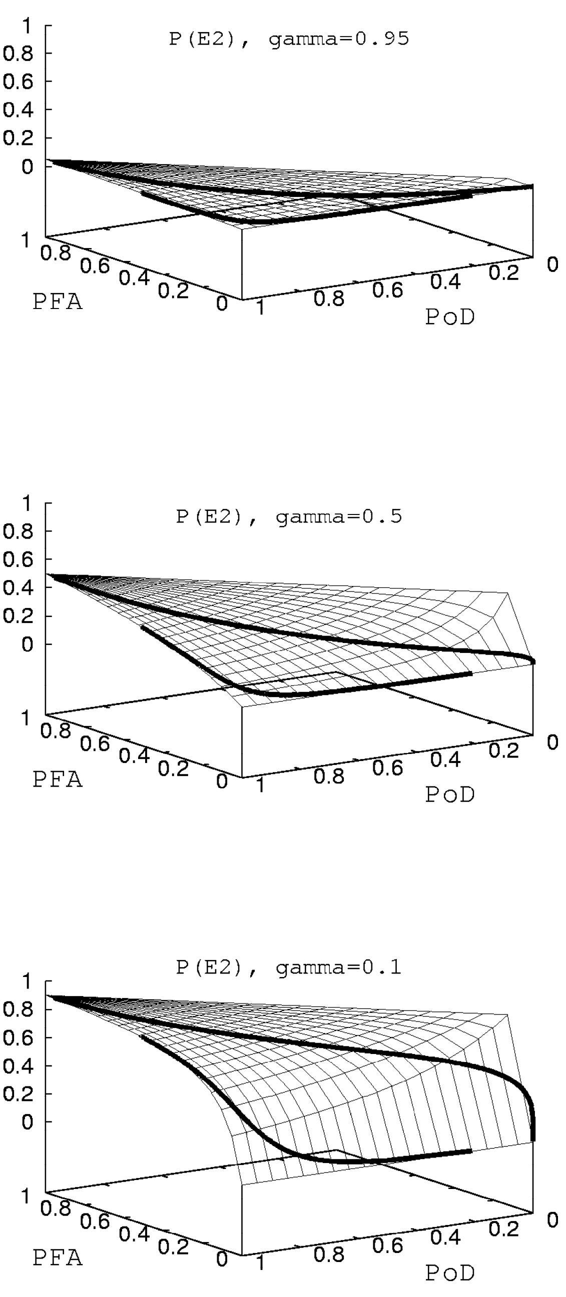

Figure 3: $P(E_2)$ *for* $\gamma = 0.95, 0.5, 0.1$.

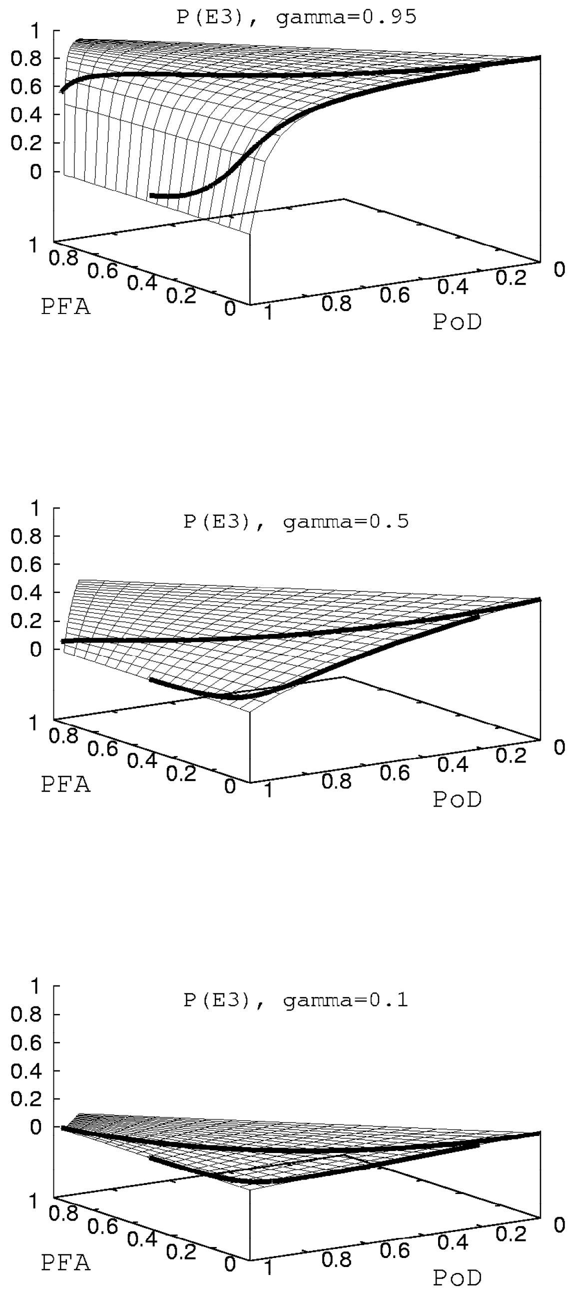

Figure 4: $P(E_3)$ *for* $\gamma = 0.95, 0.5, 0.1$

[9] Rouhan A. étude sur les probabilités de détection des contrôles non destructifs. Technical report, IFREMER, Novembre 2000. Rapport d'étude de contrat universitaire no 002021316 TMSI/RED/MS. En Cours.

[10] Rouhan A., Schoefs F., Labeyrie J. Approche probabiliste du contrôle non destructif des structures pétrolières pour l'évaluation de leur intégrité structurelle. In *INSTRUC 4, Prévision et estimation de la durée de vie des structures mécaniques*, volume 1, 11 1999.

Codified Risk Based Inspection Planning

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Keywords: Fatigue, reliability, inspection planning, codes, stochastic models

Summary

Simplified methods are described for reliability- and risk-based inspection planning of steel structures. The methods simplify the practical aspects of identifying inspection plans complying both with specific requirements to the maximum acceptable annual probability of structural collapse and member failure and at the same time minimizing overall total service life costs. Generic inspections plans can then be established for representative fatigue sensitive details in terms of characteristics such as FDF (Fatigue Design Factor), detail type, RSR (Reserve Strength Ratio) given fatigue failure and inspection / repair techniques. Generic inspection plans can be used for design of new structures as well as for requalification of existing structures. In this paper it is described how generic inspection plans can be used codification purposes in connection with inspection planning of steel structures.

1. Introduction

Reliability based inspection planning for steel offshore structures has been a topic of considerable interest during the last 10-15 years see e.g. Skjong [1], Thoft-Christensen & Sørensen [2], Fujita et al. [3] and Sørensen et al. [4]. The inspection planning problem was treated within the framework of Bayesian decision theory with the aim to minimize the service life economical risk, i.e. the service life expected costs. The basic idea is to plan inspection and maintenance activities such that legislative requirements to the safety of personnel and environment for the considered structure are fulfilled and at the same time such that the overall costs of inspections, repairs and failures are minimized. The general theoretical formulation of the decision problem is based on the framework of the Bayesian pre-posterior decision analysis see e.g. Raiffa & Schlaifer [5] and Benjamin & Cornell [6].

To solve the inspection planning problem in its most general formulation is very difficult, because it generally requires the calculation of probabilities of a large number of intersection events. FORM/SORM methods, see e.g. Madsen et al. [7] can be applied if the number of intersections does not become too large. To overcome this problem in practical applications certain simplifications to the general formulation of the inspection-planning problem have often been pursued aiming to reduce the complexity of the intersections to be evaluated. Examples hereof are the adaptive approach described in Fujita et al. [3] and inspection planning based on the assumption of "no-finds" described in Pedersen et al. [8].

In Bloch et al. [9] and Faber et al. [10] simplified and practically applicable approaches for risk based inspection planning of offshore structures are described. In this paper it is described how this methodology can be used in general for steel structures such as bridges. Each fatigue sensitive detail is categorized according to its detail type, fatigue design factor (FDF) and consequence of fatigue failure. Based on this generic description of fatigue sensitive structural details it is shown how pre-fabricated inspection plans can be established taking into account the relative cost of inspections, repairs and failures. It is shown how this format of the generic inspection plans can be applied in code making for the design and maintenance of steel structures.

2. General formulation of the optimization problem

When inspection planning for engineering systems is considered it is important to take all uncertainties into consideration, as they will strongly influence the performance of the systems. It is also important to realize that the degree of control of the engineering systems achieved by the inspections is strongly influenced by the reliability of the inspections, i.e. their ability to detect and size degradation. The reliability of inspections may be subject to large uncertainties and this must be taken into account in the planning of inspections.

The decision problem of identifying the cost optimal inspection plan may be solved within the framework of pre-posterior analysis from the classical decision theory see e.g. Raiffa & Schlaifer [5] and Benjamin & Cornell [6].

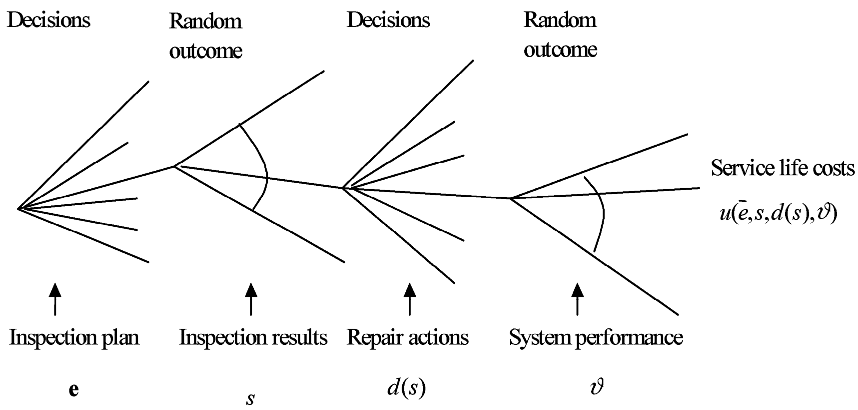

Figure 1. Inspection planning decision tree.

The inspection decision problem may be represented as shown in figure 1. In the general case the parameters defining the inspection plan are, see e.g. Sørensen et al. [4]

- the possible repair actions i.e. the repair decision rule d
- the number of inspections N in the design lifetime T_L
- the time intervals between inspections $\mathbf{t} = (t_1, t_2, ..., t_N)$
- the inspection qualities $\mathbf{q} = (q_1, q_2, ..., q_N)$.

These inspection parameters are written as $\mathbf{e} = (N, \mathbf{t}, \mathbf{q})$. The outcome, typically a measured crack size, of an inspection is modeled by a random variable S. A decision rule d is then applied to the outcome of the inspection to decide whether or not repair should be performed. The different uncertain parameters (stochastic variables) modeling the state of nature such as load variables and material characteristics are collected in a vector $\mathbf{X} = (X_1, X_2, ..., X_n)$.

If the total expected costs are divided into inspection, repair, strengthening and failure costs and a constraint related to a maximum yearly (or accumulated) failure probability $\Delta P_F^{\max}$ is added then the optimization problem can be written

$$\begin{aligned} \min_{\mathbf{e},d} \quad & C_T(\mathbf{e},d) = C_{IN}(\mathbf{e},d) + C_{REP}(\mathbf{e},d) + C_F(\mathbf{e},d) \\ \text{s.t.} \quad & \Delta P_{F,t} \leq \Delta P_F^{\max} \qquad t = 1,2,...,T_L \end{aligned} \tag{1}$$

$C_T(\mathbf{e},d)$ is the total expected cost in the design lifetime T_L. C_{IN} is the expected inspection cost, C_{REP} is the expected cost of repair and C_F is the expected failure cost. $\Delta P_{F,t}$ is the annual probability of failure in year t.The N inspections are assumed performed at times $0 \leq T_1 \leq T_2 \leq ... \leq T_N \leq T_L$.

If repair is assumed to be performed only when a crack is detected and has been measured to have a crack size a larger than a critical level a_r then the total number of repair realizations (branches) is 3^N. It is noted that generally the total number of branches can be different from 3^N if the possibility of individual inspection times for each branch is taken into account.

The total capitalized expected inspection costs are

$$C_{IN}(\mathbf{e},d) = \sum_{i=1}^{N} C_{IN,i}(\mathbf{q})\left(1 - P_F(T_i)\right)\frac{1}{(1+r)^{T_i}} \tag{2}$$

The ith term represents the capitalized inspection costs at the ith inspection when failure has not occurred earlier. $C_{IN,i}(q_i)$ is the inspection cost of the ith inspection, $P_F(T_i)$ is the probability of failure in the time interval $[0,T_i]$ and r is the real rate of interest. The total capitalized expected repair costs are

$$C_{REP}(\mathbf{e},d) = \sum_{i=1}^{N} C_{R,i}P_{R_i}\frac{1}{(1+r)^{T_i}} \tag{3}$$

$C_{R,i}$ is the cost of a repair at the ith inspection and P_{R_i} is the probability of performing a repair after the ith inspection when failure has not occurred earlier.

The total capitalized expected costs due to failure are estimated from

$$C_F(\mathbf{e},d) = \sum_{t=1}^{T_L} C_F(t)\Delta P_{F,t}P_{COL|FAT_j}(RSR)\frac{1}{(1+r)^t} \tag{4}$$

where $C_F(t)$ is the cost of failure at the time t and $P_{COL|FAT_j}(RSR)$ is the conditional probability of collapse of the structure given fatigue failure of the considered component j.

For given

- Type of fatigue sensitive detail
- Fatigue strength measured by FDF (Fatigue Design Factor)
- Importance for total structure of the considered element, e.g. measured by RSR (Reserve Strength Ratio)
- Inspection costs
- Repair costs
- Failure costs

the optimal inspection plan $(\mathbf{e},d)$ can be determined by solving (1). This inspection plan is generic in the sense that it is representative for the specified assumptions. Even though it is possible to formulate the problem in general terms theoretically, problems arise in practice when the intervals between inspections are determined. One of the problems is that the solution of (1) involves an integer optimization problem, i.e. the number of inspections superimposed on an optimization problem with continuos variables such as the inspection qualities, the repair decision rule and the intervals between inspections.

For this reason the optimization problem is often approached in a sequential manner by performing the optimization regarding the number of inspections outside the optimization of inspection intervals, repair decision rules and inspection qualities. The optimal number of inspections is then identified simply by varying the number of inspections from 0 and upwards until a minimum total cost has been identified. Even this optimization problem is numerically difficult to solve and therefore simplifications are required for practical applications. The following two simplifications are described in Bloch et al. [9] and Faber et al. [10]:

- Assuming equidistant inspection times
- Assuming constant annual failure probability at the times of inspections

Assuming equidistant inspection times reduces the inspection-planning problem to a problem of determining the optimal number of inspections, the repair decision rule and the inspection qualities. Due to the constant inspection intervals the identified inspection plan will be sub-optimal but the approach may, however, still provide a strong tool in inspection planning.

Another approach is to apply a model where the acceptable annual failure probability for joint fatigue failure is constant in time just as in the case where the acceptance criteria is given by the codes of practice. The cost optimal acceptance criteria for the annual failure probability may then be determined by cost minimization. Inspection plans derived on this basis will also be sub-optimal. A number of different maximum acceptable annual probabilities of failure $\Delta P_{F,j}^{\max}$ is selected.

Assuming no-finds the inspection times are determined such that $\Delta P_{F,t,j} \leq \Delta P_{F,j}^{\max}$ for all t the corresponding total expected costs are calculated using Equations (2) - (4). It is important to note that the probabilities entering the cost evaluation are not conditioned on the assumed no-finds at the inspection times. This in order to include all possible contributions to the failure and repair costs.

The threshold is selected as the one which results in the lowest total expected costs and at the same time satisfy a requirement to maximum acceptable annual probability of fatigue failure. The inspection plan corresponding to this threshold is the optimal plan.

The inspections will in general be performed with varying time intervals and at varying levels of the annual probability of failure. The approximation can be expected to be reasonable for components with a high reliability.

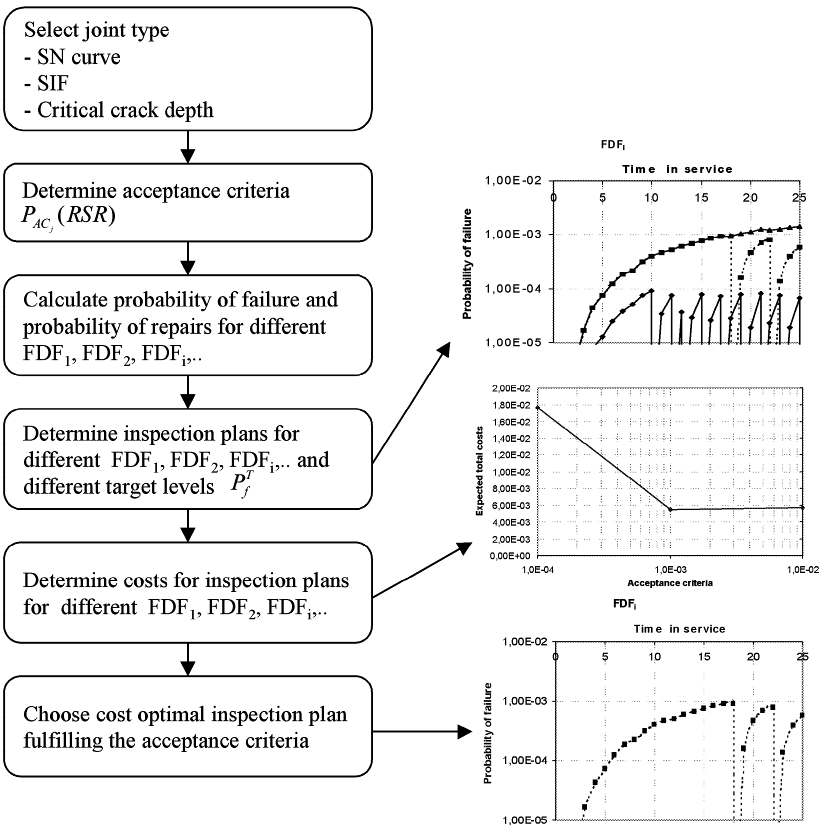

Figure 2. Scheme for generic inspection (from Faber et al. [10]).

In generic inspection planning pre-fabricated inspection plans are established for different structural details for a range of FDF's, weld qualities, RSR's given failure, repair strategies and relative costs of failure, repair and inspection, see Faber et al. [10]. The inspection plans may be established for a considered detail already at the design stage as soon as the detail type is known together with the design FDF value and the RSR for the structure given fatigue failure of the considered detail. The suggested approach for generic inspection planning is illustrated in Figure 2.

As the generic inspection plans are given for different values of the FDF it is possible to assess the effect of design changes in the design phase as well as the effect of strengthening in existing structures as such actions directly effect the FDF. Furthermore it is also interesting to observe that the effect of service life extensions on the required inspection efforts may be directly assessed through the corresponding change on the FDF. Given the required service life extension the FDF for the structural detail is recalculated and the corresponding pre-fabricated inspection plan identified.

3. Example

Variable	Unit	Distribution	Expected value	Standard deviation
Δ		LogNormal	1.0	0.30
$\log(K)$		Normal	12.4	0.2
m		Deterministic	3	
k		LogNormal	μ_k	$0.1\,\mu_k$ / $0.3\,\mu_k$
$1/\lambda$		Deterministic	2	
ν	/ year	D	$1.4 \cdot 10^6$	

Table 1. Uncertainty modeling for SN based reliability analysis.

The following example illustrates how generic inspection plans may be established for welded longitudinal stiffeners with a service life of 50 years. In deterministic, code based fatigue analyses the SN approach is used. The relation between the number of cycles to failure, N, and the stress range Δs is given by $N = K\Delta s^{-m}$ where K and m are constants depending on the material and the geometry of the considered joint. On the basis of a very large number of test results given in Sedlacek et al. [11] the statistical data for K and m in table 1 are used. These data corresponds to an equivalent fatigue stress equal to 80 MPa for $2 \cdot 10^6$ cycles and the characteristic SN curve obtained as the mean value minus 2 standard deviations.

FDF	μ_k [MPa]
1	2.707
3	1.877
5	1.583
10	1.257

Table 2. FDF and μ_k.

The fatigue load on steel bridges has been considered in Waarts & Vrouwenvelder [12] on the basis of statistical models of traffic loading. The stress ranges Δs are assumed to have a long term Weibull distribution with shape parameter λ and scale k. k is assumed to be modeled by a stochastic variable modeling the uncertainty related to the magnitude of the stress ranges. Two coefficients of variation are considered, namely 0.1 and 0.3, representing well-defined fatigue load and uncertain fatigue load. The expected values of k are calibrated such that FDF values equal to 1, 3, 5 and 10 are obtained, see table 2.

The damage is defined as the ratio between the actual number of cycles at the stress range Δs_i and the number of stress ranges, which can be sustained at the stress range Δs_i. The limit state function for the SN based reliability analysis is written

$$g(\mathbf{x}) = \Delta - D = \Delta - \nu t \frac{E\left[\Delta s^m\right]}{K} = \Delta - \nu t \frac{k^m \Gamma\left(1+\frac{m}{\lambda}\right)}{K} \tag{5}$$

where Δ is the damage limit corresponding to failure, D is the damage, ν is the intensity of stress cycles, $\Gamma(.)$ is the Gamma function, t is time, $E[\Delta s^m]$ is the mth order moment of the long term stress distribution. Table 1 summarizes the input parameters to the reliability analysis are given.

In order to perform the reliability-based inspection planning a fracture mechanics analysis is needed. The following representative limit state function based on a 1-dimensional crack growth model is used

$$g(x) = N - \nu t = \left(N_I + \int_{a_0}^{a_c} \frac{1}{CE\left[\Delta s^m\right](\pi a)^{\frac{m}{2}} Y^m} da \right) - \nu t \tag{6}$$

where N the number of cycles to failure, a is the crack depth, a_0 the initial crack depth, a_c the critical crack depth and where C and m are material parameters. The geometry function in this example is taken as Y, i.e. independent on the crack length a.

Variable	Unit	Distribution	Expected value	Standard deviation
a_0	mm	Deterministic	0.1 mm	
$\ln C$ $\sigma_k = 0.1\,\mu_k$ $\sigma_k = 0.3\,\mu_k$		Normal	-25.24* -24.49*	0.77 0.77
m		Deterministic	3	
a_c	mm	Deterministic	20.0 mm	
N_0 $\sigma_k = 0.1\,\mu_k$ $\sigma_k = 0.3\,\mu_k$	Cycles	Weibull	$200 \cdot 10^3$ * $300 \cdot 10^3$ *	$0.35 \cdot 200 \cdot 10^3$ * $0.35 \cdot 300 \cdot 10^3$ *
Y		LogNormal	1	0.1
ε	mm	Normal	0	0.1
POD – MPI	mm	see (9)	b=1 mm^{-1}	P_0=0.95
POD – visual	mm	see (9)	b=0.167 mm^{-1}	P_0=1

Table 3. Uncertainty modeling in FM based reliability analysis. *: calibrated values.

The number of cycles from construction to crack propagation is modeled by

$$N_I = N_0 (150\text{MPa})^m / E[S^m] \tag{7}$$

where N_0 is the number of stress cycles to crack propagation for $\Delta\sigma$=150 MPa, see Lassen (1997). The stochastic variables are given in Table 3. $\ln C$ and N_0 are correlated with correlation coefficient $\rho_{\ln(C),N_0} = -0.50$, see Lassen [13]. The values marked with * are determined such that the reliability index as function of time based on the fracture mechanics approach fit as close as possible the reliability index based on the SN approach. It is assumed that inspections are performed using close visual inspections or the MPI technique with the POD curve represented by:

$$POD(a) = P_0\left(1 - \exp(-a\ b)\right) \tag{8}$$

where the parameters and the probabilistic model the measurement uncertainty ε are given in Table 3. P_0 represents the probability of detecting even very large cracks.

In figure 3 the reliability index (based on the annual probability of failure) as function of time are shown for SN-approach and calibrated fracture mechanics model for FDF=3 and COV[k]=0.3. It is seen that a reasonable agreement is obtained.

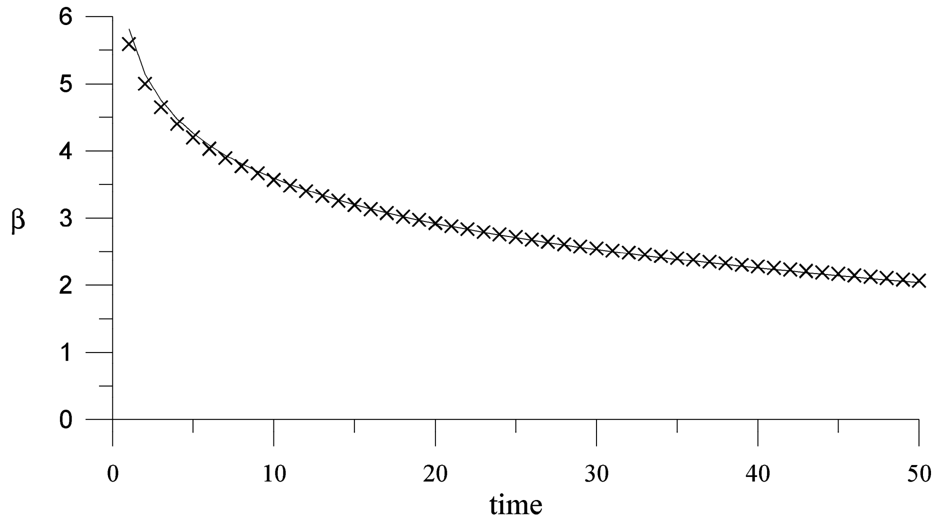

Figure 3. Reliability index as function of time in years for SN-approach (full line) and calibrated fracture mechanics model (broken line) for FDF=3 and COV[k]=0.3.

The total expected cost during the lifetime of the structure is determined on the basis of the cost of failure, $C_F(t) = C_F$, the cost or inspections, $C_{IN,i} = C_{INSP}$, the cost of repair, $C_{R,i} = C_{REP}$ and the real rate of interest, r. Basically it is assumed that

$(C_F,\ C_{INSP},\ C_{REP},\ r) = (1.0, 0.00002, 0.02, 0.06)$ for MPI inspections, and

$(C_F,\ C_{INSP},\ C_{REP},\ r) = (1.0, 0.000005, 0.02, 0.06)$ for close visual inspection

It is assumed that detected defects are repaired by welding if they exceed 1.0 mm in depth. Smaller cracks are assumed to be repaired by inexpensive grinding.

Optimal inspection plans can now be determined using equidistant inspection times. The assessment of the failure and the repair probabilities are conveniently estimated using Monte Carlo simulation, see Faber and Sørensen [14]. Monte Carlo simulation is used since it is easy to simulate the different inspection/repair scenarios and fast computers make the computation times reasonable.

In tables 4-7 optimal inspection times and related total expected costs are shown using equidistant inspection times for $\Delta P_{F,j}^{\max} = 10^{-3}$ for combinations of inspection method (close visual and MPI) and uncertainty of fatigue stress ranges (COV[k]=0.1 and COV[k]=0.3).

It is seen that the uncertainty of the fatigue stress ranges has a significant influence on the number of inspections required and as expected, that the optimum number of inspections for close visual inspection are larger than for MPI. Tables 4-7 can e.g. be used directly in design to select the optimum value of FDF for a given fatigue sensitive structural detail and to determine the optimal inspection plan.

FDF	$(C_{INSP},\ C_{REP}) =$ (0.00002, 0.02)	$(C_{INSP},\ C_{REP}) =$ (0.000002, 0.02)	$(C_{INSP},\ C_{REP}) =$ (0.00002, 0.2)

1	12 (0.000140)	24 (0.000024)	24 (0.000195)
3	0 (0.000007)	2 (0.000004)	1 (0.000006)
5	0 (0.000000)	0 (0.000000)	0 (0.000000)
10	0 (0.000000)	0 (0.000000)	0 (0.000000)

Table 4. Optimal number of equidistant inspections and related total expected costs in ( ). MPI inspections, $\Delta P_{F,j}^{\max}=10^{-3}$ and COV[k]=0.1.

FDF	(C_{INSP}, C_{REP}) = (0.00002, 0.02)	(C_{INSP}, C_{REP}) = (0.000002, 0.02)	(C_{INSP}, C_{REP}) = (0.00002, 0.2)
1	49 (0.000475)	49 (0.000419)	49 (0.000895)
3	49 (0.000079)	49 (0.000022)	49 (0.000104)
5	24 (0.000043)	49 (0.000009)	24 (0.000056)
10	7 (0.000015)	24 (0.000004)	7 (0.000018)

Table 5. Optimal number of equidistant inspections and related total expected costs in ( ). MPI inspections, $\Delta P_{F,j}^{\max}=10^{-3}$ and COV[k]=0.3.

FDF	(C_{INSP}, C_{REP}) = (0.00002, 0.02)	(C_{INSP}, C_{REP}) = (0.000002, 0.02)	(C_{INSP}, C_{REP}) = (0.00002, 0.2)
1	49 (0.000423)	49 (0.000140)	49 (0.000566)
3	0 (0.000007)	1 (0.000007)	0 (0.000007)
5	0 (0.000000)	0 (0.000000)	0 (0.000000)
10	0 (0.000000)	0 (0.000000)	0 (0.000000)

Table 6. Optimal number of equidistant inspections and related total expected costs in ( ). Close Visual inspections, $\Delta P_{F,j}^{\max}=10^{-3}$ and COV[k]=0.1.

FDF	(C_{INSP}, C_{REP}) = (0.00002, 0.02)	(C_{INSP}, C_{REP}) = (0.000002, 0.02)	(C_{INSP}, C_{REP}) = (0.00002, 0.2)
1	49 (0.004784)	49 (0.004727)	49 (0.006409)
3	49 (0.000346)	49 (0.000289)	49 (0.000479)
5	49 (0.000123)	49 (0.000066)	49 (0.000154)
10	4 (0.000025)	49 (0.000012)	4 (0.000026)

Table 7. Optimal number of equidistant inspections and related total expected costs in ( ). Close Visual inspections, $\Delta P_{F,j}^{\max}=10^{-3}$ and COV[k]=0.3.

4. Conclusions

The general formulation of reliability and risk based optimal inspection planning for steel structures is presented. A simplified approach for risk-based inspection planning of fatigue sensitive structural details is described. Fatigue sensitive details are categorized according to their Fatigue Design Factor (FDF) and SN curve. Using a fracture mechanics model calibrated on a probabilistic basis to the appropriate SN-curve, cost-optimal inspection and repair planning can be performed. The procedure is illustrated by an example considering inspection planning of welded longitudinal stiffeners in steel road bridges.

5. References

[1] Skjong, R. Reliability based optimization of inspection strategies. In Proc. ICOSSAR 85 Vol.

III, Kobe, Japan, 1985, pp. 614-618.
[2] Thoft-Christensen P. and J.D. Sørensen. Optimal Strategies for Inspection and Repair of Structural Systems, Civil Engineering Systems, Vol. 4, June 1987, pp. 94-100.
[3] Fujita, M., Schall, G. and R. Rackwitz. Adaptive reliability based inspection strategies for structures subject to fatigue. In Proc. ICOSSAR89, San Francisco, USA, 1989, pp. 1619-1626.
[4] Sørensen, J.D., Faber, M.H., Rackwitz, R. and P. Thoft-Christensen. Modeling in optimal inspection and repair. In Proc. OMAE91, Stavanger, Norway, 1991, pp.281-288.
[5] Raiffa, H. and R. Schlaifer. Applied Statistical Decision Theory. Harward University Press, Cambridge University Press, Cambridge, Mass. 1961.
[6] Benjamin, J.R. and C.A. Cornell. Probability, statistics and decision for civil engineers, McGraw-Hill, NY, 1970.
[7] Madsen, H.O., S. Krenk and N.C. Lind. Methods of Structural Safety, Prentice-Hall, 1986.
[8] Pedersen, C., Nielsen, J.A., Riber, J.P., Madsen H.O. and S. Krenk. Reliability Based Inspection Planning for the Tyra Field. Proceedings of OMAE 1992, Calgary, Canada, 1992, pp. 255-263.
[9] Bloch, A., J.D. Sørensen and M.H. Faber. Simplified Approach to Inspection Planning. Proc. ASCE Joint Specialty Conf. on 'Probabilistic Mechanics and Structural reliability', Notre Dame, Indianapolis, PMC2000-188, July 2000.
[10] Faber, M.H., Engelund, S., Sørensen, J.D. and A. Bloch. Simplified and generic risk based inspection planning. In Proc. OMAE 2000 conf. S&R paper 6143, 2000.
[11] Sedlacek, G., J. Bild & J.B. Jo. Background documentation, Eurocode no. 3 – chapter 9, document 9.02, 1989.
[12] Waarts, P.H. & A.C. Vrouwenvelder. Traffic Loads on Bridges. TNO-report B-92-011, Delft, The Netherlands, 1992.
[13] Lassen, T. Experimental investigation and stochastic modelling of the fatigue behavior of welded steel joints. PhD thesis, Structural reliability theory, paper No. 182, Aalborg University, 1997.
[14] Faber, M.H. & Sørensen, J.D. Aspects of Inspection Planning - Quality and Quantity, Proc. ICASP8, Sidney Australia, 1999.

Bridge Inspection Planning Today and in the Future

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Keywords: Bridge maintenance, bridge management, Bayesian network, decision support, inspection, condition assessment, priority ranking.

Summary

Decision support provided by today's bridge maintenance management systems is mainly based on a deterministic approach. The paper presents a probabilistic model for decision support concerning the consequences of choosing different levels of maintenance. Thus the model aims to make a projection of bridge condition depending on e.g. the level of maintenance. The model has been developed using Bayesian network and an example is illustrating the purpose and use of the model.

1. Introduction

Bridge maintenance management system is today a powerful tool for rational bridge inspection and maintenance planning. The current systems are mainly based on a deterministic approach and consequently the many important factors subjected to great uncertainty are not modelled satisfactory. In order to improve decision making and make more cost optimal decisions on the planning of inspections and maintenance it is therefore necessary to take into account the uncertainty in tomorrows bridge maintenance management systems. Existing well known probabilistic approaches provide the tools for such tasks.

Many bridge stock owners or rather all owners of e.g. infrastructure or buildings often face the problem of optimising the benefit of the often limited resources to be used on maintenance. One of the important issues in this problem is to quantify the future effect of a chosen maintenance strategy and at the same time take into account the uncertainty.

2. Today's bridge maintenance management systems

In today's bridge maintenance management systems a very common way of quantifying the condition of a bridge or a bridge element is by using a condition rating system. The condition is then expressed in terms of e.g. a condition rating mark. The objective of a condition rating mark is to reflect:

- the nature, degree and extent of damages,
- the bridge element's ability to function,
- possible harmful implication on other elements.

During a principal or special inspection the bridge inspector assess the following sub marks for each bridge element:

- damage mark,
- function mark,
- implication mark

and the condition rating mark is then derived as a sum of the sub marks.

In the present paper the generally accepted system in Denmark forms the basis but any other system could as well be implemented. The condition rating mark ranges from 0 to 5 where 0 indicates a new structure and 5 a heavily deteriorated structure ready for renewal.

One of the important features of a bridge maintenance management system is the priority ranking. The system must establish a fair and reliable basis for ensuring budgeting of maintenance works so that these can be initiated or postponed in accordance with the actual condition or capacity of the structure - both individually as well as relatively to other structures. The priority ranking of future maintenance work is based on the condition rating marks and in some cases also on a load capacity rating mark. The latter is measured on the same scale as the condition rating mark and the objective for the load capacity mark is to appoint whether sufficient load bearing capacity is present or not.

For long term budgeting it is of importance to be able to estimate the future basis for deciding the priority of maintenance work and to take into account the effect of different maintenance strategies. This is typically an integrated part of today's bridge maintenance management systems. However the inherited uncertainty in such projection is typically not taken into account and should be accounted for in the future.

In the following a short description of the priority ranking methodology is given.

2.1 Priority Ranking Model

The priority ranking model calculates the final priority ranking points based on information on the condition and the load capacity as indicated in equation (1).

$$PR = W_{road} \cdot W_{con} \cdot P_{cr} + W_{road} \cdot W_{cap} \cdot P_{lcr} \tag{1}$$

where

W_{road} is the weight on the road (depending on e.g. traffic intensity),
W_{con} is the weight on the condition (depending on e.g. the maintenance strategy),
W_{cap} is the weight on the capacity(depending on e.g. the maintenance strategy),
P_{cr} is the condition points,
P_{lcr} is the load capacity points.

The condition point is calculated on the basis of the following:

$$P_{cr} = EW_{con} \cdot 2^{cr} \tag{2}$$

where

EW_{con} is the condition element weight (the influence from a damaged bridge element to the lifetime of the bridge),
cr is the condition rating mark (0-5).

The condition rating marks are evaluated at described above.

The load capacity point is calculated on the basis of the following:

$$P_{lcr} = EW_{cap} \cdot 2^{lcr} \tag{3}$$

where

EW_{cap} is the condition element weight (the importance of an element to the safety of the bridge),
lcr is the load capacity rating mark (0⇒OK, 3⇒restricted passage or 5⇒Not OK).

The load capacity rating includes a calculation of actual load capacity for the bridge in relation to standard loads. The analytical result is adjusted to take into account the condition of the structural elements.

The weighing parameters are introduced in order to be able to weigh different roads and different bridges/bridge element to reflect their overall importance and also it is possible to weigh condition and load capacity.

In the following a method for handling the uncertainty is presented suggesting a possible way for bridge inspection planning in the future.

3. Probabilistic Model for Projection of Bridge Condition

The probabilistic model for projection of bridge condition aims to quantify the effect of different choices of the bridge owner. These choices are related to:

- maintenance level,

- reinvestment strategy,
- traffic condition.

The model is based on the same basis as a typical bridge maintenance management system, see chapter 2 thus it is possible to use the principles and experience already integrated in today's bridge management. The condition of the bridge is quantified by the condition rating mark and the load bearing capacity is described by the load capacity rating mark. Priority ranking is based on ranking point which is determined on the basis of the condition rating mark, the load capacity mark and weighing parameters. Bayesian networks have formed the basis for the probabilistic model. The uncertainty in the model is representing the statistical uncertainty arising from handling a number of bridges or bridge elements at a time

3.1 Modelling in Bayesian Networks

Bayesian network is a rational tool for handling complex decision problems making the basis for decisions very transparent. In an easy and user-friendly way it is possible to model parameters as stochastic variables and define the possible dependencies. In the following it is not aimed to give a comprehensive introduction to Bayesian networks. In such case it is referred to, e.g. Jensen [1]. Instead it is intended to give an understanding of the necessary input, the modelling by the user and the output.

A Bayesian network is build up using nodes for each uncertain parameter and logical relations is identified using arrows. This is illustrated in Figure 1.

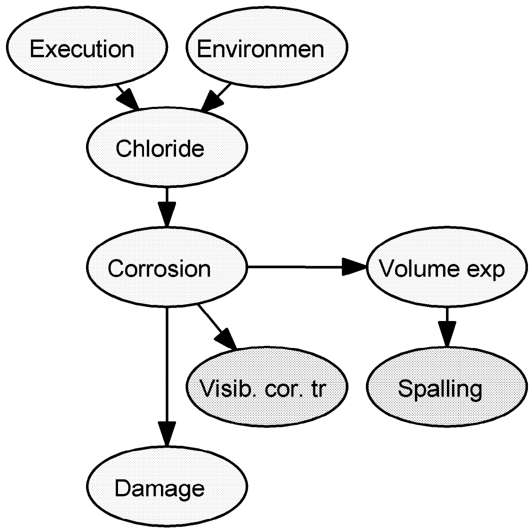

Figure 1 *Illustration of a simple Bayesian network for determination of the probability of a damage due to chloride induced corrosion.*

The necessary steps in establishing a Bayesian network is as follows. Initially a qualitative representation must be established. This involves the following:

- Identification of the necessary input in causal order (nodes)
- Logical relations between nodes
- Define states for each node

The result of a qualitative representation is presented in Figure 1 where the probability of a damage due to chloride induced corrosion is to be calculated. The probability of damage is determined from the probability of having corrosion. In this case the corrosion can only be initiated by chloride where the probability of having a critical chloride concentration at the reinforcement depends on the environment and the execution. Corrosion can lead to either a visible corrosion trace or a volume expansion leading to spalling.

A qualitative representation is followed by a quantitative representation which is established by assigning probabilities to each node. The probabilities consists of a set of conditional probability tables of the nodes.

A Bayesian network can be used to calculate the probability of different events given some observation. Using the simple example in Figure 1 it is possible to calculate the probability of damage due to corrosion given that a principal inspection has resulted in an observation of spalling or a visible corrosion trace.

3.2 Bayesian Network for Projection of Bridge Condition

The model for projection of bridge condition in Bayesian network is developed as a mock up in order to illustrate the overall principles for a projection of the condition including the uncertainty arising from considering a large number of structural elements at a time. Further it is the idea to develop the model in more detail. The model presented in the following is applicable for decision support in the top management of an organisation managing bridges. Hence it is a very general model with few entries.

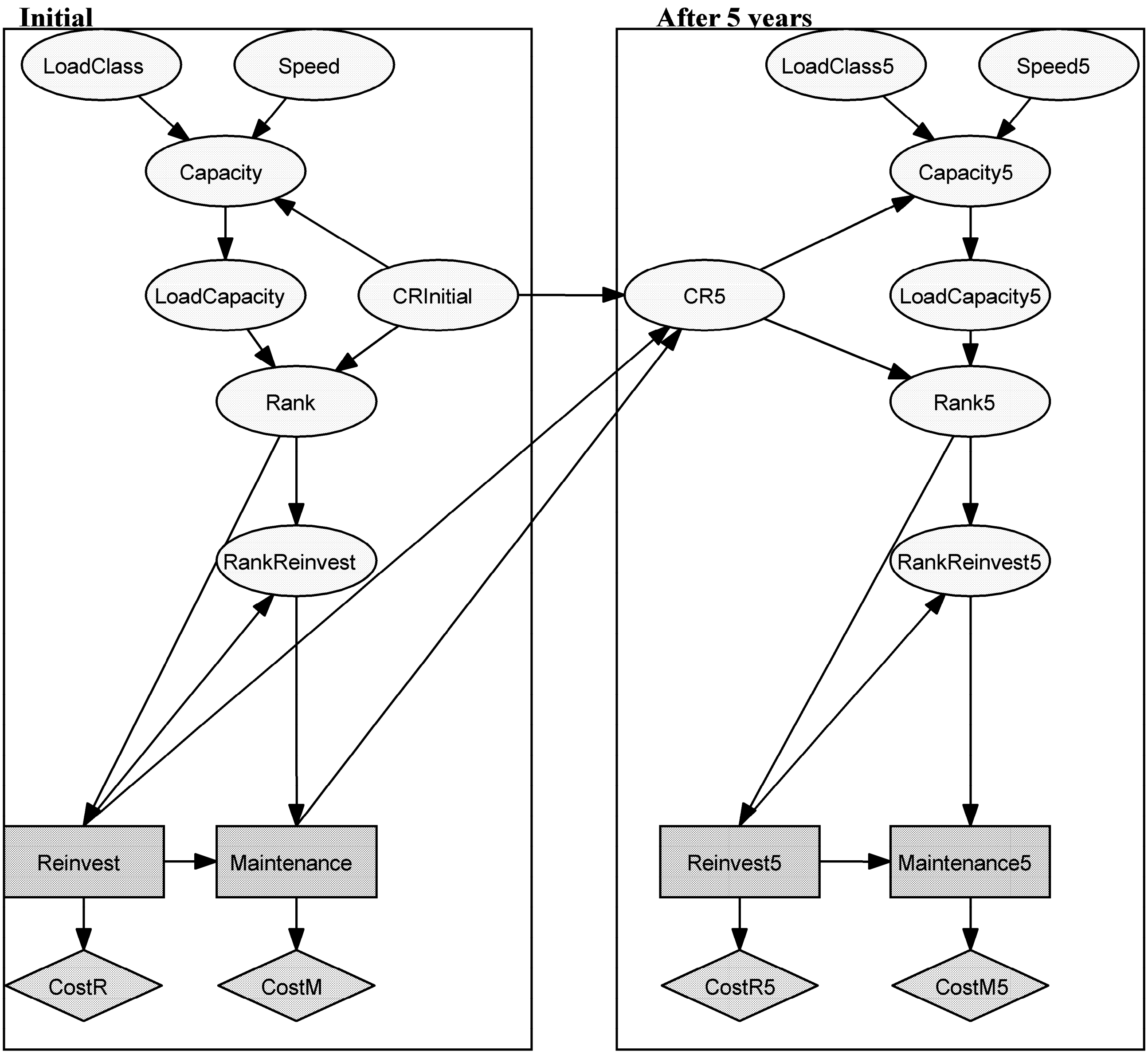

Figure 2 Bayesian network for projection of bridge condition.

In Figure 2 the Bayesian network for projection of the bridge condition is shown. The projection period is in this illustration chosen to be 5 years.

The central parameter used in the projection is the condition rating mark (*CRinitial*). The condition rating mark influences the capacity (*Capacity*) or rather the load capacity mark (*LoadCapacity*) which again is depending on the load classes (*LoadClass*) and speed (*Speed*) of the transports passing the bridge. As described in section 2.1 the load capacity mark and condition rating mark is combined and the result is the ranking points (*Rank*). The node *RankReinvest* is introduced in order to model a possible reinvestment strategy. In this case it is modelling the level of the ranking points resulting in a reinvestment. That is if the ranking point is greater than a specified level the structure is assumed to be replaced. The projection of the condition rating mark from the initial marks to the marks expected after 5 years is depending on the maintenance level and the reinvestment strategy. The network modelling, the condition and the load bearing capacity after 5 years is then established analogue to the network modelling the initial situation. Further projection could be carried out by copying the projection from initial to 5 years and then establish a projection from 5 years to for example 10 years and so on.

The best way of illustrating the outcome and the purpose of the model is to make a quantitative representation and present the results in an example.

4. Example

In the following the quantitative representation of the Bayesian network shown in Figure 2 is given and an example is illustrating the input and output of the model. However the model includes an extra time-step with a projection from 5 years to 10 years. This projection is performed analogues to the projection from the initial time to 5 years.

4.1 Input

Thirty-two railway bridges situated on the same main railway line in Denmark forms the basis for the example. The quantitative representation is based on data for this selected population of bridges and the uncertainty modelled in the network is therefore representing the fact that we are considering a group of bridges at one time. Input is established based on data from the carrying superstructure of the bridges. All bridges are an old bridge type from before 1940 with a short span width. It is therefore assumed that it is reasonable to consider the bridges as one population.

The model does in principle involve two types of decisions one on the reinvestment strategy and one on the maintenance strategy. Further it should be possible to investigate the influence from allowing different load classes and speed restrictions. In this example the reinvestment strategy is modelled by a simple decision on whether the bridge elements are replaced or not. Possible maintenance strategies are modelled by different levels of maintenance: High, normal or low corresponding to an improvement of the condition, unchanged condition or only emergency repair allowing for the element to deteriorate.

When quantifying the top input, termed the parent, basis is knowledge about the railway line (load class and speed), the load capacity and the condition of the bridges. Initial condition rating marks for the bridges is assessed by data from an existing bridge management system (Danbro) and they are based on the latest principal inspections performed. In Table 1 the probability assignment for the node *CRInitial* is given.

Condition rating mark	Probability
0	0.30
1	0.40
2	0.26
3	0.03
4	0.01
5	0.00

Table 1 Probability assignment for CRInitial.

The load capacity (*Capacity*) is determined considering the moment capacity of the bridge elements for the following cases:

- maximum allowable speed of the trains equal to 80 km/h, 120 km/h or 200 km/h
- possible load classes are D2, D3 or D4, resulting in an axle load equal to 22.5 tons and a uniformly distributed load equal to 6.4 t/m, 7.2 t/m and 8.0 t/m
- moment capacity is reduced according to the condition rating mark:
  condition mark = 0 or 1 ⇒ no reduction
  condition mark = 2 ⇒ reduction factor = 1.11
  condition mark = 3 ⇒ reduction factor = 1.33
  condition mark = 4 ⇒ reduction factor = 3.33
  condition mark = 5 ⇒ inadequate load capacity

Hence the moment capacity has been evaluated for all possible outcome of the speed, the load class and the condition rating mark. For each case the number of bridges with adequate load capacity has been identified and a conditional probability table has been established. The node *LoadCapacity* simply transforms the node *Capacity* to a load capacity rating mark; 0 if the load capacity is adequate and 5 if the load capacity is inadequate.

It is now possible to calculate the distribution of the priority ranking points (*Rank*) by the expressions given in (1)-(3). In this case the load capacity rating marks and the condition rating marks have been weighed equally (=1.0). The input for node *Rankreinvest* is modelled as an if-expression. If it is decided to replace the bridge elements the priority ranking points is assigned to 0 otherwise the *Rankreinvest* is equal to *Rank*.

A projection of the condition rating mark five years in the future will depend on the initial condition rating mark, the reinvestment strategy and the maintenance strategy. Therefore it is necessary to establish a conditional probability table. Hence the input for node *CR5* is established based on the following assumptions:

- if the bridge element is replaced the condition rating mark is always equal to 0
- maintenance level = high ⇒ 5 % of the elements increase one condition rating mark per year
- maintenance level = normal ⇒ condition rating marks are unchanged in time
- maintenance level = low ⇒ 5 % of the elements decrease one condition rating mark per year

Using these assumptions it is possible to assign probabilities to the node *CR5.* The projection from 5 years to 10 years are performed likewise.

All other probability nodes represented after 5 and 10 years are modelled equal to the initial time.

Finally only the cost nodes need to be quantified. The cost for a reinvestment is modelled as indicated in Table 2 where an interest rate of 7 % has been used.

Time for replacement	Cost [DDK]
Initial	50 mill.
After 5 years	36 mill.
After 10 years	25 mill.

Table 2Cost of replacement.

For the maintenance costs the following assumption has been applied:

Maintenance level	
High	1.5 % of the replacement cost
Normal	1.0 % of the replacement cost
Low	0.5 % of the replacement cost

Table 3Cost of maintenance.

4.2 Output

Using the input established in section 4.1 it is now possible to illustrate the output from the model. In the following different levels of maintenance have been chosen and the effect on the condition and the load bearing capacity is quantified. Hence the example is based on two scenarios:

1. No replacements are performed during the 10 year period. Maintenance is carried out at a high level, thus the condition of the bridge elements is improved.
2. No replacements are performed during the 10 year period. Maintenance is carried out at a low level, thus accepting some deterioration of the bridge elements.

In both cases the load class has been chosen to D4 and the maximum allowable speed to 120 km/h.

For the two cases the expected cost (in a 10 year period) is calculated to Case 1: 6.6 mill. DDK
Case 2: 2.2 mill. DKK

In Figure 3 and Figure 4 the effect on the load bearing capacity is illustrated.

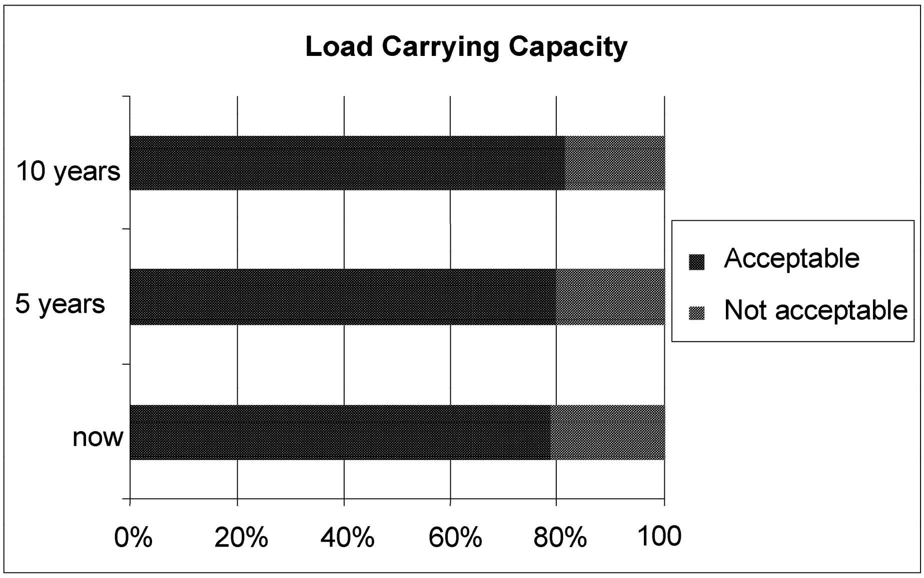

Figure 3 Load carrying capacity assuming high maintenance level.

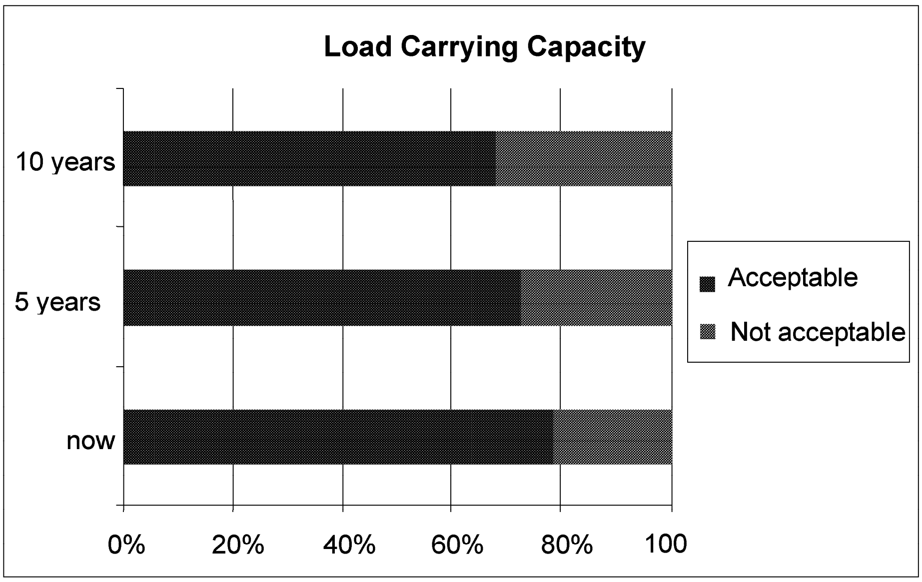

Figure 4 Load carrying capacity assuming low maintenance level

From Figure 3 it is seen that a maintenance strategy involving a high level of maintenance improves the load bearing capacity of the bridges slightly. For comparison a strategy involving a low level of maintenance is seen to result in a more pronounced tendency of reducing the number of bridges with sufficient load carrying capacity.

In Figure 5 and Figure 6 the effect on the condition is illustrated.

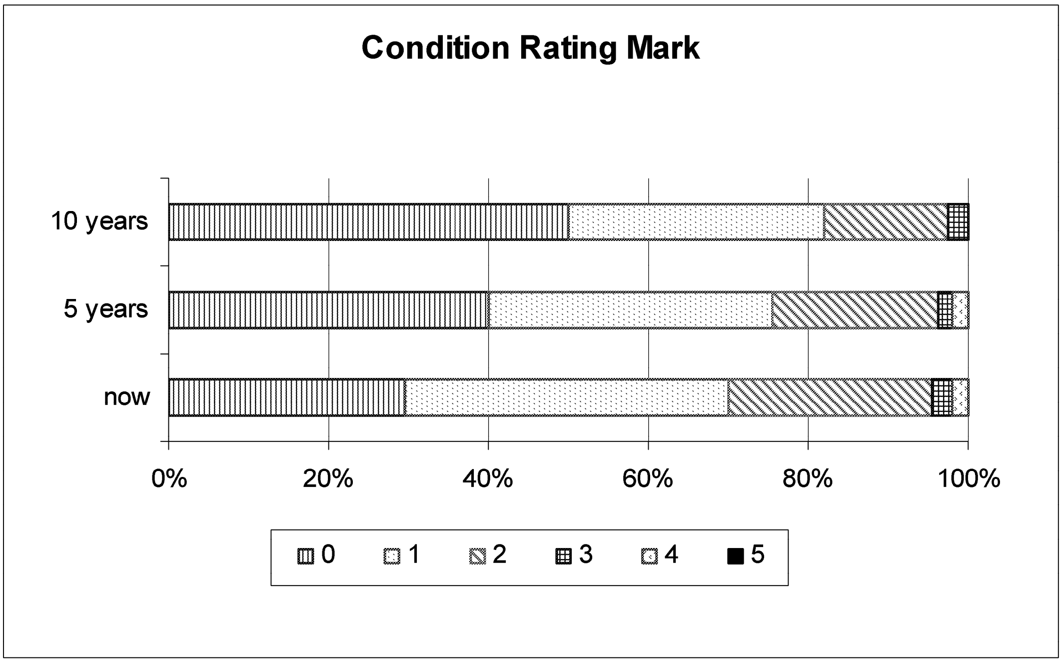

Figure 5 Condition rating mark assuming high maintenance level.

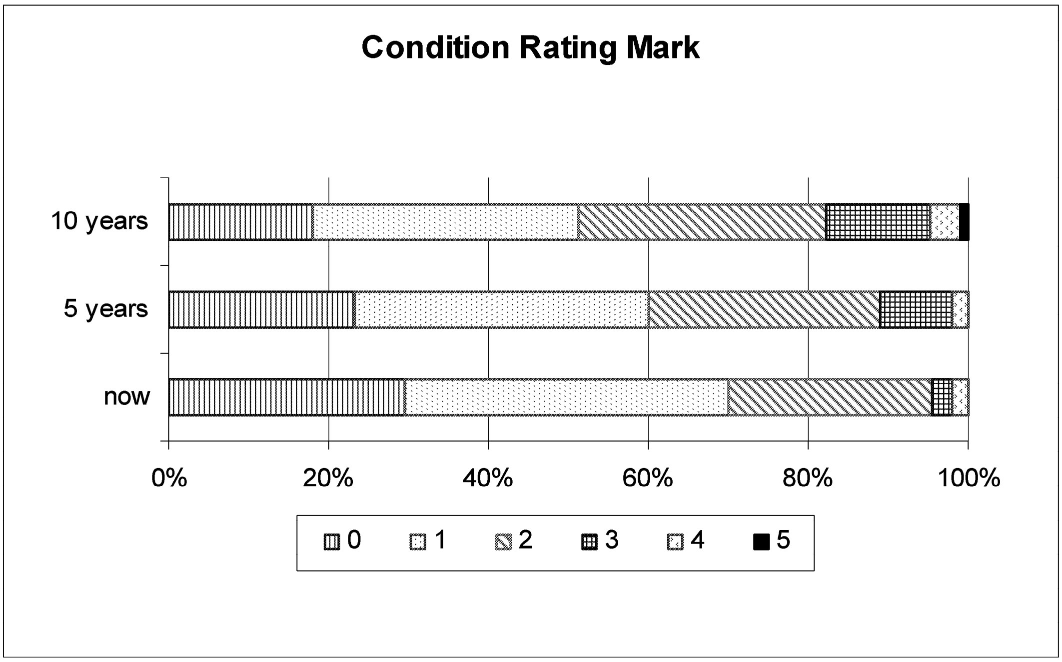

Figure 6 Condition rating mark assuming low maintenance level.

From Figure 5 and Figure 6 it is seen that there is a significant different effect of the two maintenance strategies. Using the information on the expected cost of a maintenance strategy and the derived effect on the load carrying capacity and the condition it is now possible for the bridge owner to simulate different strategies and then choose an optimal strategy.

5. Conclusion

A probabilistic model for simulating the effect of different maintenance strategies considering bridges has been presented. The model is formulated using Bayesian networks, thus making it possible to take into account the statistical uncertainty arising from handling a number of bridges or bridge elements at a time.

The presented model is based on simple relations between the input and the output. Some of the relations could be refined and adjusted by experts. For example it is possible to introduce a more realistic reinvestment strategy where the reinvestment is carried out if the ranking point exceeds a certain level.

Using the model it is possible in a quick way to evaluate the effect of different maintenance strategies, different maximum allowable speed and different load classes. The Bayesian network provides easy documentation of the decision problem which makes the establishing of the input much easy.

The present paper only introduces one of the many possibilities of introducing Bayesian network into future bridge maintenance management systems in order to take into account the uncertainties in the decision making. Another obvious application for the Bayesian network is to use the tool for identifying the most probable origins of an observed damage. Thus a future maintenance management system would have a much better basis for estimating the damage consequences.

6. References

[1] Jensen F. V., *An introduction to Bayesian networks,* UCL Press, London, 1996, 178 pp.

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